A Frequency-Domain Approach to ADC Phase-Plane Modeling and Testing

Conceição Líbano Monteiro(1), Pasquale Arpaia(2) and António Cruz Serra(1)
(1)Instituto de Telecomunicações / DEEC, IST, Universidade Técnica de Lisboa,
Av. de Rovisco Pais, 1049-001, Lisboa, Portugal
Phone: +351-218418490 Fax: +351-218417672, concha@alfa.ist.utl.pt, acserra@lx.it.pt
(2) Facoltà di Ingegneria, Università del Sannio
Piazza Roma, 82100, Benevento, Italy
Phone: +39-0824-305817 Fax: +39-0824-21866, arpaia@unisannio.it

Abstract –An alternative spectrum-based approach for determining the phase-plane error of an analog-to-digital converter is proposed. The method exploits a phase-plane error modeling, based on a dual-tone input stimulus and an output spectrum analysis [1]-[2]. In this way, the amount of data required by the dual-tone histogram test [3] is reduced.

Keywords – Analog-to-digital converters, dynamic testing, .nonlinearity, phase plane, frequency domain test, dual tone.

I. Introduction

In last years, technological enhancement in design and production generated a significant improvement in analog-to-digital converters (ADCs) metrological performance [4]-[5].
In this scenario, among other topics, a pre-eminent role has been played by phase-plane error modeling [7]-[12]. The ADC error is described as a function of the output code as well as the instantaneous input slope, i.e. in the "phase-plane". Last developments were related to the introduction of a 2-dimensional histogram test based on a dual-tone test signal [12] for its intrinsic capability of mapping the phase-plane extensively However, the statistical-domain approach needs for a large amount of data in order to reach a suitable accuracy.
In this paper, a spectrum-based approach for determining the phase-plane error is proposed. The approach exploits an analytical phase-plane error modeling based on dual-tone stimulus and output spectrum analysis [1]-[2] in order to reduce the amount of data required by the dual-tone histogram test. Results of numerical tests, aimed at characterizing the proposed procedure with reference to the state-of-the-art dual-tone histogram, are discussed.

II. The Proposal

In the following, (i) the basic idea, (ii) the ADC model, (iii) the analytical backgrounds, (iv) a phase-plane figure of merit, and (iii) a test procedure to measure the phase-plane *INL* are presented.

A. Basic Idea

Let the *INL* of an ADC with a dynamic behavior be described as a function of the output code k and input slope value s. In this paper, a dual-tone input signal is used as test signal for its capability of extensively mapping the phase plane [12]. Then, a fully based spectrum analysis of the output signal is exploited in order to measure the ADC *INL* in the phase plane. The spectral analysis allows the experimental constrains on the number of samples to be relaxed [1], [6].

B. ADC Model

Authors' past work presented a model for a nonlinear ADC with dynamic hysteresis representing the distortion as a function of input value and slope sign [1]-[2]. In particular, the ADC distortion is modeled by a two-branch function $y_\uparrow$ and $y_\downarrow$, swept with positive and negative slope, respectively. The ADC is modeled as illustrated in Fig. 1, where the non-linear block accounts for all dynamic nonlinearities, including hysteresis, whereas the ideal ADC accounts for sampling and quantization effects.
However, this is a local view over a wider reality, since experimental tests reveal that ADC *INL* values tend to increase with the input frequency. In other terms, higher input slope values stress the working conditions of the ADC, by usually degrading its performance.

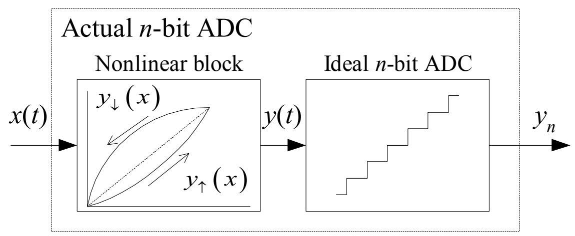

Fig. 1 ADC hysteretic model.

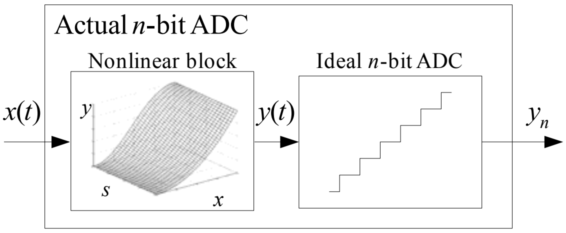

Fig. 2 ADC phase-plane model.

For this reason, the ADC behavior can be modeled in a more complete way by assuming a dependency not only on input slope sign, but also on input slope value s. This is accomplished by extending the previously presented model to the phase plane (x, s), as illustrated in Fig. 2. The nonlinear block is now graphically represented by a surface, where the output distorted value $y(t)$ a function of the instantaneous input and slope values $x(t)$ and $s(t)$, respectively.

C. Analytical Backgrounds

Let the dual-tone signal $x_{DT}(t)$ and its normalized derivative s_{DT} be expressed by

$$x_{DT}(t) = 1/2 \cdot \cos(\omega_A \cdot t) + 1/2 \cdot \cos(\omega_B \cdot t) \tag{1}$$

where $\omega_A \approx \omega_B$ with $\omega_A < \omega_B$, [3],[12], and the amplitude is normalized so that $x_{DT} \in$ [-1; 1], and

$$\begin{aligned} s_{DT}(t) &= \frac{2}{\omega_A + \omega_B} \frac{dx_{DT}}{dt}(t) \\ &= -\frac{\omega_A}{\omega_A + \omega_B} \sin(\omega_A \cdot t) - \frac{\omega_B}{\omega_A + \omega_B} \sin(\omega_B \cdot t) \end{aligned}, \tag{2}$$

where the amplitude is also normalized, so that $s_{DT} \in$ [-1; 1].

Let

- $\omega_S = 2\pi \cdot f_S$ Sampling frequency;
- $N = 2^n$ Number of samples per record ($n \in \infty$);
- $\omega_1 = \omega_S / N$ Frequency resolution, i.e. frequency of the first bin, of the FFT spectrum,

spectral leakage is avoided if the frequencies ω_A and ω_B are integer multiples of ω_1:

$$\omega_A = N_A \cdot \omega_1 \quad \omega_B = N_B \cdot \omega_1, \tag{3}$$

with N_A and N_B integers, and $0 < N_A < N_B$, whereas sampling theorem conditions are met:

$$\omega_B < \omega_S/2 \quad \Leftrightarrow \quad N_B < N/2. \tag{4}$$

From (3), it follows that x_{DT} is periodic, with frequency

$$\omega_{DT} = N_{DT} \cdot \omega_1 \quad N_{DT} = GCD(N_A, N_B) \quad 1 \le N_{DT} \le N_A \tag{5}$$

where GCD stands for the greatest common divisor of its arguments.

The resulting distorted function $y_{DT}(t)$ has necessarily the same fundamental frequency than the input, being described by the n_h-term Fourier series

$$y_{DT}(t) = Y_0 + \sum_{i=1}^{n_h} Y_i \cos(i \cdot \omega_{DT} + \alpha_i), \tag{6}$$

where Y_0 is the dc component and Y_i and α_i are the amplitude and phase of the i-th harmonic component, respectively.

According to the model of Fig. 2, the *INL* phase-plane values can be computed from $x_{DT}(t)$ and $y_{DT}(t)$. If the all instants $t_{k;s}$ when the distorted wave $y_{DT}(t)$ crosses the ideal transition levels T_k are found trough

$$y_{DT}(t_{k;s}) = T_k, \tag{7}$$

the actual slope dependent transition levels $T_{k;s}$, correspond to the value taken by the input signal and input slope just at those instants:

$$T_{k;s} = x_{DT}(t_{k;s})\big|_{s = s_{DT}(t_{k;s})}. \tag{8}$$

Notice that during one full period of the dual-tone signal, the same transition level can be crossed several times with different slope values.

Finally, the *INL* slope dependent values are directly obtainable from the transfer function transition levels:

$$INL_{k;s} = T_k - T_{k;s}. \tag{9}$$

D. Figure of Merit

For an ADC with phase-plane error, the distortion varies with the input slope value, by definition. Hence, for some code level k, The *INL* measured at different input slopes, s_1 and s_2 will present different values. Conversely, for an ADC without phase-plane error, the *INL* is slope-independent: i.e. for the same code k, the *INL* evaluated at different input slopes presents the same value. The phase plane trend of the error is defined by

$$\exists_{k \wedge s_1 \neq s_2} : INL_{k;s_1} \neq INL_{k;s_2}. \tag{10}$$

Taking the $INL_{k;0}$ values obtained for null slope as a reference, the difference $INL_{k;s}$ - $INL_{k;0}$ will be identically null for all k if and only if there is no phase plane behavior. Hence, the ratio between the root mean square (rms) of the referenced difference evaluated along all code and slope values and the rms of the $INL_{k;0}$ evaluated along all codes provides normalized information about the amount of phase-plane distortion *PPD*:

$$PPD = 20\log\frac{\text{rms}\left(INL_{k;s} - INL_{k;0}\right)}{\text{rms}\left(INL_{k;0}\right)} = 20\log\frac{\sqrt{\frac{1}{2\left(N_k - 1\right)}\sum_{k=1}^{N_k - 1}\int_{-1}^{1}\left(INL_{k;s} - INL_{k;0}\right)^2 ds}}{\sqrt{\frac{1}{N_k - 1}\sum_{k=1}^{N_k - 1} INL_{k;0}^{\ 2}}}, \quad (11)$$

where N_k is the number codes of the ADC and s stands for the normalized slope, varying between -1 and +1.
PPD value will be -∞ if and only if there is no phase plane distortion, and the higher its value, the strongest the slope dependency.

E. Test Procedure

In Fig. 3 the main steps of the procedure for computing the ADC slope dependent transition levels $T_{k;s}$, the respective $INL_{k;s}$ values, and the phase plane distortion figure of merit *PPD* are described.

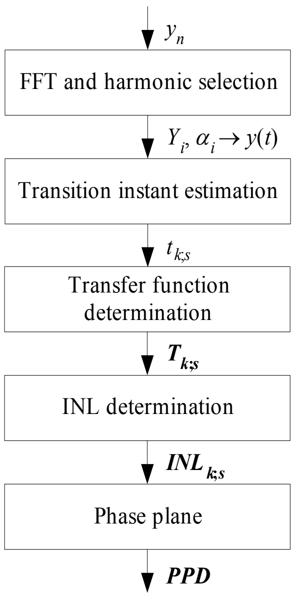

Fig. 3 Test procedure for metrological characterization of ADCs in the phase plane.

- Preliminary, N_x samples of the dual tone not normalized signal $x_{DT}'(t) = X{\cdot}x_{DT} + X_0$, with offset X_0 and amplitude X are collected into the record y_n. The choice of the input frequencies, that verify (3) and (4), can be established as a function of the slope range and quality of the phase plane coverage according to [3]. As for the standard histogram test, X_0 and X values must be known.
- An FFT is then performed on vector y_n, in order to extract the values of amplitude Y_i and phase α_i of the i-th harmonic of the reconstructed output signal $y(t)$.
- The instants $t_{k;s}$ when the distorted wave $y_{DT}(t)$ crosses the ideal transition levels T_k are found for one full period of $y(t)$. As referenced before, the same transition levels can be crossed different times with different input slope values.
- The actual transition levels are found trough (8), where $x_{DT}(t) = (x_{DT}'(t) - X_0) / X$. The normalized slope domain [-1; 1] is divided into N_s intervals with width $2 / N_s$, and the slope index SI of each calculated transition level, given by the index of the interval where $s_{DT}(t_{k;s})$ falls, computed, so that the transition levels $T_{k;SI}$ can be put into a transfer function matrix of $N_{k-1} \times N_S$. When different instants $t_{k;s}$ fall into the same transfer function matrix position, the final value of $T_{K;SI}$ is given by the geometric average of the different found solutions.
- The $INL_{k;SI}$ values are derived from (9).
- Finally, the *PPD* figure of merit is evaluated by the equivalent to (11), where the continuous integral on s is substituted by a discrete sum of values

$$PPD = 20\log\frac{\sqrt{\frac{1}{N_s\left(N_k - 1\right)}\sum_{k=1}^{N_k - 1}\sum_{SI=1}^{N_s}\left(INL_{k;SI} - INL_{k;0}\right)^2}}{\sqrt{\frac{1}{N_k - 1}\sum_{k=1}^{N_k - 1} INL_{k;0}^{\ 2}}}. \quad (12)$$

III. Numerical Results

In the following, results of simulation tests designed to (i) highlight the working of proposed approach, and (ii) establish a comparison with the bi-dimensional histogram are presented.
With this purpose, three different 12-bit ADCs were modeled resorting to suitably different bidimensional transfer functions: ADC_1 presents no phase-plane behavior, i.e, its distortion function only depends on k, ADC_2 presents moderate slope dependent behavior and ADC_3 presents strong slope dependent behavior. Polynomials of 20^{th} degree were used to generate the code dependent distortion, while slope dependent behavior was modeled by a tanh function.
For the test procedure, the full-scale dual tone signal x_{DT} with frequencies selected according to [3] was applied to all 3 ADCs and a sample of $N_x = 16\,384$ points collected.
In Fig. 4 to Fig. 6, the imposed and estimated phase plane INL of the three simulated ADCs are represented, revealing a clear match between both values. The figure of merit *PPD* has been evaluated in the three cases, assuming the values of PPD_1 = -59dB, PPD_2 = -15dB and PPD_3 = 9dB, which increase with the amount of phase plane behavior, as expected. Due to small estimation errors in INL of ADC_1, PPD_1 does not take the theoretical value -∞.

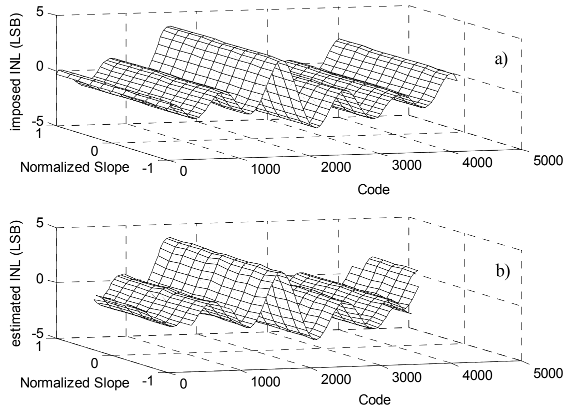

Fig. 4 Imposed (a) and estimated (b) $INL_{k;SI}$ values of simulated ADC_1.

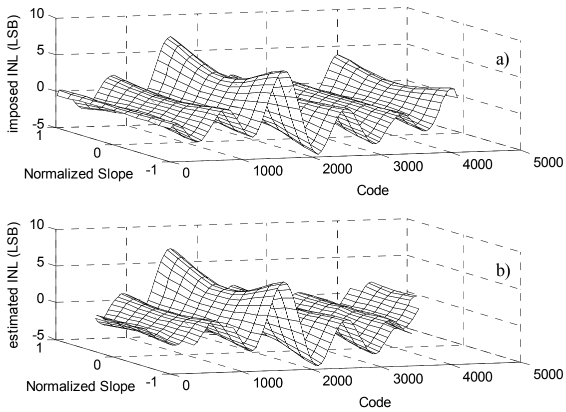

Fig. 5 Imposed (a) and estimated (b) $INL_{k;SI}$ values of simulated ADC_2.

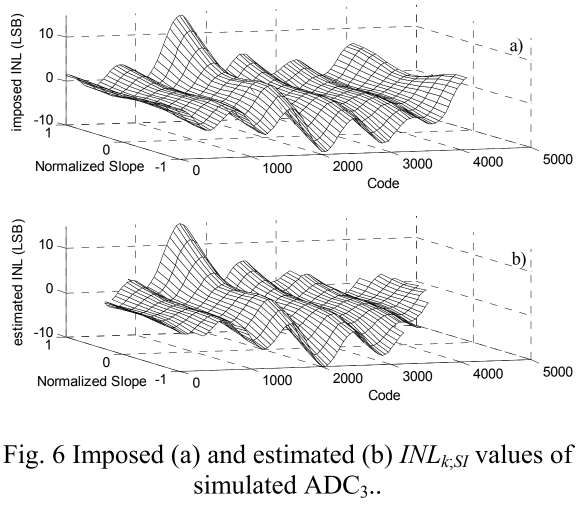

Fig. 6 Imposed (a) and estimated (b) $INL_{k;SI}$ values of simulated ADC_3..

The rms error of the INL estimation was also computed for the three cases: 0.17 LSB for ADC_1, 0.26 LSB for ADC_2 and 0.51 LSB for ADC_3.

IV. CONCLUSIONS

In this paper, a frequency-domain approach to the ADC metrological characterization in the phase plane Has been proposed. A dual-tone signal and an output spectrum analysis are carried out in order to derive the INL. Preliminary simulations provide encouraging results for further experimental investigations.

REFERENCES

[1] C.L.Monteiro, P.Arpaia, A.C.Serra, "Phase-spectrum analysis for detection of ADC hysteretic distortion", *Proc. of IEEE IMTC 2001*, vol. 3, pp. 1677-1683, May 2001, in press on *IEEE Trans on Instrum. and Meas.*

[2] P.Arpaia, A.C.Serra, P. Daponte, C.L.Monteiro, "A Critical Note to IEEE 1057-94 Standard on Hysteretic ADC Dynamic Testing", *IEEE Trans on Instrum. and Meas.,* Vol. 50, No. 4, August 2001, pp. 941-948.

[3] S.Acunto, P.Arpaia, F.H.Irons, D.M.Hummels, "A new bidimensional histogram for the dynamic characterization of ADCs", *Proc. of IEEE IMTC 2001*, vol.3, pp. 2015-2020, May 2001, in press on *IEEE Trans on Instrum. and Meas.*

[4] P. Arpaia, P. Daponte, S. Rapuano, "A State of the Art on ADC Modeling", (invited paper), *Proc. of joined IEE ADDA 2002 Conf. and IMEKO TC-4 Workshop on ADC Modeling and Testing,* Prague (CZ), 2002.

[5] P. Arpaia, F. Cennamo, P. Daponte, "Metrological Characterization of Analog-to-digital Converters: A State of the Art", *Proc. of IEE ADDA 1999 Conf.*, Glasgow (UK), September, 1999.

[6] C.L.Monteiro, P.Arpaia, A.C.Serra, "Dual-Tone Analysis for Phase-Plane Coverage in ADC Metrological Characterization", *Proc. of IEEE IMTC 2003*, vol. 3, pp. 1677-1683, May 2003.

[7] F.H.Irons, "Dynamic characterization and compensation of analog-to-digital converters", *Proc. of IEEE Int. Symp. on Circ. and Sys.*, May, 1986, pp.1273-1277.

[8] J.P.Deyst, T.M.Souders, "Phase plane compensation of the NIST sampling comparator system", Proc. of IMTC/94, vol. 2, pp. 914-916, May 1994.

[9] J.Tsimbinos, K.V.Lever, "Improved error table compensation of analog-to-digital converters using pseudo-random calibration signals", Electronic Letters, vol. 30, No. 6, pp-461-462, Mar. 1994.

[10] D.M.Hummels, S.P.Kennedy, "Improved dynamic compensation of ADCs using an iterative estimate of the ADC calibration signal", *Proc. of 35th Midwest Symposium on Circuits and Systems*, pp. 68-71, Aug. 1992.

[11] P.Arpaia, P.Daponte, L.Michaeli, "Analytical a priori approach to phase-plane modeling of SAR A/D converters", *IEEE Trans. on Instrum. and Meas.*, Vol. 47, No.4, pp. 849-857, Aug. 1998.

[12] J. Blair, "Selecting test frequencies for two-tone phase-plane analysis of ADCs", *Proc. of IEEE IMTC 2001*, vol.3, pp. 1843-1846, May 2001.