

# SHORT-TERM FREQUENCY STABILITY OF DIGITIZER TESTING SIGNALS

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## ABSTRACT

In this paper we will discuss methods that can be readily used for frequency (phase) stability measurements of testing signals. To illustrate the case we will describe the measurements on two function generators for ADC testing, a HP33120A and Agilent 33250A, carried out at the Standard Time and Frequency Laboratory of the Institute of Radio Engineering and Electronics (IREE), Prague.

*Keywords:* ADC testing, testing signal, short-term frequency stability.

variances) or in the frequency domain (in terms of spectral densities).

In the time domain we measure the average of relative (fractional) frequency difference

$$\bar{y}(\tau) = \frac{1}{\tau} \int_{t_0}^{t_0+\tau} y(t) dt = \frac{x(t_0 + \tau) - x(t_0)}{\tau} \quad (1)$$

where  $\tau$  is the averaging interval. The recommended statistical measure of frequency stability in the time domain is the Allan deviation

$$ADEV(\tau) = \sqrt{\frac{1}{2} \langle (\bar{y}_{i+1} - \bar{y}_i)^2 \rangle}, \quad (2)$$

## 1. INTRODUCTION

Digitizers are commonly tested by using sine wave signals which should have the signal-to-noise and distortion ratio (SINAD) by about 10 dB larger than signal-to-noise Ratio (SNR) of an ideal digitizer. It follows that for testing the A/D converters with SNR = 86 dB (corresponding to fourteen resolution bits), the value of SINAD > 96 dB is needed. It turns out that commercial systems commonly used for calibration of the ADC testing signals do not meet these requirements when applied to signals of high spectral purity ([1], [2], [3]). In this case the testing signals can be characterized by their short-term frequency (phase) stability.

## 2. OVERVIEW OF METHODS

Generally, we measure the random variations in either the phase difference,  $\varphi(t)$ , or in related magnitudes as the phase-time difference  $x(t) = \varphi(t)/2\pi\nu_0$ , where  $\nu_0$  is the frequency of reference source, or relative frequency difference  $y(t) = dx/dt$ . The variations in these quantities can be characterized either in the time domain (in terms of

where the symbol  $\langle \rangle$  denotes the average over all samples. Thus  $ADEV$  is calculated from the adjacent samples  $\bar{y}_i(\tau)$ ,  $\bar{y}_{i+1}(\tau)$  that are measured with a time-interval counter (with no dead time between the measurements).  $ADEV$  can also be calculated from the time-difference samples,  $x(t)$ , using (1).  $ADEV$  has an advantage over the standard deviation in that it is convergent for all kinds of noises that occur in frequency sources. For the white frequency noise it is equal to standard deviation.

The recommended measures in frequency domain are one-sided “power” spectral densities made out of the variations, specially  $S_\varphi(f)$  in  $\text{rad}^2/\text{Hz}$  or  $L(f) \approx \frac{1}{2} S_\varphi(f)$  in  $\text{dBc}/\text{Hz}$  where  $f$  is the Fourier frequency. The conversion can be made between the stability measures in the frequency and time domains for the noises common in frequency sources [4]

$$ADEV^2(\tau) = \int_0^\infty \left(\frac{f}{\nu_0}\right)^2 S_\varphi(f) \frac{2 \sin^4(\pi f \tau)}{(\pi f)^2} df, \quad (3)$$

The internal frequency sources in function generators from which ADC testing signals are being derived are mostly high-quality quartz oscillators that commonly operate at standard frequencies of 5 MHz or 10 MHz. Thus if these signals are accessible they can be readily measured against high-precision 5/10 MHz references by employing a variety of methods in the time domain based on time-interval counter measurements [5]. By making use of the sensitivity-enhancement methods based on the multiplication of time difference or frequency difference, the stability can be measured to a high degree of accuracy if appropriate reference is available. For instance the one-sigma uncertainty of the IREE time-domain stability-measurement system at 5/10 MHz shows  $1 \times 10^{-13}$  at  $\tau = 1$  s in terms of  $ADEV$ .

In the frequency domain the phase-detection method is commonly used, which is based on the comparison of two signals in quadrature with following low-noise amplification and FFT analysis.

The difficulty arises if evaluation of testing signals at non-standard frequencies is needed. If no low-noise frequency conversion to these frequencies is available (which is usually the case) one has to turn to “classical” methods of direct measurement with the time-interval counter.

Obviously, the factor limiting the uncertainty in these direct measurements is the noise floor of both the counter and the reference. The counter-trigger timing jitter has a character of white phase modulation giving

$$ADEV(\tau) = \sqrt{3} \frac{\sigma_x}{\tau} \quad (4)$$

where  $\sigma_x$  is the standard deviation of the time difference  $x(t)$ . The value of  $\sigma_x = \sigma_v/S$ , where  $\sigma_v$  is the standard deviation of the counter-input voltage noise and  $S$  is the slew rate (V/s) at zero crossing. The reference-signal noise for  $\tau > 1$  s is white frequency if a cesium-beam standard is used ( $ADEV(\tau) \sim \tau^{-1/2}$ ). If making use of a high-precision quartz oscillator as reference, flicker frequency ( $ADEV(\tau) = \text{const}$ ), random-walk frequency ( $ADEV(\tau) \sim \tau^{1/2}$ ), and aging effects ( $ADEV(\tau) \sim \tau$ ) are gradually manifested as  $\tau$  increases. Thus, qualitatively, the counter instability prevails at smaller  $\tau$  while the reference instability at larger  $\tau$ .

### 3. DIRECT FREQUENCY MEASUREMENT

The block diagram of the direct frequency method used in the measurement of the testing signals is shown in Fig.1.

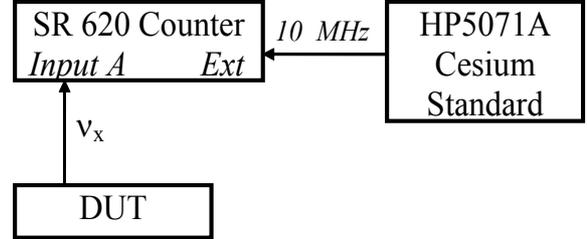


Fig.1. Direct frequency measurement.

Each time we measured  $m$  samples of  $\bar{y}_i(\tau)$ , at  $\tau = 0.1, 0.2, 0.5, \dots$  s, out of which  $ADEV(\tau)$  has been estimated. The impact of the dead-time has been neglected, given the measurement response of the counter of about 10 ms. The results with 68% uncertainties are shown in Fig.2. Apparently the contribution of the measurement-system background noise can be neglected.

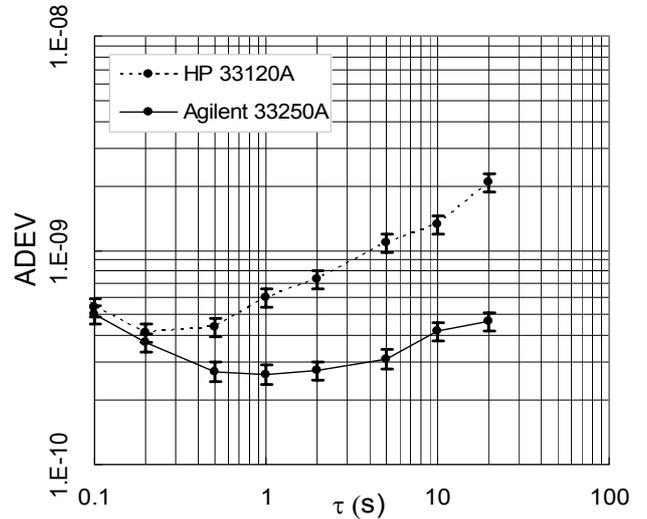


Fig.2.  $ADEV(\tau)$  plots obtained from direct frequency measurement.

A high-resolution reciprocal Stanford Research SR620 counter was used with a 5071A high-performance cesium-beam standard as external reference. The counter was set to the “Frequency”

mode. Prior to the measurement the SR620 counter had been tested in the “Frequency” mode with a 5 MHz / +7 dBm sine-wave signal (i.e. with  $S = 22$  mV/ns at 50  $\Omega$ ) from an ultra-stable BVA oscillator. It has been found that the counter shows  $ADEV(\tau) = 2 \times 10^{-11} / \tau$  which corresponds to  $\sigma_x = 12$  ps. From earlier measurements the cesium standard shows  $ADEV = 4 \times 10^{-12}$  at  $\tau = 1$  s and  $ADEV(\tau) = 8 \times 10^{-12} \tau^{-1/2}$  for  $\tau > 10$  s.

#### 4. DIRECT TIME-INTERVAL MEASUREMENT

The block diagram of the direct time-interval measurement is depicted in Fig.3.

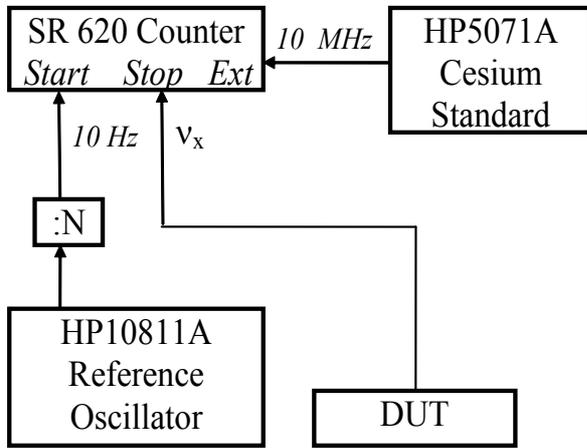


Fig.3. Direct time-interval measurement.

In the setup in Fig. 3 the counter was operating in the “Time” mode. The reference frequency,  $\nu_0 = 5$  MHz, was firstly divided to 10 Hz in a low-noise divider to define the basic sampling interval,  $\tau_0 = 100$  ms, and then applied to the START input. This allowed to measure the samples of the time differences  $x(t)$ . The resulting  $ADEV(\tau)$  plots with the 68% uncertainties are shown in Fig. 4.

Compared with the previous method, the counter noise contribution is by  $\sqrt{2}$  larger because of the trigger-noise from two uncorrelated inputs. On the other hand, the knowledge of  $x(t)$  gives advantageously more information about the noise process than merely the knowledge of its derivative,  $y(t)$ . Also, because of the continuous measurement there is no dead time involved.

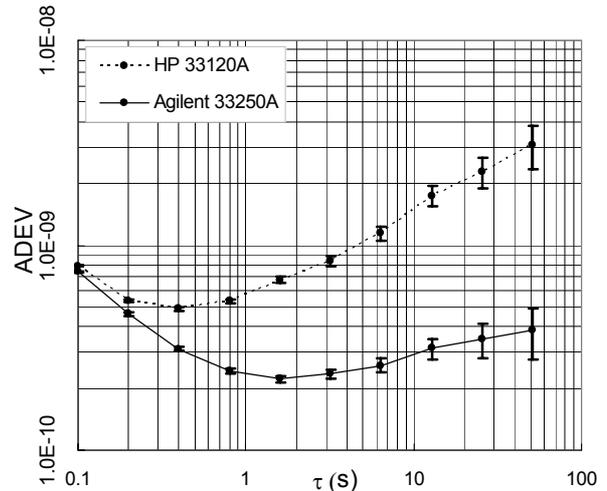


Fig.4.  $ADEV(\tau)$  plots obtained from direct time-interval measurement.

#### 5. PHASE-DETECTION MEASUREMENT

The phase-detection method allows to characterize the phase variations (phase noise) in the frequency domain. It is done in terms of power spectral densities  $S_\phi(f)$  in  $\text{rad}^2/\text{Hz}$  or  $L(f) \approx \frac{1}{2} S_\phi(f)$  in dBc/Hz. The block diagram of the system used in our measurement is shown in Fig. 5.

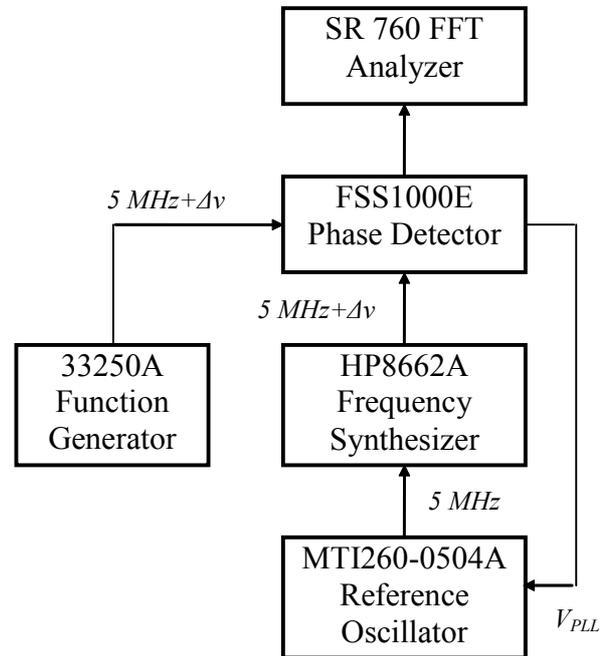


Fig.5. Phase-detection measurement.

The setup is based on a Femtosecond Systems FSS1000E Phase Detector whose principal element is a low-noise double-balanced mixer followed with a low-pass filter. The output voltage,  $V(t) = K_d \varphi(t)$ , where  $K_d$  is the conversion factor, is proportional to the phase difference between the measured and reference sine-wave signals. The FSS1000E also contains a 60-dB DC amplifier used to achieve an appropriate signal level for the FFT analysis. In addition, the FSS1000E provides a control voltage that allows to maintain the compared signals in quadrature (i.e. shifted by  $\pi/2$ ) through a phase-locked loop (PLL). The PLL control voltage was applied to a 5-MHz Milliren MTI260-0504A quartz oscillator to which an auxiliary HP8662A synthesizer was referenced. The use of the synthesizer was needed to compensate for a large frequency offset of the function generator which was about  $3 \times 10^{-7}$  i.e. too large to ensure a phase-lock with the Milliren oscillator. The plot of  $L(f)$  for the Agilent 33250A function generator appears in Fig. 6.

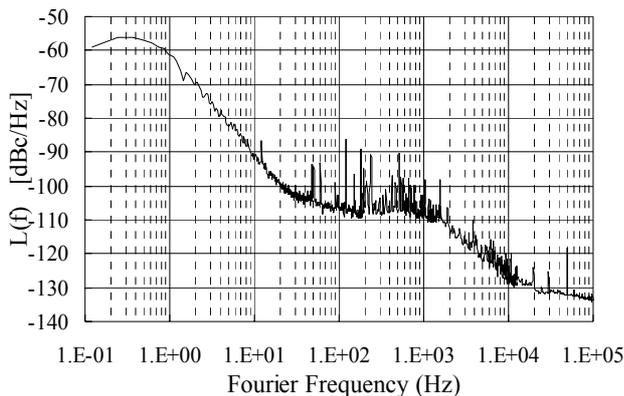


Fig.6.  $L(f)$  plot obtained from phase-detection measurement.

The Femtosecond Systems specifies the noise floor of the measurement system at 5 MHz as  $L(f) = -145 \text{ dBc/Hz}$  at  $f = 1 \text{ Hz}$ , and  $L(f) = -177 \text{ dBc/Hz}$  at  $f = 100 \text{ kHz}$ , with  $\pm 3 \text{ dBc/Hz}$  (95% uncertainty). A SR760 Analyzer was employed to perform the FFT analysis.

It can be seen in Fig. 6 that the prevailing noise around the Fourier frequency of 1 Hz behaves as  $S_{\varphi}(f) \sim 1/f^3$  which characterizes the flicker-frequency modulation. The unwanted departure from the flicker-frequency slope  $1/f^3$  at the Fourier frequencies  $f < 1 \text{ Hz}$  is caused by PLL since the

loop bandwidth had to be set somewhat larger than optimum in order to ensure the phase lock.

## CONCLUSIONS

The measured value of  $L(f=1 \text{ Hz}) = -62 \text{ dBc/Hz}$  for the 33250A unit in the frequency domain (see Fig. 6) corresponds, by making theoretical conversion based on (3), to  $ADEV(\tau=1 \text{ s}) = 2.6 \times 10^{-10}$  in the time domain. This value of ADEV is in agreement with the measured values shown in the stability plots in Fig. 2. and Fig. 4, respectively. Thus the consistency of the above methods in both time and frequency domains has been verified. Evidently, the 33250A unit shows better frequency stability than 33120A.

Apparently at small averaging intervals the direct time-domain methods are limited by the counter noise-floor which, in addition, depends on the testing-signal frequency. Thus in the case of high-precision testing signals the use of the direct methods is limited and one has to turn to sensitivity-enhancement methods.

## ACKNOWLEDGMENT

The authors wish to thank Dr. Ludvík Šojdr of IREE for his contribution to the measurements. This work was supported by the Czech Grant Agency, Project #101/02/0672 and Czech Technical University Internal Grant CTU0309513.

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