A Risks Assessment and Conformance Testing of Analog–to–Digital Converters

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Abstract – The conformance test to which electronic devices are subjected after the manufacturing process, indicates if the device complies with an a priori given requirement set. On the basis of the test result, the component is considered to be working or not–working. However, because of the measurement uncertainty introduced by the testing bench assessment and by the chosen estimation algorithm, the manufacturer could include in the production process a component which does not respect the given requirements or could reject a working–device, thus affecting both testing and productivity costs. In this paper, it is considered the problem of the estimation of spectral parameters of analog–to–digital converters (ADCs). In particular, the risks to which both manufacturers and consumers of ADCs are subjected, are explicitly evaluated.

Keywords – ADC–testing, Hypothesis testing, Measurement uncertainty, Conformance testing.

I. Introduction

Overall performances of analog–to–digital converters (ADCs) are evaluated by means of figures of merit such as signal–to–random–noise ratio (SRNR), signal–to–non harmonic distortion ratio (SINAD), spurious free dynamic range (SFDR) and total harmonic distortion (THD). Each parameter can by estimated by means of various testing algorithms, which usually process N acquired samples obtained by applying a single or dual tone signal at the ADC input. The noise introduced by the employed acquisition system and the values chosen for the algorithm parameters, influence the value and the measurement uncertainty of the corresponding figures of merit. Thus, the test result strongly depends on the employed test method and on the values of the algorithm parameters used for obtaining the estimates [1], [2].

In order to evaluate if a device meets given requirements, the converter is subjected to a conformance test, in which usually the measured figure of merit is compared with a nominal reference value. The result of the comparison indicates if the device is conforming to the given specification requirement set. However, the conformance test could induce the producer to an incorrect assessment. In fact, because of the measurement uncertainty, the manufacturer could discard a properly working device or could accept a device which does not meet the established requirements. As a consequence, as indicated in the norm ISO 14235-1 [3], which gives information on the decision rules for proving conformance or non–conformance of workpieces and measurement equipment with respect to specifications, the measurement uncertainty should be taken into account when performing conformance tests. In particular, for quantifying the risk to which the manufacturer or the consumer are subjected, the probability density function (pdf) of the employed parameter estimator should be available.

In this paper, an estimator of the $SRNR$ of an ADC, evaluated by means of the Fast Fourier Transform (FFT)–based algorithm, is analyzed. In particular, the Chi–Square Goodness-of-Fit Test is applied in order to validate the hypothesis of the estimator having a normal distribution with known standard deviation. This in-

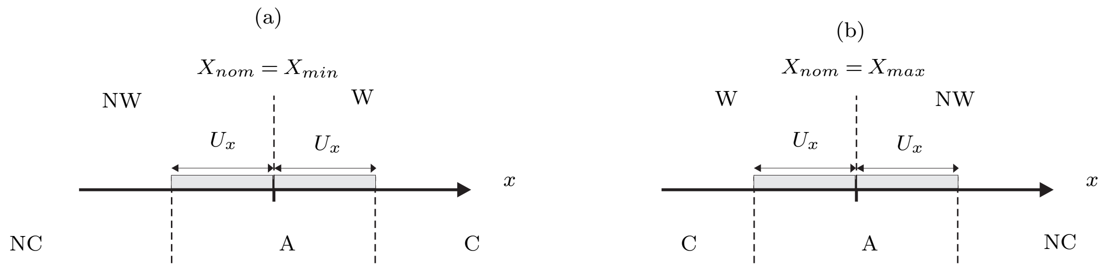

Fig. 1. Representation of the working (W), not–working (NW), conformance (C), non–conformance (NC) and ambiguity (A) zones usually employed in a device conformance test for giving confidence that the device meets its requirements with respect to a given value X_{nom} of a parameter x measured with an expanded measurement uncertainty U_x. X_{nom} represents the minimum X_{min} (a) or the maximum X_{max} (b) value which should be guaranteed in order to declare the device to be in conformity.

formation is finally employed for calculating the corresponding manufacturer and consumer risks when a specific device is subjected to a number N_{REC} of $SRNR$ measurements.

II. The product conformance zone

ADCs specifications are usually of the one–sided kind, i.e. the converter should present an $SRNR$ higher than a minimum specified value and a THD lower than a maximum reference value. In order to determine if a parameter x of the ADC satisfies the required performances, it is first estimated on the basis of known algorithms. Then, the resulting value, $\widehat{x}$, is compared to a nominal value X_{nom}, which represents the minimum or the maximum value that includes the given parameter tolerance and that the manufacturer must guarantee for that product. It should be noticed that the value of the parameter x differs from X_{nom} because of device mismatchings introduced by the manufacturer process. If the measurement result is out of the required range, the device is considered not satisfying the requirement. Correspondingly, the two working (W) or not–working (NW) regions are defined as shown in Fig. 1(a) and (b) when X_{nom} is the minimum X_{min} (a) or the maximum X_{max} (b), tolerated parameter value.

The measured value can be expressed as $\widehat{x} \triangleq x + \varepsilon$, where ε represents the deviation induced by the measurement uncertainty sources, which is usually modeled by means of a given pdf, $f_\varepsilon(\cdot)$, with mean and standard deviation equal to $\mu_\varepsilon = 0$ and σ_ε, respectively. It follows that, in order to determine if the estimated device parameter is within the maximum given tolerance, the measurement uncertainty should be taken into account.

In particular, ISO 14235-1 states that the measurement result has to be given along with the measurement uncertainty expressed in terms of the expanded uncertainty U_x. This has to be calculated by following directions given in the Guide to the Expression of Measurement Uncertainty (GUM)[4]. Accordingly, $U_x = ku_c$, where u_c is the combined standard measurement uncertainty, and k is the chosen coverage factor, usually equal to 2 or 3. When the measurement result belongs to the interval $X_{nom} \pm U_x$, the manufacturer could discard an ADC satisfying the given prerequisites or could accept a device which does not work properly. Accordingly, a customer could buy a non–working converter which has been declared to be in conformance to specifications. The $X_{nom} \pm U_x$ interval is indicated in Fig. 1(a) and (b) as the "ambiguity region" (A), while the zones on its left and on its right are called the "non–conformance" (NC) and the "conformance" (C) zones. It follows that the amplitude of the ambiguity zone depends on the estimated value of the measurement uncertainty, i.e. on the chosen estimator and on the employed algorithm parameters.

In [3] directions are given on the criterion to be followed for managing relationships between manufacturers and customers. Such information can be used, in particular, for quantifying the so called "consumer risk" (CR) and "producer risk" (PR), which may become significant whenever the measured value belongs to the interval A in Fig. 1. Both points of view, that of the manufacturer and that of the consumer should be taken into account. As an example, the case of a parameter which should not fall below a given minimum value will be considered. Whenever a manufacturer intends to prove the conformance of the device with respect to a given parameter x, CR and PR can be defined, respectively, as:

$$\mathrm{CR_C} = \Pr\{\widehat{x} \geq X_{nom} + U_x | x \leq X_{nom}\} \quad (1)$$

$$\mathrm{PR_C} = \Pr\{\widehat{x} < X_{nom} + U_x | x \geq X_{nom}\} \quad (2)$$

On the other hand, when a customer tries to prove the non–conformance of the product with respect to the same parameter x, the CR and PR are defined, respectively, by the following expressions:

$$\mathrm{CR_{NC}} = \Pr\{\widehat{x} \geq X_{nom} - U_x | x \leq X_{nom}\} \quad (3)$$

$$\mathrm{PR_{NC}} = \Pr\{\widehat{x} < X_{nom} - U_x | x \geq X_{nom}\} \quad (4)$$

Expressions (1)-(4) can be calculated if the pdf of the parameter estimator, $f_{\widehat{x}}(\cdot)$, or equivalently of the measurement deviation $f_\varepsilon(\cdot)$, is known. Since variations of x and $\widehat{x}$ from their nominal value are due to different physical phenomena, such random variables can be assumed to be statistically independent. By indicating with $f_x(\cdot)$ the pdf of x with mean and standard deviation values equal to μ_x and σ_x, respectively, the consumer and producer risks can then be calculated as:

$$\mathrm{CR_C} = \frac{\int_{-\infty}^{X_{nom}} \int_{X_{nom}+U_x-x}^{\infty} f_x(x) f_\varepsilon(\varepsilon) \mathrm{d}\varepsilon \mathrm{d}x}{\int_{-\infty}^{X_{nom}} f_x(x) \mathrm{d}x} \quad (5)$$

$$\mathrm{PR_C} = \frac{\int_{X_{nom}}^{\infty} \int_{-\infty}^{X_{nom}+U_x-x} f_x(x) f_\varepsilon(\varepsilon) \mathrm{d}\varepsilon \mathrm{d}x}{\int_{X_{nom}}^{\infty} f_x(x) \mathrm{d}x} \quad (6)$$

$$\mathrm{CR_{NC}} = \frac{\int_{-\infty}^{X_{nom}} \int_{X_{nom}-U_x-x}^{\infty} f_x(x) f_\varepsilon(\varepsilon) \mathrm{d}\varepsilon \mathrm{d}x}{\int_{-\infty}^{X_{nom}} f_x(x) \mathrm{d}x} \quad (7)$$

$$\mathrm{PR_{NC}} = \frac{\int_{X_{nom}}^{\infty} \int_{-\infty}^{X_{nom}-U_x-x} f_x(x) f_\varepsilon(\varepsilon) \mathrm{d}\varepsilon \mathrm{d}x}{\int_{X_{nom}}^{\infty} f_x(x) \mathrm{d}x} \quad (8)$$

In the next section it is demonstrated that the estimator of the $SRNR$ of an ADC, based on the FFT–algorithm, presents a normal distribution. Such a result is then employed for clearly evaluating the producer and consumer risks.

III. A DFT–based $SRNR$ estimator

Spectral figures of merit are often calculated using frequency domain algorithms. The N–length ADC output sequence obtained by applying a single tone to the ADC input, is weighted by a suitable window $w[\cdot]$ in order to reduce the spectral leakage of the non–coherently sampled frequencies which are eventually present in the output spectrum. Then, the FFT-based algorithm is applied to the windowed acquired data for estimating the powers of the wide–band noise, $\widehat{\sigma}_R^2$, and of the L narrow–band tones, $\widehat{\sigma}_{X_i}^2, i = 1, ..., L$, which are composed by the fundamental ($i = 1$), harmonic and spurious ($i = 2, ..., L$) tones.

The estimator of the ADC $SRNR$ expressed in dB, can then be written as [5]:

$$\widehat{SRNR}|_{dB} \triangleq 10 \log_{10} \frac{N_R}{N_R + ENBW_0} \frac{\widehat{\sigma}_{X_1}^2}{\widehat{\sigma}_R^2}, \quad (9)$$

with $ENBW_0 \triangleq N \sum_{n=0}^{N-1} w^4[n] / (\sum_{n=0}^{N-1} w^2[n])^2$ being the equivalent–noise bandwidth of the squared window, $w^2[\cdot]$, and N_R is the number of frequency bins associated to the discrete spectrum of the wide–band noise.

Such estimator presents a theoretical standard deviation equal to [6];

$$\mathrm{std}\{\widehat{\mathrm{SRNR}}|_{\mathrm{dB}}\} = \sigma_{\widehat{\mathrm{SRNR}}|_{\mathrm{dB}}} = \sqrt{\frac{18.9\mathrm{ENBW_0}}{\mathrm{N_R}}}. \quad (10)$$

Thus, the expanded uncertainty associated to (9) is equal to $U_{\widehat{SRNR}|_{dB}} = k \cdot \sigma_{\widehat{SRNR}|_{dB}}$.

In order to validate the hypothesis that (9) presents a normal pdf, the hypothesis testing method has been applied to the $SRNR$ estimator. To this purpose, (9) has been employed for calculating the $SRNR$ values of $N_{REC} = 5 \cdot 10^3$ records of simulated data, each of length $N = 2^{14}$. Each record of data is composed by the following signal:

$$y[n] = \sum_{i=1}^{5} A_i \cdot \sin(2\pi f_i n T_c) + e[n], \quad (11)$$
$$n = 1, ..., N$$

where $f_i = i \cdot 1$ kHz, $A_1 = 10$ V, $A_2 = 0.5$ V, $A_3 = 1$ V, $A_4 = 0.1$ V, $A_5 = 0.3$ V, and $e[\cdot]$ is a zero–mean normally distributed sequence with a variance σ_e^2 representing the quantization noise power of the ADC under test [7]. In particular, a converter has been considered with a full–scale (FS) equal to 10 V, and a resolution of 16–bit.

The normal probability plot of the N_{REC} estimates obtained by using (9) has been graphed

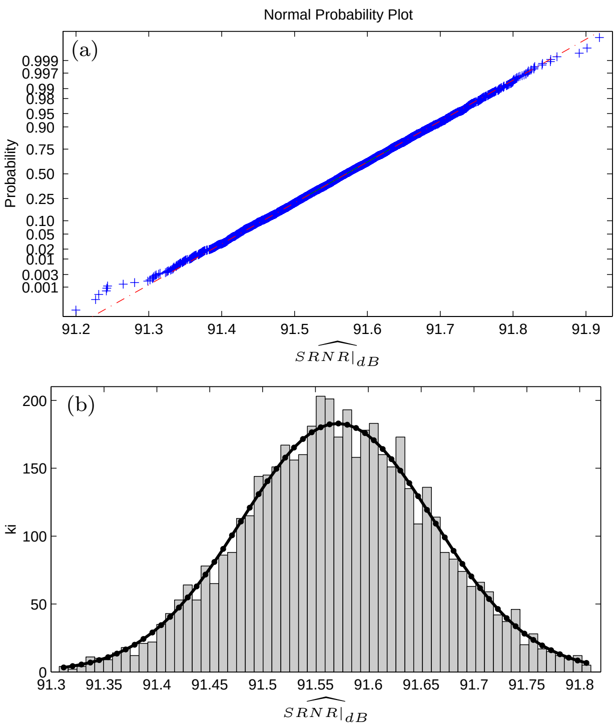

Fig. 2. Simulation results of $N_R = 5 \cdot 10^3$ values of the $SRNR$ estimated by means of $\widehat{SRNR}|_{dB}$. (a): the normal probability plot. (b): the grey bar–plot represents the histogram bins, k_i, $i = 1, ..., m$, evaluated on $m = 61$ equally spaced intervals, of the estimated quantities. Circled–solid line represents the probability p_{oi} of a normal density function of taking values in the i–th interval.

in Fig. (2)(a), which shows that the behaviour of the estimates well approximates a normal distribution.

To verify the hypothesis of (9) having a normal distribution with standard deviation equal to (10), the Chi-Square Goodness-of-Fit Test has been applied. In particular, it has been proved that (9) has a normal distribution with a standard deviation equal to (10) (null hypothesis H_0) against the assertion that (9) presents a different distribution (alternative hypothesis H_1).

To this aim, the null hypothesis has been assumed true with a significance level $\alpha = 5\%$, and the histogram of the available estimates evaluated, over number of equally spaced intervals $m = 2N_{REC}^{2/5} = 61$, each containing at least 5 estimates, has been calculated [8]. Moreover, for each interval, the probability p_{oi} of taking values in the i–th interval, $i = 1, ..., m$, of a normal distribution with mean equal to $\mu = \left(\sum_{j=1}^{N_{REC}} \left(\widehat{SRNR}(j)\right)/N_{REC}\right)|_{dB}$ and standard deviation provided by (10), has been calculated. The k_i, $i = 1, ..., m$ histogram bins have been graphed in Fig. 2(b) with a grey bar–plot along with the p_{oi} values multiplied by the number of employed estimates N_{REC} (circled–solid line). In order to apply the Chi–squared test, the k_i values have been used for calculating the Pearson's test statistic, q, which is known to have a $\chi^2(m-2)$ distribution with $m-2$ degrees of freedom [8]. The value of the Pearson's statistic, $q = 57.2$, has been compared to the critical value of the test c, which is equal to the $(1-\alpha)$–percentile $\chi^2_{1-\alpha}(m-2) = 77.9$. Since $q < c$, the null hypothesis can not be rejected.

As a consequence, a normal distribution with standard deviation equal to (10) can be employed for evaluating the consumer or manufacturer risks with a significance level $\alpha = 5\%$.

Such information has been employed for explicitly calculating the consumer and producer risks by considering that the measurement deviation ε presents the same pdf of the estimator $\widehat{x} = \widehat{SRNR}$, with the same standard deviation but with zero-mean. As an example, the measurand $x = SRNR$ has been assumed to be uniformly distributed in the interval $[\mu_x - \Delta_x, \mu_x + \Delta_x]$, with $\Delta_x \triangleq \sigma_x\sqrt{3}$ being a parameter characterizing the ADC manufacturing process. By substituting the pdf of x and ε in (5)–(8) and by evaluating the resulting expressions, we obtain:

$$
\begin{aligned}
\mathrm{CR_C} \;=\;& \frac{1}{2}\frac{\mu_x - U_x - \Delta_x - X_{nom}}{X_{nom} + \Delta_x - \mu_x} \cdot \\
& \cdot \mathrm{erf}\left(\frac{\sqrt{2}}{2}\frac{U_x + \Delta_x - \mu_x + X_{nom}}{\sigma_\varepsilon}\right) + \\
& + \frac{1}{2}\frac{X_{nom} + \Delta_x - \mu_x + U_x \mathrm{erf}\left(\frac{\sqrt{2}}{2}\frac{U_x}{\sigma_\varepsilon}\right)}{X_{nom} + \Delta_x - \mu_x} + \\
& - \frac{\sigma_\varepsilon}{\sqrt{2\pi}}\frac{1}{\Delta_x - \mu_x - X_{nom}} \cdot \\
& \cdot \left(e^{-0.5\frac{U_x^2}{\sigma_\varepsilon^2}} - e^{-0.5\frac{(U_x + \Delta_x - \mu_x + X_{nom})^2}{\sigma_\varepsilon^2}}\right)
\end{aligned} \qquad (12)
$$

$$
\begin{aligned}
\mathrm{PR_C} \;=\;& \frac{1}{2}\frac{U_x - \Delta_x - \mu_x + X_{nom}}{X_{nom} - \Delta_x - \mu_x} \cdot \\
& \cdot \mathrm{erf}\left(\frac{\sqrt{2}}{2}\frac{U_x - \Delta_x - \mu_x + X_{nom}}{\sigma_\varepsilon}\right) +
\end{aligned} \qquad (13)
$$

$$
\begin{aligned}
& +\frac{1}{2}\frac{X_{nom}-\Delta_x-\mu_x-U_x\text{erf}\left(\frac{\sqrt{2}}{2}\frac{U_x}{\sigma_\varepsilon}\right)}{X_{nom}-\Delta_x-\mu_x}+\\
& +\frac{\sigma_\varepsilon}{\sqrt{2\pi}}\frac{1}{\Delta_x+\mu_x-X_{nom}}\cdot\\
& \cdot\left(e^{-0.5\frac{U_x^2}{\sigma_\varepsilon^2}}-e^{-0.5\frac{(U_x-\Delta_x-\mu_x+X_{nom})^2}{\sigma_\varepsilon^2}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\text{CR}_{\text{NC}} = & \frac{1}{2}\frac{\mu_x+U_x-\Delta_x-X_{nom}}{\mu_x-X_{nom}-\Delta_x}\cdot \qquad (14)\\
& \cdot\text{erf}\left(\frac{\sqrt{2}}{2}\frac{(U_x-\Delta_x+\mu_x-X_{nom})}{\sigma_\varepsilon}\right)+\\
& +\frac{1}{2}\frac{\mu_x-\Delta_x-X_{nom}+U_x\text{erf}\left(\frac{\sqrt{2}}{2}\frac{U_x}{\sigma_\varepsilon}\right)}{\mu_x-\Delta_x-X_{nom}}+\\
& -\frac{\sigma_\varepsilon}{\sqrt{2\pi}}\frac{1}{\Delta_x-\mu_x-X_{nom}}\cdot\\
& \cdot\left(e^{-0.5\frac{U_x^2}{\sigma_\varepsilon^2}}-e^{-0.5\frac{(U_x-\Delta_x+\mu_x-X_{nom})^2}{\sigma_\varepsilon^2}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\text{PR}_{\text{NC}} = & \frac{1}{2}\frac{X_{nom}-U_x-\Delta_x-\mu_x}{X_{nom}-\Delta_x-\mu_x}\cdot \qquad (15)\\
& \cdot\text{erf}\left(\frac{\sqrt{2}}{2}\frac{X_{nom}-U_x-\Delta_x-\mu_x)}{\sigma_\varepsilon}\right)+\\
& +\frac{1}{2}\frac{X_{nom}-\Delta_x-\mu_x-U_x\text{erf}\left(\frac{\sqrt{2}}{2}\frac{U_x}{\sigma_\varepsilon}\right)}{X_{nom}-\Delta_x-\mu_x}+\\
& -\frac{\sigma_\varepsilon}{\sqrt{2\pi}}\frac{1}{X_{nom}-\Delta_x-\mu_x}\cdot\\
& \cdot\left(e^{-0.5\frac{U_x^2}{\sigma_\varepsilon^2}}-e^{-0.5\frac{(X_{nom}-U_x-\Delta_x-\mu_x)^2}{\sigma_\varepsilon^2}}\right)
\end{aligned}
$$

IV. Conclusions

In this paper, the pdf of an FFT-based estimator of the $SRNR$ of an ADC has been analyzed. In particular, the Chi–squared of Goodness fit test has been applied in order to validate the hypothesis of the estimator having a normal distribution. This result has been employed for quantifying the producer and consumer risks, which can be employed to evaluate both testing and productivity costs of the components.

References

[1] *Standard for Digitizing Waveform Recorders*, IEEE Std. 1057, Dec. 1994.

[2] *Standard for Terminology and Test Methods for Analog-to-Digital Converters*, IEEE Std 1241, Oct. 2000.

[3] ISO, *"ISO 14253-1, Geometrical Product Specifications (GPS) – Inspection by measurement of workpieces and measuring equipment – Part 1: Decision rules for proving conformance or non-conformance with specifications"*, 1998.

[4] ISO, "Guide to the Expression of Uncertainty in Measurement," Geneva, 1995.

[5] P.Carbone, E.Nunzi, D.Petri, "Windows for ADC Dynamic Testing via Frequency–Domain Analysis," *IEEE Trans. Instrum. and Meas. Tech.*,vol. 50, n. 6, pp. 1571–1576, Dec. 2001.

[6] D.Petri, "Frequency-Domain Testing of Waveform Digitizers," *IEEE Trans. Instrum. and Meas. Tech.*,vol. 51, pp. 1571–1576, Dec. 2001.

[7] M. Bertocco, C. Narduzzi, P. Paglierani, D. Petri, "A noise model for quantized data," *IEEE Trans. Instrum. and Meas. Tech.*,vol. 49, n. 1, pp. 83–86, Feb. 2000.

[8] E. S. Keeping, "Introduction to Statistical Inference," *Dover Publications,* NY, 1995.