Time Series Prediction for Biomedical Measurements using Fuzzy Logic

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Abstract - In this paper is proposed an algorithm of prediction fuzzy for chaotic time series. This approach has been select because, in presence of specific pathologies, biomedical data may be represented as a chaotic time series [1]. In particular, we are interested in monitoring the intracranial pressure (IP) of some patients in a state of coma who were suffering from intracranial hypertension syndrome. In these particular cases, prediction is necessary (from a diagnostic point of view) if you want to operate at the right moment on IP abnormal conditions. The proposed approach is based on a prediction multi-factor algorithm which doesn't need the knowledge of the mathematical working model of the biologic phenomenon, translating the real time series into a fuzzy time series.

I. Introduction

Several studies have recently been carried out about intracranial pressure monitoring. They refer to both the instrumentation and the measurement techniques in data processing of biomedical data. As far as the instrumentation is concerned, a relatively recent technique consists in using an optical fibre internal probe to determine the alterations in light reflected by a pressure-sensitive diaphragm localized in the end point of the probe; it has showed its stability in laboratory sensing. Regarding the data processing we have followed a fuzzy logic approach. In this case fuzzy systems (as neural systems) are particularly indicated, as they are dynamics systems and are able to learn when trained.

The inconvenient of traditional prediction methods is in their incapability to deal with problems in which historical data are represented by "linguistic values" (see [2]). We have so proposed a new multi-factor fuzzy (time variant) series model to overcome this difficulty. First of all we notice that our database just consists in a mono-dimensional vector. To make of some sense to apply the multi-factor algorithm, at least two time-series are necessary: the principal factor, on which we make our predictions, and the secondary factor. The basic idea is to get all the necessary information from intracranial pressure data, in order to be able to forecast the requested data. We have so thought to take the results of statistical calculation on data groups as secondary factors.

II. Linguistic variables and fuzzy logic

Fuzzy measures can be introduced for two different uses: either they can represent a concept imprecisely known (although well defined) or a concept which is vaguely perceived such as in the case of a linguistic variable ([3], [4]). In the first case they represent possible values, while in the second they are better understood as a continuous truth valuation (in the interval [0, 1]). To be more precise:

- in the first case we associate a possibility distribution (an ordinal distribution of uncertainty) to classical logic formulas;
- in the second case we have a multi-valued logic where the semantics allow values in the entire interval [0, 1].

We interpret a property as fuzzy if a precise measurement of this property can be obtained in principle. Many researchers interpret the term vagueness as a sub-category of fuzziness. They consider vagueness to be distinct from fuzziness. In contrast to fuzzy terms, we call those terms vague for which no measurement process can exist. In "People feel uncomfortable when it is hot" the term "hot" is fuzzy, but "uncomfortable" is vague: we have no dependable way of measuring discomfort. Part of the research on uncertainty should be aimed at reducing vagueness by developing new measurement processes.

Now we call linguistic variables all ones "whose values are not numbers, but words or sentences in a natural or artificial languages". For example, the linguistic variable Intracranial Pressure is a linguistic variable with a Universe of Discourse, which is the range of numerical values it can assume. On a linguistic variable a term set T may be created: T= {very low, low, medium, high, very high}.

Linguistic expression are taken from natural language to emulate the human way of making a decision; a construction “IF antecedent THEN subsequent” is performed. But contrary to the classical logic (where logical predicates and rules have binary interpretation: a specific object satisfies a predicate or it does not) in fuzzy logic an object satisfies a predicate to some degree (fuzzification), and this is the concept of fuzzy number (see also [4] for more details).

III. The proposed model

The proposed model uses a prediction technique based on the fuzzification of difference estimation between successive intracranial pressure samples. It works as follows:

$$F(t) = F(t-1) \circ R \qquad (1)$$

where:

F (t-1)	fuzzified data at t-1;
F (t)	data prediction at time t;
R	fuzzy relations union;
$\circ$	Max-Min composition operator.

Long time is necessary to compute the fuzzy relations union R, but the calculation is performed once at all.

A. Time series fuzzification

Let’s assume y(t) (t=0,1,2,…) is a subset of R and let U be the Universe of Discourse. Let F(t) and G(t) (t=1,2,…) be two fuzzy time series on y(t), where F(t)=$\{$ μ11(t), μ12(t),…, μ1n(t)$\}$ and G(t)=$\{$ μ21(t), μ22(t),…, μ2n(t)$\}$ and μ1i and μ2i are two fuzzy sets on y(t), for $1 \le i \le n$.

If we want to foresee F(t) and to use G(t) to improve the prediction on F(t), then F(t) and G(t) are respectively called principal factor fuzzy time series and secondary factor fuzzy time series.

We can describe the fuzzified variation f(t) (between time t and t-1) of principal factor time series F(t) and assume that the Universe of Discourse U has been divided into m intervals (for example u1, u2,…,um). Let’s assume there are k linguistic terms (A1, A2,…, Ak), which are described by fuzzy sets Ai; the maximum membership value of Ai is verified in ui interval and $1 \le i \le k$. If x is the difference between the historical data at time t and t-1 and $x \in$ ui, the fuzzified variation f(t) is Ai, where $1 \le i \le k$, that is f(t)=[μ Ai (u1) μ Ai (u2) … μ Ai (um)] .

Analogously, the historical data of secondary factor fuzzy time series g(t) may be described assuming there are k linguistic terms (for example B1, B2,…,Bk) represented by fuzzy sets, as follows:

B1=μ B1 (u1)/ u1+μ B1 (u2)/u2+…+μ B1(um)/um
B2=μ B2 (u1)/ u1+μ B2 (u2)/u2+…+μ B2(um)/um
...
Bk=μ Bk (u1)/ u1+μ Bk (u2)/u2+…+μ Bk(um)/um

B. Data prediction analysis

To foresee data at time t we have to define the width w of an observation window [2]. Then we can define the criterion matrix C(t) and the operative matrix Ow(t), expressed as:

C(t)=f(t-1)=[C1 C2 … Cm]

$$O^{w}(t) = \begin{bmatrix} f(t-2) \\ f(t-3) \\ . \\ . \\ . \\ f(t\text{-}w) \end{bmatrix} = \begin{bmatrix} O_{11} & O_{12} & \dots & O_{1m} \\ O_{21} & O_{22} & \dots & O_{2m} \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ O_{(w-1)1} & O_{1(w-1)2} & \dots & O_{(w-1)m} \end{bmatrix} \qquad (2)$$

where f(t-1) is the fuzzified variation at time (t-1); m is the elements numbers of the Universe of Discourse; Cj and Oij :

$$0 \leq Cj \leq 1, \; 0 \leq Oij \leq 1, \qquad 1 \leq i \leq w\text{-}1, \; 1 \leq j \leq m.$$

Following [1] we define the secondary factor fuzzy vector, as S(t)=g(t-1)=[S1 S2 ... Sm], $S_i \in [0,1]$ with $1 \leq i \leq m$ and g(t-1) represents the fuzzified data of G(t) at time (t-1).

Now we have to decide the fuzzy relationship among the criterion vector, the operative matrix and the secondary factor vector. The fuzzy relationship matrix R(t) is R(t)= Ow(t) ⊗ C(t), that is:

$$R(t) = \begin{bmatrix} O_{11}\times S_1\times C_1 & O_{12}\times S_2\times C_2 & \cdots & O_{1m}\times S_m\times C_m \\ O_{21}\times S_1\times C_1 & O_{22}\times S_2\times C_2 & \cdots & O_{2m}\times S_m\times C_m \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ O_{(w-1)1}\times S_1\times C_1 & O_{(w-1)2}\times S_2\times C_2 & \cdots & O_{(w-1)m}\times S_m\times C_m \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ R_{(w-1)1} & R_{(w-1)2} & \cdots & R_{(w-1)m} \end{bmatrix} \quad (3)$$

where Rij= Oij x Sj x Cj $1 \leq i \leq w\text{-}1$, $1 \leq j \leq m$. The forecasted fuzzified variation f(t) is f(t)=[Max(R11,…,R(w-1)1)…Max(R1m,…,R(w-1)m)]

The algorithm complexity is dominated by the computation of R(t)= Ow(t)⊗S(t)⊗C(t); it's a O(wm) complexity. We propose to use more than a single secondary factor to improve the capability in data prediction.

IV. Experimental results

Several experimental tests have been performed monitoring intracranial pressure on some adult patients who were suffering from intracranial hypertension syndrome.

In a first time we have used no-filtered data. The Universe of Discourse U= [DL-D1,DR+D2] where D2 and D1 are two positive real numbers, DR and DL are respectively the maximum increase and the minimum decrease (valued between every sample and the next one). We have chosen 7 partitions on U, so to obtain 7 equi-spaced intervals: $u_{1\ldots}u_7$. Let's define these fuzzy sets (expressed linguistic terms):

A1=(very big decrease)
A2=(big decrease)
A3=(decrease)
A4=(no variation)
A5=(increase)
A6=(big increase)
A7=(very big increase)

To fuzzify these linguistic expressions, it is sufficient to translate them in terms of membership functions, according to the fuzzy sets theory:

A1=1/ u1+0.5/ u2+0/ u3+0/ u4+0/ u5+0/ u6+0/ u7
A2=0.5/ u1+1/ u2+0.5/ u3+0/ u4+0/ u5+0/ u6+0/ u7
A3=0/ u1+0.5/ u2+1/ u3+0.5/ u4+0/ u5+0/ u6+0/ u7
A4=0/ u1+0/ u2+0.5/ u3+1/ u4+0.5/ u5+0/ u6+0/ u7
A5=0/ u1+0/ u2+0/ u3+0.5/ u4+1/ u5+0.5/ u6+0/ u7
A6=0/ u1+0/ u2+0/ u3+0/ u4+0.5/ u5+1/ u6+0.5/ u7
A7=0/ u1+0/ u2+0/ u3+0/ u4+0/ u5+0.5/ u6+1/ u7.

The secondary factor time series is constituted by the standard deviation computed data groups which cover the temporal interval of two seconds (for example).

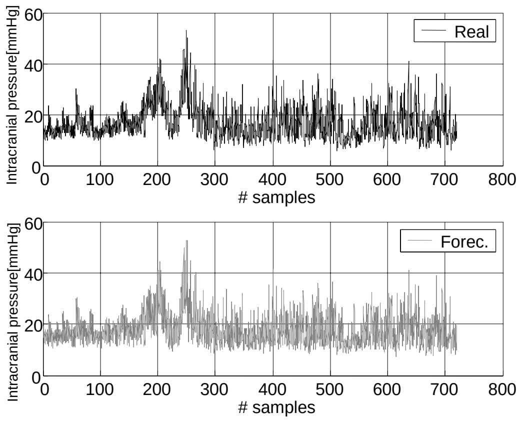

Figure 1. Historical (black) and forecasted (grey) data representation.

The secondary factor time series is constituted by the standard deviation computed data groups which cover the temporal interval of two seconds (for example).

As the secondary factor is concerned, we can define the following linguistic expressions:

B1=B7=(very big standard deviation);
B2=B6=(big standard deviation);
B3=B5=(discrete standard deviation);
B4=(no standard deviation);

bearing in mind that B_i has to imply A_i.

In the preliminary inspection, we have seen that a big increase or decrease (big variations) corresponds to a large value in standard deviation. In terms of membership functions:

B1=0/ u1+0/ u2+0/ u3+0.5/ u4+1/ u5+1/ u6+1/ u7
B2=0/ u1+0/ u2+0.5/ u3+1/ u4+1/ u5+1/ u6+1/ u7
B3=0/ u1+0.5/ u2+1/ u3+1/ u4+1/ u5+0.5/ u6+0/ u7
B4=1/ u1+0.5/ u2+0/ u3+0/ u4+0/ u5+0/ u6+0/ u7
B5=0/ u1+0.5/ u2+1/ u3+1/ u4+1/ u5+0.5/ u6+0/ u7
B6=0/ u1+0/ u2+0.5/ u3+1/ u4+1/ u5+1/ u6+1/ u7
B7=0/ u1+0/ u2+0/ u3+0.5/ u4+1/ u5+1/ u6+1/ u7

In Fig. 1 are reported the real data and the forecasted data. The real data have been acquired with a sampling time of 80 ms for a window of four hours. To evaluate the predictions goodness we take into consideration the root mean square error value. Since the cardiac and respiratory activity (the patient is under drug effect) can produce a disturbance on the Intracranial Pressure signal, a filtering step is necessary. In Fig. 2 are reported the spectra of the real and the filtered signal. After the filtering activity we repeat the analysis and respect the first result there is an improvement of the root mean square error.

It's obvious that a lonely secondary factor based on variance calculus has not a great credibility, so to obtain a further improvement of the prediction algorithm performances, it's necessary the secondary factor is a multi-parameter statistical variable, jointly defined on average, variance, and kurtosis values.

The problem is to define with precision the fuzzy rules: they become more complicated and surely less intuitive. Using three statistical parameters we obtain a further improvement in the accuracy of prediction algorithm, but we increment the complexity in constructing our model. We had to build 3 matrixes, each of which presents in the position (i, j) the membership degree of the j^{th} partition of the universe of discourse U to the i^{th} fuzzy set. In phase of construction of the matrix R, we will use the usual method of max-prod composition, as already seen (for more details about the operations on fuzzy numbers, see [5]-[8]).

This approach makes possible a further reduction of the root mean square error value.

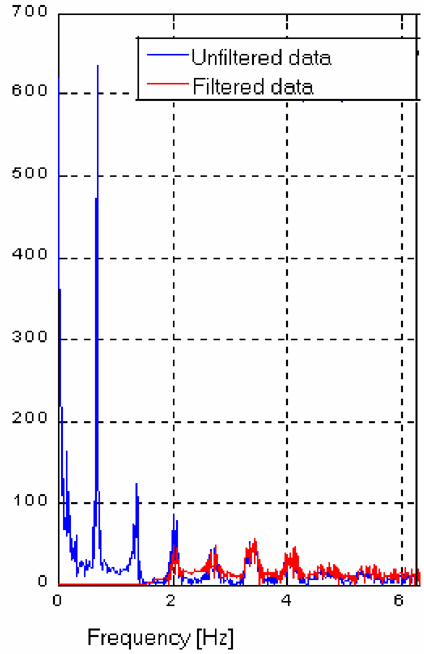

Figure 2 . Noisy (black) and filtered (grey) signal FFT.

V. Conclusions

In this paper we have proposed a new technique of prediction on sequences of biomedical measurement data. The proposed approach is based on a prediction multi-factor algorithm which doesn't need the knowledge of the mathematical working model of the biologic phenomenon. This technique is obtained translating the real time series into a fuzzy time series: it represents a faster and easier way to predict a development of biomedical phenomena with a very low computational cost.

The proposed approach is of great interest when we can find two or more secondary biological factors influencing the intracranial pressure, and so the use of statistical parameters is not necessary and the knowledge of an expert is sufficient to build a discrete numer of simple rules.

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