MEASURING DYNAMIC NONLINEARITY OF A/D CONVERTERS VIA SPECTRAL METHODS

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Abstract – In this paper an extension of a previously developed and well-proven technique for the estimate of the INL of an ADC via spectral analysis methods is presented and discussed. Until now the authors applied this technique only to estimate the static INL, but further in-depth analyses show that it can be used to measure the ADC response to a sine wave even if the dynamic nonlinearity cannot be neglected (i.e. when the frequency of the input signal is higher than a given limit). To prove the validity of the method the authors compare their results with those obtained by the a time-consuming but certainly accurate method, i.e. direct comparison of the average of many output records with the input sine wave.

Keywords: A/D Converters, Spectral Techniques, Dynamic INL

1. Introduction

Measuring the nonlinearity of an analog-to-digital converter (ADC) is quite simple in principle but may be often difficult in practice. The standard measurement methods are the well-known servo-loop (or static) test and the histogram (or dynamic) test [1], [2]. The first one has some serious drawbacks, i.e. it requires always a big amount of time to complete even for low resolution ADCs, and it does not yield any information about the dynamic behavior of the device. The second method is much more popular, but it becomes all the same lengthy and unpractical when testing converters with a very high number of codes (i.e. when resolution ≥ 16 bit). A more subtle but important drawback of the histogram test is that the measured nonlinearity in dynamic conditions does not represent the actual characteristic of the converter if the common phenomenon of dynamic hysteresis is present [3].

Starting from these considerations, in former works the authors have developed a frequency-domain approach to the problem of measuring systematic errors in ADCs [4], [5], [6]. The proposed *Chebyshev test* yields the best polynomial approximation (of any given degree, up to 50-80) to the nonlinearity of the device, using as few as 8,000 samples, even for 20 bits converters. The polynomial approximation, even if not as accurate as the result yielded by the histogram test, is often quite sufficient for purposes of uncertainty evaluation, and even for correcting most of the systematic error [4]. However, in its present version, the test (like the servo-loop and the histogram methods) is not trustworthy when considerable dynamic nonlinearities affect the ADC. The chief aim of this research is to circumvent this problem, in order to achieve an accurate measurement of the nonlinearity error, also in dynamic conditions, via the same spectral approach.

2. Defining and measuring the nonlinearity error with a sinusoidal input

We start considering the very basic problem of defining the nonlinearity error introduced by an ADC when stimulated by a sinusoidal input, for example a cosine function of the form

$$x(t) = V \cos 2\pi f_x t \tag{1}$$

The general expression of the nonlinearity error is

$$\begin{aligned} e(n) &= y(n) - [Gx(nT_s + t_0) + O] = \\ &= y(n) - GV\cos(2\pi F_x + \varphi_0) - C \end{aligned} \tag{2}$$

where $y(n)$ is the ADC output sequence, T_s is the sampling period, t_0 is a temporal shift introduced to take into account signal delays in the ADC and G and O account for gain and offset errors respectively; of course $F_x = f_x T_s$ is the normalized frequency of the input signal and $\varphi_0 = 2\pi f_x t_0$ is the phase shift equivalent to the delay t_0. The parameters G, O, φ_0 (and also F_x if unknown) must be chosen so that $e(n)$ is as small as possible; if we compute the error in the root mean square sense it accounts also for actual deformations of the digitized sine wave, and not only for simple shifts or attenuations (i.e. linear distortions).

Most ADCs, when stimulated with a low enough input signal frequency f_x, behave as nearly static systems. This happens when the periodic output has the form

$$y(n) = \frac{a_0}{2} + \sum_{k=1}^{\infty} d_k \cos k(2\pi F_x n + \varphi) \qquad (3)$$

that is, all the harmonics are "in phase" and the sequence is a cosine series with a temporal shift. In this case the device may be represented as a linear dynamic system with frequency response $H(f)$ cascaded with a single-valued nonlinear function $g(x)$ (Fig. 1). Of course only the latter contributes to the nonlinearity error defined by (2): the first is responsible only for the temporal shift and for an amplitude variation of the input.

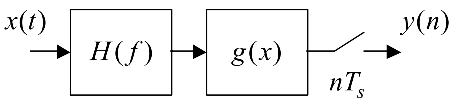

Fig. 1. Model of a perfectly static ADC

Measuring the nonlinearity error is in this case quite simple and does not require explicitly the construction of the term $x(nT_s + t_0)$ in (2). The probability distribution of the ADC output $y(n)$ yields, indeed, the nonlinear function $g(x)$ and consequently the nonlinearity error $e = g(x) - (Gx + O)$, as a single-valued function of *x*. This is the standard technique called histogram test [1], [2].

Unfortunately, with the currently available technologies any actual ADC is prone to nonlinear dynamic effects when the input frequency is above a certain limit value (which can be quite low for some device). In this case the model in Fig. 1 is not acceptable and the nonlinearity error $e(n)$ is not a single-valued function of the input *x*. In other words, $e(n)$ and $x(nT_s + t_0)$ go along a closed cycle (an *hysteresis)* in the (*x*, *e*) plane, for any choice of t_0. The situation is illustrated by Fig. 2, where it is clear that $y_{dyn}(n)$ (unlike $y_{stat}(n)$) is not symmetric and cannot be seen as an instantaneous function of $x(n)$.

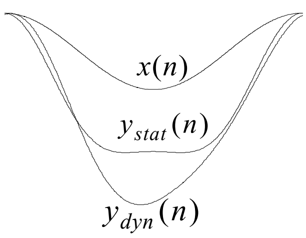

Fig. 2. Illustration of static and dynamic nonlinearity. Curve $x(n)$ is one period of the cosine signal ADC input; curve $y_{stat}(n)$ is a possible output if the ADC is static (note the "symmetry" of the curve); curve $y_{dyn}(n)$ is a possible output if the ADC is dynamic.

When the ADC is dynamic, the conventional histogram test makes very little sense, as it gives anyway a single-valued function of *x*. To deal with this difficulty, a modified histogram test based on the analysis of two separate probability distributions, the first for *rising* values, and the second for *falling* values of $y(n)$ has been proposed [7]. Figure 2 helps to understand that, unfortunately, the proposed approach is not always correct and that, sometimes, it can give approximate or totally wrong results. Indeed, due to the asymmetry of $y_{dyn}(n)$, the negative-slope part of it is *not* always the response to the negative-slope part of $x(t)$: as it is shown in Fig. 2 the response to the first half of $x(t)$ is in fact a non-monotonic curve (due to the nonlinearities of the ADC $y_{dyn}(n)$ begins to fall prior the instant in which $x(n)$ assumes its most negative value), and this is a case to which the *classic* histogram test is not applicable. Besides, it is possible to have an asymmetric $y_{dyn}(n)$ whose positive- and negative-slope parts have exactly the same probability distribution – a response that appear static to the test described in [7].

A sure way to measure the nonlinearity error in dynamic conditions is as follows:

- acquire a large number of records of the input sine wave $x(t)$ with the same phase, adding a dither signal like Gaussian noise with $\sigma \cong 0.5$ LSB (this is often already present in the system);
- average the records obtaining a very "clean" $y(n)$, free from both system noise and quantization noise;
- perform a sinusoidal best-fit on $y(n)$, in order to find the proper values of φ_0, *G*, *O* and F_x in (2);
- evaluate the error according to (2).

This procedure is certainly correct but can be very time-consuming, as it requires a large number of records (especially if the ADC is prone to a substantial system noise) and of samples per record (in order to test the characteristic in many points). In the next section a much faster test method is illustrated, the *Chebyshev test for dynamic nonlinearity error*: the "brute force" method described here will be used, however, to validate the results of the new test.

3. Chebyshev test for dynamic nonlinearity error

The first step in the "Chebyshev test", already proposed by the authors, is stimulating the ADC under test with a sinusoidal signal $x(t) = V\cos\omega t + C$. If a purely static model of the systematic error is assumed, the output may be represented by the equation:

$$y(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + e(t) \qquad (4)$$

where $e(t)$ takes into account all the random errors (noise, etc.). Of course, if one wants that the static model could be appropriate, the test must be performed at a conveniently low frequency ω .

If the hypothesis of purely static error is relaxed, the model of the converter output is given by the slightly more complex expression:

$$y(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) \; + \; e(t) \qquad (9)$$

where the out-of-phase harmonics b_n are caused by the dynamic nonlinearity. Due to these harmonics, the input-output characteristic relevant to the selected sinusoidal input is a *two-valued* function (instead of the single-valued function $y = g(x)$ of the static case). The ADC behavior is therefore represented by a pair of functions:

$$y = g_1(x) \qquad (10)$$

$$y = g_2(x) \qquad (11)$$

The first one describes the ADC output corresponding to *falling* values of the sinusoidal input, and the second one is relevant to *rising* values of the input. It can be easily demonstrated, with a few algebra similar to that employed for the static case [4], [5], [6], that the functions $g_1(x)$ and $g_2(x)$ are given by

$$g_1(x) = g(x) + h(x) \qquad (12)$$

$$g_2(x) = g(x) - h(x) \qquad (13)$$

where $g(x)$ is given by

$$g(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n C_n\left(\frac{x-C}{V}\right) \qquad (14)$$

while $h(x)$ is given by

$$h(x) = \sqrt{1 - \left(\frac{x-C}{V}\right)^2} \sum_{n=1}^{\infty} b_n D_n\left(\frac{x-C}{V}\right). \qquad (15)$$

The terms $C_n(z)$ and $D_n(z)$ are the *Chebyshev polynomials*, *of the first and of the second kind respectively*.

It is worth noting that:

- the input-output characteristic of the ADC with the considered sinusoidal stimulus, described by $g_1(x)$ and $g_2(x)$, is an hysteresis cycle;
- the *average characteristic*, $g(x)$, is obtained from the cosine coefficients a_n of the output and from the Chebyshev polynomial of the first kind $C_n(x)$;
- the *deviation-from-the-average characteristic*, $h(x)$, is obtained from the sine coefficients b_n of the output and from the Chebyshev polynomial of the second kind $D_n(x)$.

4. Experimental results

To prove the validity of the previous analyses the authors carried out a number of experiments on real-world digitizers (oscilloscopes). The results given by the dynamic Chebyshev test are compared with those obtained with the method described in Section 2, consisting in the direct comparison of the sinusoidal best-fit input with the average of many output records. The histogram test, as we said in the same section, cannot be used as a reliable tool in this situation.

Fig. 3 reports the nonlinearity error, obtained with the averaging-and-best-fit method, of an 8-bit digital oscilloscope stimulated by 15 kHz sinusoidal test signal. The graph shows two curves hardly distinguishable one from each other: the first one (black) refers to the positive slope of the input signal and the second one (gray) refers to the negative slope. Due to the low frequency used for the test signal the behavior of the digitizer is clearly static and the slight differences between the two curves are surely ascribable to secondary sources of inaccuracy (mainly residual noise after the averages).

Fig. 4 shows the results obtained on the same digitizer when the stimulus signal frequency grows to 6 MHz. In this case the two curves (corresponding to the functions $g_1(x)$ and $g_2(x)$) are clearly different and this is proves the dynamic behavior of the DUT.

Figures 5 and 6 show the result of the dynamic Chebyshev test, compared with those of the average-and-best-fit method. In Fig. 5 the function $g(x)$, derived by the a_n spectral coefficients, is compared with the expression $[g_1(x) + g_2(x)]/2$, where $g_1(x)$ and $g_2(x)$ are the same of Fig. 4. Likewise, Fig. 6 compares the function $h(x)$, derived by the b_n spectral coefficients, with the expression $[g_1(x) - g_2(x)]/2$, where again $g_1(x)$ and $g_2(x)$ come from Fig. 4. It is apparent that the spectral approach, which required less the 1/200 of the samples employed by other test, gives correct results.

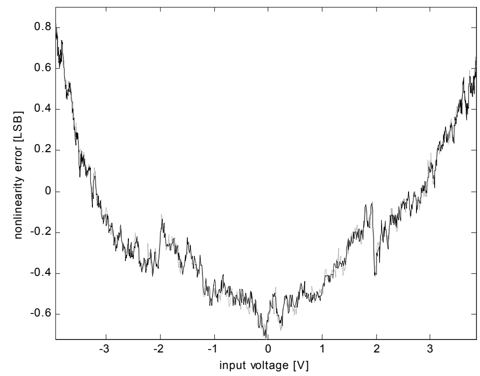

Fig. 3. Nonlinearity error (measured via sine wave fit on the average of 1,000 output records) of an 8-bit digital scope with a sinusoidal input @ 15 kHz. The black line is relevant to the positive slope of input, the

grey line to the negative slope. Here the device is static.

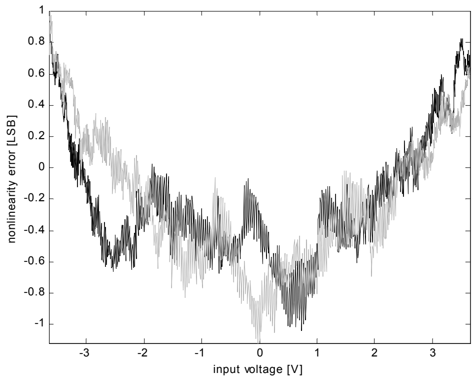

Fig. 4. Nonlinearity error of the same ADC of Fig.3, with a sinusoidal input @6 MHz. The difference between the lines clearly shows that the device has intrinsic dynamic nonlinearity.

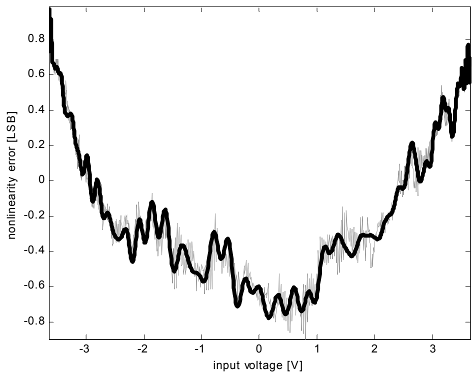

Fig. 5. Average nonlinearity error in the same conditions of Fig. 3. The grey line is the "true" error derived by the semi-sum of the curves in Fig. 3, while the black line is the "g" function from the Chebyshev test with 100 harmonics.

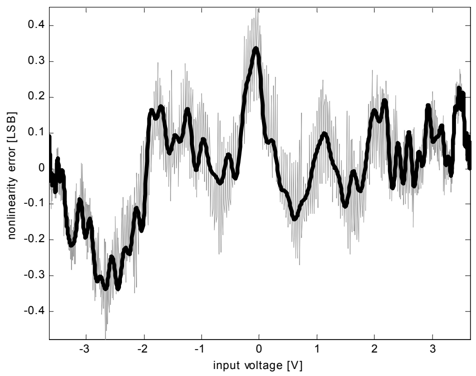

Fig. 6. Deviation-from-the-average nonlinearity error in the same conditions of Fig. 4. The grey line is the "true" error derived by the semi-difference of the curves in Fig. 4, while the black line is the "h" function from the Chebyshev test with 100 harmonics.

5. Conclusions

The theory outlined in this paper shows that it is possible to perform, with very simple and elegant mathematical means, a frequency-domain measurement of an ADC nonlinearity even when it is not purely static. The result of the test, consisting of the two functions $g(x)$ and $h(x)$ (or, equivalently, $g_1(x)$ and $g_2(x)$), is the same obtainable with the direct comparison of the ADC averaged output with the sinusoidal input.

The frequency-domain test discussed here, however, is incomparably faster (especially for high-resolution converters), even if this velocity is paid in terms of reduced accuracy (due to the used polynomial approximation of the characteristic). Besides being fast, it should be also remembered that the dynamic Chebyshev test is more mathematically rigorous of the modified histogram test, which is also used to examine dynamic nonlinearity errors.

In the authors' opinion the proposed test method is very promising for modelling and testing ADC in dynamic conditions. Besides allowing the fast determination of the hysteresis error and of other figures of merit relevant to dynamic nonlinearity, it could be easily embedded, for example, in a state-space compensation technique [8].

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