Compensation of Digital Voltmeters Nonlinearities by means of a Quantum Standard

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Abstract – The high linearity and stability of modern top-level Digital Voltmeters (DVM) can be exploited to measure voltage ratios with very low uncertainty by making two separate absolute voltage measurements. Yet, to obtain the very low uncertainties required to use a DVM as a voltage *ratio standard*, the intrinsic linearity of the instrument is not sufficient and techniques to determine and correct the residual non linearities must be adopted.

The Josephson Array Voltage Standard (JAVS) is well known as a voltage standard with nearly ideal accuracy. JAVS can can be also used for voltage *ratio* calibration, provided a source with high short-term stability is available. JAVS are then the best way to measure the very small linearities of a DVM, being the short-term requirement generally fulfilled by high quality voltmeters.

We performed several measurements for the evaluation of the linearity of DVMs in use in our laboratories, aimed at determining the performances of commercial multimeters as voltage ratio standards for the dissemination of the volt and for replacing ordinary techniques based on resistive dividers in primary metrology. The results appear to be encouraging for both applications: after calibration, uncertainty below 0.1 ppm can be obtained, with a stable profile of non linearity correction over time.

Keywords – Digital Voltmeters, Josephson Array, Voltage Standard, Linearity, Voltage Ratio.

I. Introduction

Modern state of the art digital voltmeters (multimeters) can perform measurements with accuracies very close to the highest levels attained by primary institutes. Since the task of a DVM is to accept an analog quantity at the input and provide its numerical value as output, the instrument performances mainly rely on the accuracy of the internal analog to digital converter. Indeed the ADC is the fundamental core of the DVM circuit, and the whole instrument can be described by a few blocks built around it.

1. The input preconditioning stage, responsible for adapting the signal to be measured to the ADC range by attenuating or amplifying it. Sometimes this stage does some preprocessing in order to reduce input-related errors like offsets and drifts [1].
2. The ADC itself, which can be further subdivided into a reference circuit and a *ratio evaluation* circuit, the latter providing the numerical output.
3. The stage for post processing of the digital signal for proper display or mathematical analysis

It is important to stress the separation between the reference and ratio part of ADC when dealing with linearity analysis, since, for this purpose the only reference specification of relevance is the short term stability. Usually, this reference error contribution can be neglected.

Being only interested in the ratio part, it is possible to exploit the most accurate feature of a DVM: its intrinsic linearity. Since the ADC-based ratios are fundamental for DVM operations like autocalibration, conversion techniques have been developed that use integrating method [2] to attain accuracy levels in the voltage ratios that can exceed the resistor-based techniques usually adopted in Metrology, like Kelvin-Varley dividers [3]. DVMs are thus interesting candidates for the realization a practical voltage ratio standard.

Several field of application in Metrology could benefit of direct measurements of a voltage ratio. In primary laboratories, a DVM calibrated with high accuracy has proven to be an effective way to maintain the resistance scale and for the calibration of a primary resistance standard against the quantum Hall resistance. Secondary laboratories, usually having one or more multimeters can use a DVM with calibrated linearity curve for the maintenance and traceability of the voltage scale and for the calibration of resistance ratios.

When aiming at such applications, the accuracy of ratio provided by DVMs – although intrinsically high

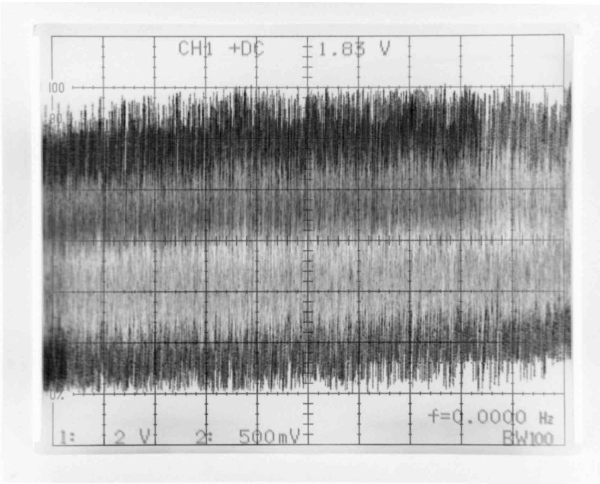

Figure 1: *I/V characteristic (vert:* 10 μA*/div, horiz:* 2 V*/div) of a JAVS, showing the quantized voltage steps generated in the* ±10 V *range. Steps are separated about* 150 μV *from each other.*

– is not sufficient, and special techniques to measure and correct the residual non linearities are needed. This is is a rather difficult task, owing to the sub-ppm level of the errors to be detected. In practice, the accuracy level provided by a quantum standard like a Josephson Array Voltage Source (JAVS) is needed.

JAVS are based on the Josephson effect, named after B.D. Josephson, who predicted in 1962 the electrical properties of a junction composed by two superconductors separated by a thin dielectric barrier. According to the theory, such a junction, irradiated by an RF field and biased by a DC current, generates a constant voltage, whose value is exactly equal to the radiation frequency, times a physical constant. Thus the voltage can be determined immediately from a frequency measurement.

However, a single junction is capable of producing just a few millivolt, a major disadvantage in high accuracy experiments. Nowadays standards are arrays of thousands of series-connected junctions giving a 10 V output; this is a value well suited to the input required for DVM calibration and verification.

II. Measurement Method

The used setup is very straightforward: the output of JAVS is directly connected to the DVM to apply known voltage steps to the meter. Provided that offsets and thermal e.m.f.s in the measurement circuit are constant, the JAVS can be treated as a voltage source with exactly known voltage increments, and DVM linearity can be evaluated simply by biasing the array to give voltages within the range, then reading the value displayed by the DVM.

If offsets and thermal e.m.f.s are not constant, but linearly varying, their effect on measurement data can still be canceled out. Otherwise a linear drift would modify the apparent DVM gain or even introduce, if test voltages are not proportional to the lapse of time, artifacts in the measured nonlinearity profile.

Although the determination of the DVM gain is not strictly required to evaluate nonlinearities, and it is possible to carry out correct measurements in presence of linear drifts by a suitable timing of test voltages, in practice it is easier to adopt a simple measurement technique to null the effect of drifts. To this aim, a first set of readings with positive increments, then, a second one, backwards, with negative increments are taken. With this procedure, changes of offset voltages over time are balanced out for every sample, provided that the instants of the increasing and decreasing samples with same test voltages are symmetrical with respect to the mean of the measurement time. Another advantage is the cancellation of the effects on the observed DVM gain, which can then be measured correctly.

In our measurements any relevant drift was assumed to be linear; samples were read with equal time interval separation to preserve linearity *vs.* sample number, as explained above.

Another fundamental assumption that must be made to justify the described procedure is the negligible effect of the fluctuations of the ADC reference voltage during the procedure. In DVM specifications terminology this short-term behaviour of the internal reference is described through the *Transfer Accuracy* parameter. However, the typical values of the Transfer Accuracy don't provide a complete justification of the assumption: indeed the best manufacturers' figures are about some parts in 10^{-7} for the ±1 V range and less than $1 \cdot 10^{-7}$ for the ±10 V range. These values turn out from a conservative estimate and can be regarded as a starting point for confirming our hypothesis. Since we aim at estimating/correcting nonlinearities to provide accuracies below $1 \cdot 10^{-7}$, reference stabilities higher than those reported in multimeter specifications are needed. The short-term stability hypothesis is then verified only *a posteriori*, from measurement results.

In DVM linearity analysis one is interested in integral linearity errors, and the whole scale for the selected measurement range is to be considered. The range is typically ±10 V [4], since this is the fundamental operating region for A/D converters of DVMs [5]. Nevertheless, there are conditions where a voltage limit exist for operation, and the lower ±1 V range, although not the most accurate, is better suited to measurement. Such constraints on applicable voltage may occur for instance while using the DVM as a ratio standard for the determination of a resistance ratio: when one of the resistors has low

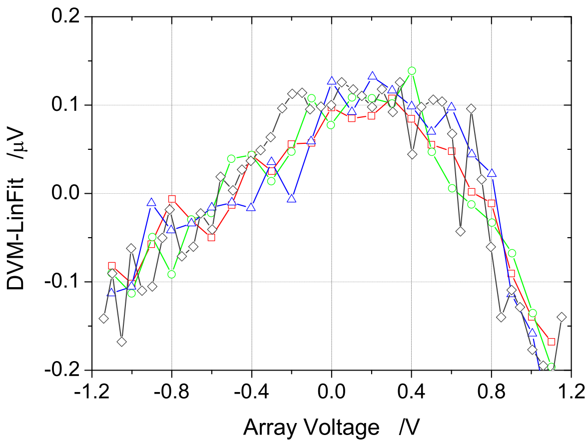

Figure 2: *Absolute deviation from least squares fit of an HP3458-Opt.002 operating in the 1 V range. To show repeatability, measurements spanning about 1 yr are reported.*

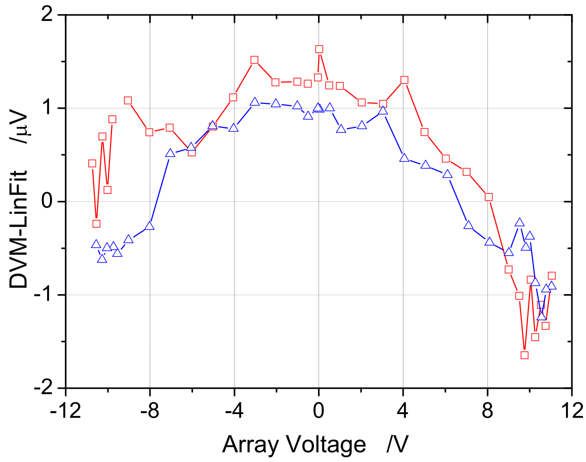

Figure 3: *Absolute deviation from least squares fit of the same multimeter as in Fig. 2, operating in the 10 V range.*

value (typically less than 100 Ω) and power should be limited to avoid self-heating; or when comparing a resistor against quantum Hall resistance, because of limitations of the test current. Both ranges have been analysed in our experiments.

III. Data Analysis

Within the measurement range, a linear fit of the DVM readings *vs.* the array voltage, is performed. The required step number determination is accomplished, assuming the DVM reading maximum error to be lower than 1/2 step (a self calibration check is required). The step number is thus the one giving a JAVS voltage closer to the reading. The knowledge of the step number provides immediately the JAVS voltage, given by the irradiation frequency times the step number times the fundamental constant $h/2e$, where h is the Plank constant and e is the electron charge. Finally the differences between the actual readings and the value given by the fit line, represent the nonlinearities.

As explained previously, after averaging the forward-backward results, the only effect of the drift is on the observed voltage offset i.e. in the constant term of the fit line. However due to changes of DVM offset with time, this is not a relevant parameter: it is assumed that it is either evaluated or canceled out when the multimeter is used as a ratio standard. As a by-product of the analysis, the voltage gain is obtained, being given by the slope of the fit line. Yet, also the multimeter gain is not required for voltage ratio measurements, although important for calibration and the evaluation of the instrument performances

Several approaches are possible to express the correction to be applied on *vs.* the voltage readings, the optimum choice being dependent on the specific measurement in which the calibrated ratio will be used. A linear fit within a short interval of voltage values may be appropriate in some circumstances when a very specific ratio is to be considered and obtained with the lowest uncertainty [6]. However in the majority of cases, the correction curve is to be evaluated over the whole instrument range and a more elaborate technique is required. We didn't find a general solution to the problem, which should be analysed considering many factors: mathematical issues, uncertainty evaluation and ease of use for the customer. The topic is still under investigation and an heuristic approach was adopted whenever the correction for nonlinearities had to be expressed as a curve.

IV. Results

Depending on the manufacturer, the model, and even the additional multimeters options, performances may vary, however the observed deviations from linearity were in general within a few parts in 10^7, in agreement with literature [7, 8]. Even when measurement are separated by several months, a good repeatability of the nonlinearity error profile was observed (Fig. 2). This property allow for correction of linearity, after deviations are calibrated. Also the gain correction obtained from the fit shows a significant repeatability, although lower (several parts in 10^{-7}). In any case, a long-term variation of gain do not affect results in ratio measurements, and can be neglected.

Quite remarkably, the profiles observed for the 1 V and 10 V are very similar, showing a high accuracy of the font-end amplifiers, with negligible effect on linearity (Fig. 3).

Besides, the very high linearity and the repeatability of results confirm the hypothesis on the stability of the ADC reference beyond the figures reported in the specifications, as it was assumed to justify the measurement method.

A JAVS-calibrated DVM has a direct application for the traceability of the 10 kΩ DC resistance standard since it can be used for the measurement of the $12,9\ \mathrm{k}\Omega/10\ \mathrm{k}\Omega$ resistance ratio for scaling down from the quantum Hall resistance value to a standard resistor with decadic value. Experiments carried out at the IEN have shown [6] an agreement of this method with conventional ones to better than $1 \cdot 10^{-7}$.

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