Estimation of A/D Converter Nonlinearities from Complex Spectrum

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Abstract: *The purpose of this paper is to provide a new method, which permits to estimate the integral nonlinearities of an A/D converter. This estimation uses the complex spectrum of a digitized sine wave, combined with a Fourier series expansion of the A/D converter integral nonlinearities. Thanks to this approach, an accurate estimation is obtained using fewer data than a classical method.*

Keywords: *A/D Converter, Integral Nonlinearity, A/D Converter Testing.*

1. Introduction

The Integral Non Linearity (INL) is used to describe the overall shape of the transfer function of an Analog-to-Digital Converter (ADC). To achieve the INL evaluation, an input signal with known Probability Density Function (PDF) is applied to the ADC and the INL is then evaluated by comparing the theoretical and the actual PDF obtained at the output of the converter. This approach has been intensively used in the field of ADCs characterization and has given accurate results. Although the performances of the converters have significantly increased, in both speed and accuracy, the method has maintained the possibility to evaluate the INL with a satisfactory resolution. However, the size of the samples dramatically increases as soon as the accuracy of the INL estimations becomes more critical. To obtain a tolerance of $\pm 25\%$ with respect to the code width and a confidence of 99% on the estimated INL of a 14 bits ADC, at least 8M samples[1] have to be acquired [1]. Although the computation capability of nowadays PC based test set-up is able to handle this amount of data, the test time is dramatically increased due to the data transfer time and it has a significant impact on the device cost. To decrease this test time several methods have been developed [2-3], mainly based on FFT. Although these methods give satisfactory results, the INL estimation is limited to polynomial approximation and cannot describe sharp transition usually encountered for actual INL.

As the influence of the A/D converter nonlinearities may be perceived in the spectral domain, an analytic study which links the INL to these dynamic parameters has been achieved, as shown in Section 2. Although this approach is not a new one, this formalism based on Fourier expansion led to accurate results. Thanks to these results, the A/D converter nonlinearities can be evaluated from complex spectrum by using the model which links the INL and the spectral components, then a singular value decomposition to solve a set of linear equations. This method is presented in Section 3. The advantage of the spectral approach is that only small amount of data is needed to achieve the estimation. Thus, the measurement time is drastically reduced and requires few memory capacity for satisfactory results.

2. Analytic Modeling of the INL influence

Considering the periodic extension, *f(x)*, of the INL associated with a N bits A/D Converter defined by

$$\begin{cases} f(x) = INL(x), 0 \le x \le 2^N - 1 \\ f\left(x + p.2^N\right) = INL(x), p \in \mathbb{Z} \end{cases} . \tag{1}$$

The Fourier series expansion of this function[2] leads to

$$\begin{cases} f(x) \sim \dfrac{a_0}{2} + \displaystyle\sum_{k=1}^{\infty}\left(a_k \cos(kwx) + b_k \sin(kwx)\right) \\ w = 2p / 2^N \end{cases} \tag{2}$$

where $x=0,\dots,2^N-1$ is the digital output code. We consider a sine wave, with input frequency f_0, applied at the input of an A/D converter, and sampled M times with a given frequency f_S. In order to avoid spectral leakage, which may be encountered in spectral analysis, these two frequencies have to satisfy the relation [4]

$$M.f_0 = J.f_S \tag{3}$$

where M and J are two relatively prime numbers. As a consequence, the nominal sampling phase of the input signal is given by the well-known expression

[1] we use the relation 1M= 2^{20}.

[2] The introduction of the periodic extension, *f(x)*, is only used to allow the Fourier series expansion of the INL, in practice, the interval of interest will be restrained to $0 \le x \le 2^N - 1$.

$$q_n = 2p\left(\frac{J}{M}\right)n + q_0\,, \tag{4}$$

where n represents the sample index, and q_0 the initial phase of the input sine wave. Using (3) and (4), an ideal quantized sine wave can be expressed by

$$x(n) = 2^N\left(\frac{V_0}{V_{FS}}\right)\cos(q_n) + 2^N\left(\frac{V_{DC}}{V_{FS}}\right) + q(n) \tag{5}$$

where V_0 and V_{DC} are respectively the amplitude and the DC component of the input signal, while V_{FS} is the full scale voltage of the converter, and $q(n)$ the quantization uncertainty term. Thanks to (2) and (5), the A/D converter produces a digitized signal $s(n)$ given by

$$s(n) = x(n) + \frac{a_0}{2} + \sum_{k=1}^{\infty}\left\{a_k \cos\left[2p\,k\left(\frac{V_0}{V_{FS}}\right)\cos(q_n) + 2p\,k\left(\frac{V_{DC}}{V_{FS}}\right)\right] + b_k \sin\left[2p\,k\left(\frac{V_0}{V_{FS}}\right)\cos(q_n) + 2p\,k\left(\frac{V_{DC}}{V_{FS}}\right)\right]\right\} \tag{6}$$

where $q(n)$ has been neglected, considering that quantization does not produce significant harmonics (this assumption is easily fulfilled for high-resolution converters and signals close to full-scale range). Using the relations

$$\begin{cases} \cos\left[a\cos(p)\right] = J_0(a) + 2\sum_{h=1}^{\infty} J_{2h}(a)(-1)^h \cos\left[2hp\right] \\ \sin\left[a\cos(p)\right] = 2\sum_{h=0}^{\infty} J_{2h+1}(a)(-1)^h \cos\left[(2h+1)\,p\right] \end{cases} \tag{7}$$

where $J_h(a)$ is the Bessel Function of the first kind with order h, it can be shown that the signal $s(n)$ is expressed by

$$s(n) = x(n) + \sum_{l\geq 0} S_l \cos(l\,q_n) \tag{8}$$

where S_l represents the amplitude of the l^{th} harmonic. The analytic expression of the harmonics S_l are expressed, using (6) and (7), by

$$\begin{cases} S_{2l}^{Th.} \underset{l\geq 1}{=} 2(-1)^l \sum_{k=1}^{\infty}\left[a_k\cos(k\,a_2) + b_k \sin(k\,a_2)\right] J_{2l}(ka_1) \\ S_{2l+1}^{Th.} \underset{l\geq 0}{=} 2(-1)^l \sum_{k=1}^{\infty}\left[b_k\cos(k\,a_2) - a_k \sin(k\,a_2)\right] J_{2l+1}(ka_1) \end{cases} \tag{9}$$

where

$$INL \xrightarrow[\text{Expansion}]{\text{Fourier Series}} \left\{a_0, a_1, \ldots, b_1, \ldots\right\} \xrightarrow{\mathrm{T}} \left\{S_0^{Th}, \ldots, S_l^{Th}, \ldots\right\}$$

Figure 1: Analytic Modeling of INL influence.

$$a_1 = 2p\,\frac{V_0}{V_{FS}} \quad a_2 = 2p\,\frac{V_{DC}}{V_{FS}}\,, \tag{10}$$

Thanks to (8) and (9), the dynamic parameters such as SFDR, THD may be estimated from the INL of the A/D converter. From a functional point of view, the mathematical developments can be represented using the diagram given in Figure 1. To avoid infinite dimensions, the number of harmonics is arbitrary set to H_{max} while the expansion order used for the Fourier series is set to K_{max}. As a consequence, the transformation $\mathbf{T_{Kmax}}$, associated with equation (9), can be rewritten as a $(H_{max}+1, 2\,K_{max}+1)$ matrix

$$\mathbf{T}_{Kmax} = \begin{pmatrix} 1/2 & A_1^0 & \cdots & A_{K_{max}}^0 & B_1^0 & \cdots & B_{K_{max}}^0 \\ 0 & A_1^1 & \vdots & A_{K_{max}}^1 & B_1^1 & \vdots & B_{K_{max}}^1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & A_1^{H_{max}} & \cdots & A_{K_{max}}^{H_{max}} & B_1^{H_{max}} & \cdots & B_{K_{max}}^{H_{max}} \end{pmatrix} \tag{11}$$

where the $2K_{max}$ column vector $\mathbf{A}_k$ and $\mathbf{B}_k$ are expressed by

$$\mathrm{A}_k = \begin{bmatrix} J_0(ka_1)\cos(ka_2) \\ -2J_1(ka_1)\sin(ka_2) \\ \vdots \\ (-1)^l\,2J_{2l}(ka_1)\cos(ka_2) \\ (-1)^{l+1}\,2J_{2l+1}(ka_1)\cos(ka_2) \\ \vdots \end{bmatrix}, \tag{12}$$

$$\mathrm{B}_k = \begin{bmatrix} J_0(ka_1)\sin(ka_2) \\ 2J_1(ka_1)\cos(ka_2) \\ \vdots \\ (-1)^l\,2J_{2l}(ka_1)\sin(ka_2) \\ (-1)^l\,2J_{2l+1}(ka_1)\cos(ka_2) \\ \vdots \end{bmatrix}. \tag{13}$$

Once the matrix $\mathbf{T_{Kmax}}$ is fully determined, using (11) (12) and (13), the harmonics components are estimated for a given INL Fourier series expansion by means of the matrix product

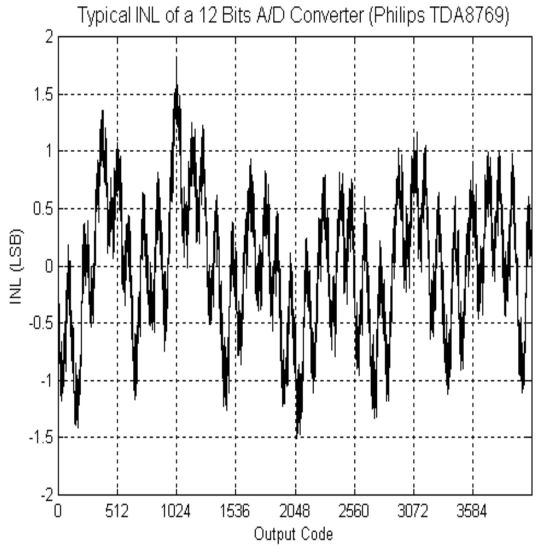

Figure 2: Typical INL of Philips TDA8769.

$$\begin{bmatrix} S_0 & \cdots & S_{H\max} \end{bmatrix}^T = \mathbf{T}_{K\max} \cdot \begin{bmatrix} a_0 \\ \vdots \\ b_{K\max} \end{bmatrix}. \qquad (14)$$

In order to confirm the validity of (14), a real digitized sine wave was firstly acquired with a classical dynamic test setup [4]. An A/D converter (Philips TDA8769) whose INL (measured using cumulative histogram) is given in Figure 2, was used to performed the digitizing of the sine wave. The input and sampling frequencies were respectively set to 70 MHz and 40 MHz, and the sine wave peak-to-peak amplitude was adjusted to reach –0.1dB of the A/D converter full-scale range. As the INL of the converter has been measured, the harmonics evaluated thanks to (14) may be directly compared this the ones evaluated from FFT applied to the digitized sine wave. The results are presented in Table 1, as a function of the Fourier series expansion order K_{max},

	THD (dB)	SFDR (dBc)
Ref.	**69.79**	**76.61**
T_{10}	67.84	74.43
T_{15}	68.70	76.43
T_{20}	68.83	76.69
$\mathbf{T_{25}}$	**70.07**	**76.41**
$\mathbf{T_{50}}$	**69.67**	**76.42**
$\mathbf{T_{100}}$	**69.75**	**76.63**

Table 1: Real Dynamic Parameters vs Analytic Modeling of INL.

where the reference line corresponds to the experimental results. Thanks to these results, it appears that satisfactory estimation is obtained with expansion order $K_{max} \geq 25$. In order to validate the estimation of spectral components, individual estimations are given in Table 2, using the same conditions than proceeding. Once again, an expansion order $K_{max} \geq 25$ leads to accurate results. The Fourier series expansion, and its main conclusion, given by (14), led to accurate modeling of the INL influence on sine wave signals. Although the modeling approach of INL is not a new one, a useful application will be derived as explained in the following section.

3. Estimation of Nonlinearities from Complex Spectrum

As the harmonic components $\{S_0, S_1, \ldots, S_{Hmax}\}$ of a real digitized sine wave can be evaluated from an FFT applied to the samples provided by the converter, to estimated the A/D converter non-linearity Fourier series expansion, one has to invert (14). Due to dimension mismatch the matrix inversion of $\mathbf{T_{Kmax}}$ is not mathematically defined except for $H_{max}=2K_{max}$. Even for this particular case, the determinant of the $\mathbf{T_{Kmax}}$ matrix was found sufficiently close to zero not to allow direct inversion (as the numerical stability of the inversion algorithm is not ensured). To avoid such numerical difficulty, an approach based on Singular Value Decomposition (SVD) [5] is used and lead to accurate results. As a consequence, once $\mathbf{T_{Kmax}^{-1}}$ has been numerically evaluated, the Fourier expansion is given by

$$\begin{bmatrix} \hat{a}_0 \\ \vdots \\ \hat{b}_{K\max} \end{bmatrix} = \mathrm{T}_{\mathrm{Kmax}}^{-1} \cdot \begin{bmatrix} \hat{S}_0 & \cdots & \hat{S}_{H\max} \end{bmatrix}^T, \qquad (15)$$

where the spectral components $\hat{S}_l$ $(0 \leq l \leq H_{max})$ are estimated from the FFT. From the vector $\mathbf{F_c}$ expressed by

	H_2 (dBc)	H_3 (dBc)	H_4 (dBc)	H_5 (dBc)
Ref.	**-85.62**	**-78.47**	**-76.61**	**-78.21**
T_{10}	-86.58	-74.43	-76.98	-78.21
T_{20}	-84.74	-79.01	-76.69	-92.74
$\mathbf{T_{25}}$	**-86.13**	**-78.64**	**-76.41**	**-90.56**
$\mathbf{T_{50}}$	**-85.08**	**-77.71**	**-76.42**	**-89.94**

Table 2: Individual Estimation of Spectral Components.

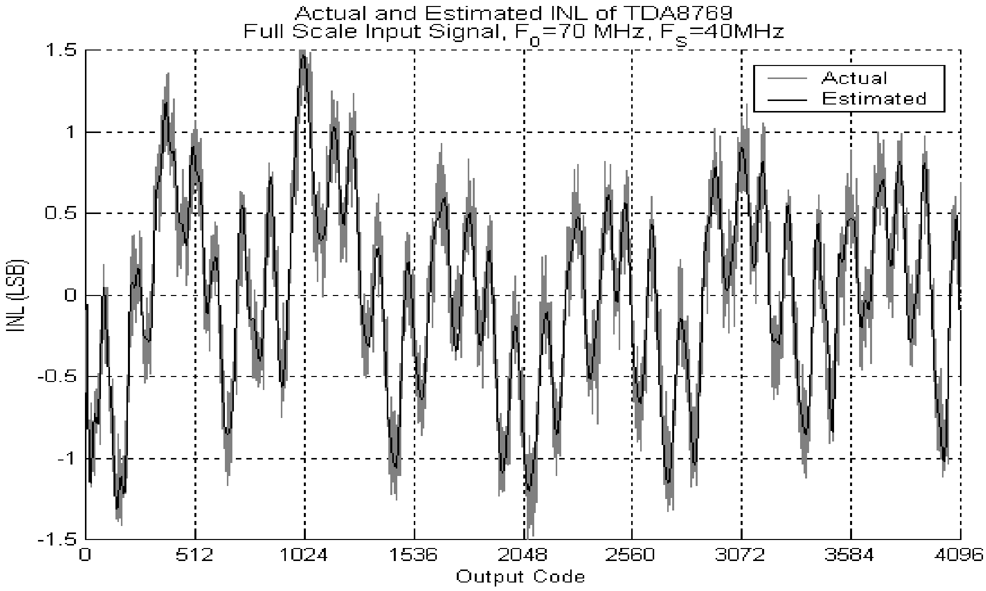

Figure 3: Actual and Estimated INL of TDA8769 from Complex Spectrum.

$$\mathbf{F_c} = \left[\hat{a}_0, \hat{a}_1, \hat{a}_2, \cdots, \hat{b}_1, \hat{b}_2, \cdots\right]^T, \qquad (16)$$

the A/D converter nonlinearities can be reconstructed using (2). As the SVD algorithm is equivalent to a least mean squares minimization [5], $\mathbf{F_c}$ is the vector, which minimizes the estimation error. An example of the A/D converter nonlinearities estimated thanks to the new method, is given in Figure 3, for A/D converter TDA8769 with N_0=8192, K_{max}=100, H_{max}=200, with a sine wave input signal with peak-to-peak amplitude set to full scale voltage. As seen in Figure 3, the estimated INL is smoothed version of the actual INL. The estimation error, defined as the difference between the actual INL and the estimated one obtained from FFT, was found to have a Gaussian probability density function with a null mean value and a standard deviation of 0.17 LSB as shown in Figure 4. This standard deviation permits to verify that the peak value of the INL estimation error is lower than 1 LSB for the chosen configuration.

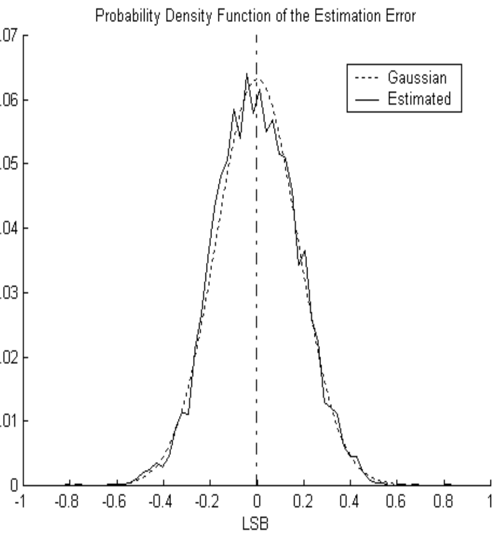

Figure 4: PDF of the Estimation Error.

4. Conclusion

An FFT approach to estimate the INL of an A/D converter has been presented. This new method led to accurate estimation, while requiring few samples with respect to the classic methods. As mentioned in [2], this method will permit to implement fast and computationally inexpensive linearization algorithm to maximize the spurious dynamic range of A/D Converters.

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