# Stochastic Approach for Memoryless Nonlinearity Measurements


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**Abstract -** In this paper we will present a simple and cost effective, yet accurate, setup for measuring the amplitude distribution of a signal and its probability density function (*pdf*). From these, it will be shown how to measure and model linear and memoryless nonlinear errors using both a power and an orthogonal series representation of the distorted signal, and how to use this knowledge to improve the resolution of an ADC characterization. Finally, we will give an usage application, with experimental results, in which the nonlinearities of a low grade stimulus are 'subtracted' in the histogram method for the characterization of ADCs.



**Keywords -** nonlinear measurements, *pdf* distortion, adc characterization


## I. Introduction

The performance assessment of an Analog to Digital Converter (ADC) usually entails inferring the downgrade suffered by the stimulus at the output of the converter, *i.e.*, by tracking changes in the properties of the signal. The evaluation of the downgrade requires either the complete knowledge of the stimulus signal or that it adequately resembles an ideal stimulus, in which case it is assumed as such. The standard procedure is to opt for the ideal stimulus assumption, establishing a sufficient requirement setting for the stimulus that assures a proper measurement. To this end, however, care must be taken to assure that the metric chosen to assess the stimulus is meaningful, which, for instance, is not entirely the case of the total harmonic distortion when paired with the histogram method, as is shown in [1–4]. Related still with the histogram method, we would like to (*i*) have a knowledge of the real stimulus amplitude distribution, (*ii*) to assess this knowledge by means of a metric and, if possible, (*iii*) to use this knowledge in an *a posteriori* correction of an ADC characterization, so as to suppress the stimulus distortions. This would allow, for instance, the use of lower grade, cheaper, signal generators. If properly exploited, the knowledge of the stimulus amplitude distribution should also allow (*i*) the assessment of its sustainability throughout the bandwith of interest and (*ii*) to set a nonlinearity metric for stochastic signals. The use of Power Spectral Density (PSD) related metrics to ascertain the quality of a given stimulus in the histogram method is not appropriate since (*i*) it is a bit cumbersome for frequency rich signals and (*ii*) does not account for the known sensitivity of this method to the phase at time zero of the harmonic distortion [2, 4].

In this paper we will layout the means to describe memoryless nonlinearities with the dual goal of accessing the stimulus adequacy for ADC characterization, and to correct *a posteriori* the stimulus distortion, without masking the ADC nonlinearities. Our approach to the aforementioned problem can be stated, in a simplistic way, as being a four stage endeavor,

1. Measurement of stimulus amplitude distribution.
2. Establishment of a parametric model to describe the nonlinearities.

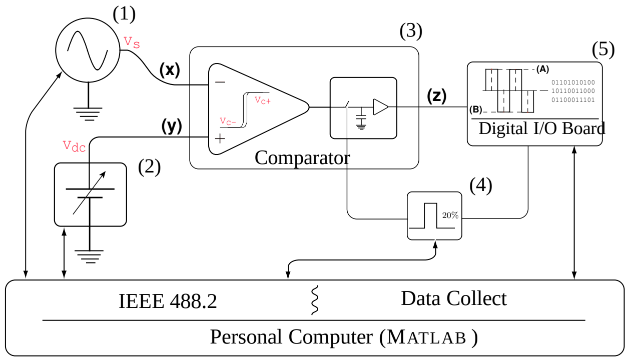


Fig. 1. PDFmeter - experimental setup used in the measurement of the amplitude distribution. (1) Waveform Generator. (2) Wavetek 9100 Calibrator. (3) Ultra High-Speed ECL Comparator. (4) HP3314 Clock Generator. (5) High-Speed Digital I/O Board. This experimental setup is fully automated through a custom developed system in MATLAB .

3. Identification of the model parameters from the amplitude distribution data.
4. Correction of the ADC characterization.

The layout of this paper closely matches this staging.

## II. Amplitude Distribution Measurement

To measure the amplitude distribution we have devised a conceptually very simple experimental setup shown in fig. 1 that, for ease of referencing, we have baptized as PDFmeter [2].The core of this experimental setup is the comparator, the output of which *(3)* [see *(3)* and *(5)* in fig. 1] relates to the amplitude distribution of the generator output *(1)*, $x$, for a given threshold amplitude *(2)*, $y$,

$$F_{\mathbf{x}}(y) = \mathcal{P}\{x < y\}, \tag{1}$$

through,

$$\begin{aligned} \mathcal{P}\{z = A\} = \mathcal{P}\{x < y\} &= \quad F_{\mathbf{x}}(y), \\ \mathcal{P}\{z = B\} = \mathcal{P}\{x > y\} &= 1 - F_{\mathbf{x}}(y). \end{aligned} \tag{2}$$

From these simple probabilities, its straightforward to express both the distribution and density functions of $z$, $F_{\mathbf{z}}$ and $p_{\mathbf{z}}$ respectively. The mean or average of $z$, $\mu_{\mathbf{z}}$, and the variance, $\sigma^2_{\mathbf{z}}$ are now also straightforward to compute,

$$\begin{aligned} \mu_{\mathbf{z}} &= \mathcal{E}\{z\} = F_{\mathbf{x}}(y)(A - B) + B, \\ \sigma^2_{\mathbf{z}} &= F_{\mathbf{x}}(y)(1 - F_{\mathbf{x}}(y))(A - B)^2. \end{aligned} \tag{3}$$

It is easy to perceive from (3) that the average of the output from the comparator, for a given reference voltage, $y$, gives the distribution of $x$ scaled by a known factor. Assuming $x$ to be a strict sense stationary and independent (SSSI) process, as long the comparator does not exhibit memory and the clock frequency *(4)* is not coherent with the frequency of the stimulus *(1)*, the output $z$ will also be SSSI. In this context we can further assess the estimates for $\mu_{\bar{\mathbf{z}}}$ and $\sigma^2_{\bar{\mathbf{z}}}$, where $\bar{z}$ is the point estimate of $\mu_{\mathbf{z}}$,

$$\mu_{\bar{\mathbf{z}}} = \frac{1}{n}\sum_{l=1}^{n} \mathcal{E}\{z_l\} = \mu_{\mathbf{z}}, \tag{4}$$

$$\sigma^2_{\bar{\mathbf{z}}} = \mathcal{E}\left\{(\bar{z}-\mu_{\mathbf{z}})^2\right\} = \frac{\sigma^2_{\mathbf{z}}}{n}. \tag{5}$$

Now we can compute the required number of samples for a given tolerance interval with a certain confidence, $\gamma$, that is, find $n$ such that,

$$\mathcal{P}\left\{F_{\mathbf{x}}(y) - \Xi_{F_{\mathbf{x}}} < \hat{F}_{\mathbf{x}}(y) < F_{\mathbf{y}}(x) + \Xi_{F_{\mathbf{x}}}\right\} > \gamma, \tag{6}$$

where $\Xi_{F_{\mathbf{x}}}$ is the absolute tolerance for $F_{\mathbf{x}}(y)$ and $\hat{F}_{\mathbf{x}}(y)$ stems directly from (3) by replacing the true average by the point estimator,

$$\hat{F}_{\mathbf{x}}(y) = \frac{\bar{\mathbf{z}} - B}{A - B}. \tag{7}$$

It can be shown that to satisfy (6),

$$n > \frac{2F_{\mathbf{x}}(y)\left(1 - F_{\mathbf{x}}(y)\right)\left[\mathrm{erf}^{-1}(\gamma)\right]^2}{\Xi^2_{F_{\mathbf{x}}}}. \tag{8}$$

With the setup shown in Fig. 1, the sample vector gathered by the acquisition channel, $z[\,]$, is pre-processed in the following manner,

$$\begin{aligned} l = 1 \ldots \mathrm{n}, \quad & z[l] > \frac{A+B}{2} \Rightarrow z[l] := 1, \\ & z[l] < \frac{A+B}{2} \Rightarrow z[l] := 0. \end{aligned} \tag{9}$$

In this way we reduce the effects of limited slewrate in the comparator, suppress resolution limitations and simplify the determination of $F_{\mathbf{x}}[y]$ through (3),

$$F_{\mathbf{x}}(y) = \frac{\mu_{\mathbf{z}} - B}{A - B} \quad \xrightarrow[\bar{z} \sim \mu_z]{A=1,\, B=0,} \quad F_{\hat{\mathbf{x}}}(y_i) = \bar{z}_i, \tag{10}$$

where $F_{\hat{\mathbf{x}}}(y_i)$ is the experimental distribution of the stimulus $x$ evaluated at the i$^{th}$ step of the calibrator. The experimental density, $p_{\hat{\mathbf{x}}}$, can be found through numerical differentiation,

$$p_{\hat{\mathbf{x}}}\left(\frac{y_{(i+1)} + y_i}{2}\right) = \frac{F_{\hat{\mathbf{x}}}(y_{i+1}) - F_{\hat{\mathbf{x}}}(y_i)}{y_{(i+1)} - y_i}. \tag{11}$$

## III. Linear and Nonlinear Measurements Quantification, Spectral Representation

We need now to choose a parametric model for the nonlinearities of the stimulus. To this end we will consider both a power series representation and one based on orthogonal polynomials, conceiving the stimulus generator under scrutiny to be the cascading of an ideal, conceptual, source, followed by a distortion circuitry. The output of the generator, $y$, can thus be expressed as,

$$y = \mathrm{f}(x) = \sum_{m=0}^{\infty} b_m x^m = \check{\mathrm{f}}(x) = \sum_{m=0}^{\infty} \check{b}_m\, \phi_m(x), \tag{12}$$

where $b_m$ are the coefficients of the power series description, $\check{b}_m$ the weights of the orthogonal representation and $\{\phi_m\}$ the orthogonal base. The chosen orthogonal polynomial is a function of the stimulus: Chebyshev polynomials of the first kind, $\phi_m \equiv T_m$, for the sine wave, Hermite polynomials, $\phi_m \equiv H_m$, for the gaussian noise and Legendre polynomials, $\phi_m \equiv P_m$, for the triangular stimulus. To quantify these errors we must analytically be able to describe the distribution of the distorted signal, $y$, in terms of the distribution of the ideal, $x$. It can be shown that for *small* nonlinearities the distribution of $y$ reduces to,

$$F_{\mathbf{y}}(a) = F_{\mathbf{x}}\left(\mathrm{f}^{-1}(a)\right), \tag{13}$$

where $a$ is a transition level and $\mathrm{f}^{-1}$ is the inverse of $\mathrm{f}(x)$, which naturally may also be described through a series,

$$x = \mathrm{f}^{-1}(y) = \sum_{m=0}^{+\infty} c_m y^m = \check{\mathrm{f}}^{-1}(y) = \sum_{m=0}^{+\infty} \check{c}_m \phi_m(y). \tag{14}$$

Solving for the inverse nonlinear model in (13) we get,

$$\mathrm{f}^{-1}(y) = r_{?}\left(F_{\mathbf{y}}(y)\right), \tag{15}$$

where $r_{?}$ stands for the analytical inverse of $F_{\mathbf{x}}$ defined, for the three aforementioned stimulus, by,

(sine wave)

$$\mathrm{f}^{-1}(y) = \mathrm{r}_s\left(F_{\mathbf{y}}(y)\right) = -A\cos\left(\pi F_{\mathbf{y}}(y)\right) + O, \tag{16}$$

(triangular wave)

$$\mathrm{f}^{-1}(y) = \mathrm{r}_t\left(F_{\mathbf{y}}(y)\right) = 2A\left(F_{\mathbf{y}}(y) - \frac{1}{2}\right) + O, \tag{17}$$

(gaussian noise)

$$\mathrm{f}^{-1}(y) = \mathrm{r}_g\left(F_{\mathbf{y}}(y)\right) = \sqrt{2\sigma^2_{\mathbf{x}}}\,\mathrm{erf}^{-1}\left(2F_{\mathbf{y}} - 1\right) + \mu_{\mathbf{x}}, \tag{18}$$

in which $A$ stands for the amplitude, $O$ and $\mu_{\mathbf{x}}$ for the offset and $\sigma^2_{\mathbf{x}}$ for the power of the stimulus signal, $x$.

As we saw in the previous section, using the PDFmeter we may acquire an experimental estimate of $F_{\mathbf{y}}$, $\hat{F}_{\mathbf{y}}$. Considering a power series limited to the $s^{\mathrm{th}}$ order, from this measurement we can estimate the coefficients, $c_0 \ldots c_s$, of the inverse nonlinear model (14), by means of a linear, in the coefficients, regression of the data, (see also (15)),

$$r_{?}\left(\underset{\sim}{\hat{F}}_{\mathbf{y}}\right) = \underset{\sim}{Y} \cdot \underset{\sim}{c} + \underset{\sim}{\varepsilon} \quad \vee \quad r_{?}\left(\underset{\sim}{\hat{F}}_{\mathbf{y}}\right) = \check{\underset{\sim}{Y}} \cdot \check{\underset{\sim}{c}} + \check{\underset{\sim}{\varepsilon}} \tag{19}$$

where,

$$\underset{\sim}{c} = \begin{bmatrix} c_0 & \ldots & c_s \end{bmatrix}^T,\ \check{\underset{\sim}{c}} = \begin{bmatrix} \check{c}_0 & \ldots & \check{c}_s \end{bmatrix}^T,\ n = \dim(\underset{\sim}{\hat{F}}_{\mathbf{y}}), \tag{20}$$

$$\underset{\sim}{Y} = \begin{bmatrix} 1 & y_1 & \cdots & y_1^s \\ \vdots & \vdots & \vdots & \vdots \\ 1 & y_n & \cdots & y_n^s \end{bmatrix},\ \check{\underset{\sim}{Y}} = \begin{bmatrix} 1 & \phi_1(y_1) & \cdots & \phi_s(y_1) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \phi_1(y_n) & \cdots & \phi_s(y_n) \end{bmatrix}. \tag{21}$$

Solving (19) through standard regression analysis means we get,

$$\hat{\underset{\sim}{c}} = (\underset{\sim}{Y}^T\underset{\sim}{Y})^{-1}\underset{\sim}{Y}^T \cdot r_{?}(\underset{\sim}{\hat{F}}_{\mathbf{y}}) \vee \hat{\check{\underset{\sim}{c}}} = (\check{\underset{\sim}{Y}}^T\check{\underset{\sim}{Y}})^{-1}\check{\underset{\sim}{Y}}^T \cdot r_{?}(\underset{\sim}{\hat{F}}_{\mathbf{y}}). \tag{22}$$

It is very important to note that these coefficients might be used directly, through (14), to correct the stimulus distortion in a time or frequency domain ADC characterization. In a similar way, a communication channel may also be compensated for its transmission nonlinearities. However, we are also interested in using an r$^{\mathrm{th}}$ order series both to describe the memoryless nonlinear model of the stimulus and to use it in stochastic ADC characterization methods. In both cases we need to obtain the direct nonlinear model, $b_0 \ldots b_r$ or $\check{b}_0 \ldots \check{b}_r$, (12), which requires a series inversion. For the power series this inversion can be conveniently accomplished through [5],

$$\underset{(s+1,1)}{\underset{\sim}{p}} = \underset{(s+1,r+1)}{\underset{\sim}{C}^{-1}} \cdot \underset{(r+1,1)}{\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T}, \tag{23}$$

| | | |
|---|---|---|
| $x^n = \frac{n!}{2^{n/2}} \sum_{k=0}^{[n/2]} \frac{1}{k!(n-2k)!} H_{n-2k}(x/\sqrt{2})$ | $-\infty < x < \infty$ | [8] |
| $x^n = \frac{1}{2^{n-1}} \sum_{k=0}^{[n/2]} \binom{n}{k} T_{n-2k}(x) - \frac{1}{2^n}$ | $-1 \leq x \leq 1$ | [9] |
| $x^n = \frac{n!}{2^n} \sum_{k=0}^{[n/2]} \frac{n+1/2-2k}{k!\left(\frac{1}{2}\right)_{n+1-k}} P_{n-2k}(x)$ | $-1 \leq x \leq 1$ | [8] |

$(z)_n$ - Pochhammer factorial, $[n/2]$ - largest integer smaller or equal to $n/2$

TABLE I

EXPRESSIONS TO OBTAIN ELEMENTS OF CONVERSION MATRIX, $\utilde{\psi}$.

where,

$$\utilde{C} = \left[\utilde{c}^{(*0)} \quad \utilde{c}^{(*1)} \quad \cdots \quad \utilde{c}^{(*r)}\right], \tag{24}$$

$$\utilde{c}^{(*n)} = \left[1 \quad c_1^{(*n)} \quad \cdots \quad c_{s\cdot n}^{(*n)} \quad \underset{(1,s(r-n))}{\utilde{0}}\right], \tag{25}$$

and [6],

$$c_k^{(*n)} = \begin{cases} 1 & \Leftarrow k = 0 \\ \sum_{l=1}^{k} \left(\left(\frac{n+1}{k}\right) l - 1\right) c_l \, c_{k-l}^{(*n)} & \Leftarrow k > 0 \end{cases}. \tag{26}$$

To invert the orthogonal representation, we first need to convert it into a power series through,

$$\check{\utilde{c}} = \utilde{\psi}\, \utilde{Q}_x \, \utilde{c}, \tag{27}$$

where $\utilde{\psi}$ is the conversion matrix between representations, whose column $n$ corresponds to the orthogonal description of $x^n$,

$$\utilde{\psi}_n \rightsquigarrow \utilde{\psi}_{0n} \phi_0(x) + \cdots + \utilde{\psi}_{nn} \phi_n(x) = x^n, \tag{28}$$

numerically derivable through table I. In (27), $\utilde{Q}_x$ is a normalization matrix required to guarantee the argument of the orthogonal polynomials remains within it's orthogonality domain. The $n^{\text{th}}$ column of $\utilde{Q}_x$, $\utilde{Q}_{xn}$, corresponds to the coefficients of the polynomial,

$$\utilde{Q}_{xn} \rightsquigarrow (A_x\, v + O_x)^n, \tag{29}$$

where $A_x \equiv \sigma_{\mathbf{x}}$ is the stimulus amplitude or standard deviation, $O_x \equiv \mu_{\mathbf{x}}$ its offset or mean, and $v$ just a dummy variable. We need also the reverse direction, the conversion of a power series into a orthogonal representation. This can be achieved with,

$$\utilde{b} = \utilde{Q}_y^{-1} \; \utilde{\psi}^{-1} \; \check{\utilde{b}}, \tag{30}$$

where the columns of $\utilde{Q}_y^{-1}$, $\utilde{Q}_{yn}^{-1}$, are given by,

$$\utilde{Q}_{yn}^{-1} \rightsquigarrow \left(\frac{v - O_y}{A_y}\right)^n, \tag{31}$$

being $v$ a dummy variable, and the columns of $\utilde{\psi}^{-1}$, $\utilde{\psi}_n^{-1}$, correspond to the decomposition of $\phi_n(x)$ into a power series,

$$\utilde{\psi}_n^{-1} \rightsquigarrow \utilde{\psi}_{0n}^{-1} x^0 + \cdots + \utilde{\psi}_{nn}^{-1} x^n = \phi_n(x). \tag{32}$$

The coefficients, $\utilde{\psi}_{mn}^{-1}$, can be numerically derived using tableb II together with the following recurrent equation [7],

$$a_{1n}\, \phi_{n+1}(x) = a_{2n}\, x\, \phi_n(x) - a_{3n}\, \phi_{n-1}(x). \tag{33}$$

Putting it all together, it can be shown that orthogonal series representation, $\check{\utilde{b}}$ is equal to,

$$\check{\utilde{b}} = \check{\utilde{C}}^{-1}\, \check{\utilde{w}}, \qquad \underset{(s+1,\, s+1)}{\check{\utilde{C}}} = \left(\utilde{\psi}\, \utilde{Q}_x \, \utilde{C}\, \utilde{Q}_y^{-1}\, \utilde{\psi}^{-1}\right), \tag{34}$$

| $\phi_n(x)$ | $a_{1n}$ | $a_{2n}$ | $a_{3n}$ | $\phi_0(x)$ | $\phi_1(x)$ |
|---|---|---|---|---|---|
| $H_n(x/\sqrt{2})$ | 1 | $\sqrt{2}$ | $2\,n$ | 1 | $\sqrt{2}\,x$ |
| $T_n(x)$ | 1 | 2 | 1 | 1 | $x$ |
| $P_n(x)$ | $n+1$ | $2\,n+1$ | $n$ | 1 | $x$ |

TABLE II

COEFFICIENTS CONNECTED WITH THE RECURRENT EQUATION 33.

with $\utilde{C}$ given by (24), requiring previously, $\check{\utilde{c}} \xrightarrow{(27)} \utilde{c}$, and,

$$\begin{aligned} \phi_n(x) \to \{T_n(x); P_n(x)\} &\Rightarrow \underset{(s+1,\,1)}{\check{\utilde{w}}} = \left[O_x \quad A_x \quad 0_x \quad \cdots \quad 0\right]^T, \\ \phi_n(x) \to \{H_n(x)\} &\Rightarrow \underset{(s+1,\,1)}{\check{\utilde{w}}} = \left[O_x \quad A_x/\sqrt{2} \quad 0 \quad \cdots \quad 0\right]^T. \end{aligned} \tag{35}$$

This method of orthogonal inversion differs in key points from [5], and should be preferred when establishing the nonlinearity model.

When the bidimensional density function of the stimulus can be expressed in the diagonal form [10],

$$p_{\mathbf{x},\mathbf{x_d}}(\tau) = p_{\mathbf{x}}(x) p_{\mathbf{x_d}}(x_d) \sum_{m=0}^{\infty} \frac{a_m(\tau)}{\sigma_m^2}\, \phi_m(x)\, \phi_m(x_d), \tag{36}$$

with $x_d$ being a delayed replica of $x$, $p_{\mathbf{x}}$ the first-order distribution of $x$, $a_m$ the autocorrelation of the orthogonal base, $a_m = \mathcal{E}\{\phi_m(x)\phi_m(x_d)\}$ and $\sigma_m$ the base normalization coefficient, $\sigma_m = \mathcal{E}\left\{\phi_m(x)^2\right\}$, then the power spectral density of the distorted signal, $S_y$, can be expressed through,

$$S_y(f) = \sum_{m=0}^{\infty} b_m^2\, \sigma_m^2\, \mathcal{F}\{a_m(\tau)\}. \tag{37}$$

In [10] was shown that (36) applies to the sine wave and to the gaussian noise. It can be shown that its not the case of the triangular wave. Nevertheless, the error made in assuming it to be diagonal is small enough to be neglected. In table III we give the Fourier transforms of $a_n$ for the sinusoidal and gaussian stimulus, and the first 4 terms for the triangular wave. Eq. (37) together with table III allow for the spectral description of the distorted signal considering an orthogonal representation. By converting the power series representation to the corresponding orthogonal representation the same equation can be used in association with the power series. It can be shown that when correcting a time or frequency domain characterization using the method proposed in this paper, there is no significant difference between the power series and orthogonal representations. However, when establishing the nonlinearities model or compensating a stochastic ADC characterization method, the orthogonal representation provides significant improvements.

| $\phi_n$ | $\sigma_n^2$ | $\hat{a}_n(f) = \mathcal{F}\{a_n(\tau)\}$ |
|---|---|---|
| $T_n$ | $\frac{1}{2} \Leftarrow n \neq 0$<br>$1 \Leftarrow n = 0$ | $\frac{1}{2}\left[\delta_d\left(f + n\, f_s\right) + \delta_d\left(f - n\, f_s\right)\right]$ |
| $H_n$ | $n!2^n$ | $\hat{a}_n(f) = \frac{S_x \star \overset{\cdot n}{\cdots} \star S_x}{\sigma_{\mathbf{x}}^{2n}} = \frac{S_x^{(*n)}}{\sigma_{\mathbf{x}}^{2n}}$, $S_x = \mathcal{F}\{R_x(\tau)\}$ |
| $P_n$ | $\frac{1}{2n+1}$ | $\hat{a}_1(f) = \frac{48}{\pi^4} \sum_{k=-\infty}^{\infty} (2k+1)^{-4} \delta_d\left(f + (2k+1)\, f_s\right)$<br>$\hat{a}_2(f) = \frac{45}{\pi^4} \sum_{k=-\infty}^{\infty} k^{-4} \delta_d\left(f + 2k\, f_s\right)$<br>$\hat{a}_3(f) = \frac{4032}{\pi^8} \sum_{k=-\infty}^{\infty} \frac{(10-((2k+1)\pi)^2)^2}{(2k+1)^8} \delta_d\left(f + (2k+1)\, f_s\right)$<br>$\hat{a}_4(f) = \frac{225}{\pi^8} \sum_{k=-\infty}^{\infty} \frac{(21-(2k\pi)^2)^2}{k^8} \delta_d\left(f + 2k\, f_s\right)$ |

TABLE III

FOURIER TRANSFORMS OF $a_n$ FOR THE 3 STIMULUS.

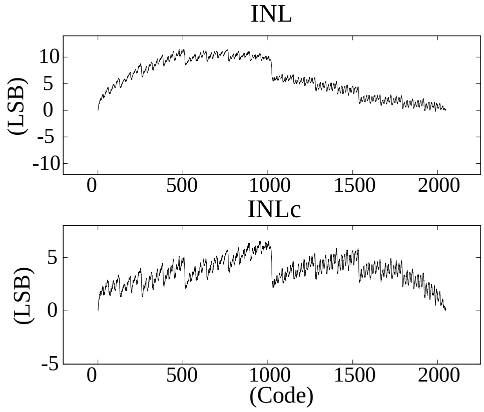


Fig. 2. Integral nonlinearity vector (INL) of a VX4240 VXI 12 bit acquisition channel using a HP3310 waveform generator (triangular output). Below, the *a posteriori*corrected INL (INLc) having the generator distortion supressed.

## IV. Experimental histogram correction

The removal of the stimulus nonlinearities in the histogram method can be accomplished by measuring the nonlinear coefficients of the series (14) associated with the generator, through the aforementioned experimental setup (PDFmeter). After gathering the cumulative histogram with the standard procedure, it's enough to 'subtract' the nonlinear behavior of the generator to obtain the nonlinearity of the DUT. Let us set, as before, $F_y$ to be the amplitude distribution of the stimulus signal, $F_x$ the amplitude distribution of the ideal, undistorted stimulus and $\mathrm{r}_? = F_{\mathbf{x}}^{-1}$ its inverse function. In a histogram test we have the following relation between the cumulative histogram at bin $i$, $H_c(i)$, and $F_{\mathbf{y}}$,

$$H_c(i) = F_{\mathbf{y}}(T[i+1]), \tag{38}$$

where $T[i+1]$ stands for the upper transition level for bin $i$. Using (13) and (38) we have the means to correct the histogram,

$$\begin{aligned} F_{\mathbf{y}}(T[i+1]) = F_{\mathbf{x}}(\mathrm{f}^{-1}(T[i+1])) \Leftrightarrow \\ T[i+1] = \mathrm{f}(F_{\mathbf{x}}^{-1}(H_c(i))) = \mathrm{f}(\mathrm{r}_?(H_c(i)). \end{aligned} \tag{39}$$

We have applied this idea to the characterization of a VX4240, VXI 12 bit acquisition channel, using the triangular output of a HP3310A generator. Employing the PDFmeter we measured, for the HP3310 with an amplitude of $3\,\mathrm{V}$ and a frequency of $1126\,\mathrm{Hz}$,

$b_2 = -8.86^{-4}$(dist. aprox. equ. to $-\ 57.5\,\mathrm{dBc}$ $\ 2^{nd}$ harm.),
$b_3 = 10.80^{-4}$ (dist. aprox. equ. to $-\ 52.1\,\mathrm{dBc}$ $\ 3^{rd}$ harm.).

Afterwards we applied the generator to the VX4240 and compiled an histogram with a total of 1.27 million samples at a rate of $10\,\mathrm{Msps}$. Both the resulting integral nonlinearity vector (INL), as well as the corrected (INLc), can be seen in Fig. 2. The difference of the corrected INL to the true INL as measured by a Wavetek M39 is very small, lower than 0.4LSB, as can be perceived from Fig. 3. It is also important to point out that this error vector does not exhibit beating. Extensive characterization done through static and quasi-static methods [11] show good agreement with these results.

## V. Conclusions

In this paper we have presented a simple and cost effective setup for measuring the amplitude distribution and the

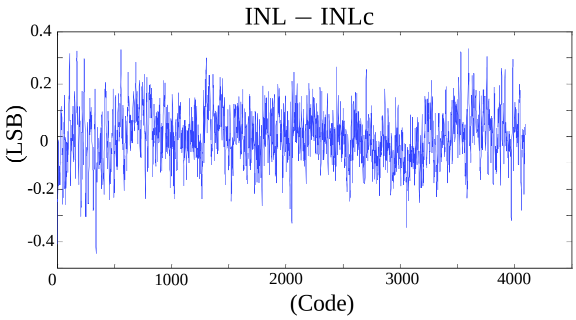


Fig. 3. The difference between the compensated INL vector of fig. 2 and the reference, taken with an Wavetek M39.

*pdf* of a stimulus signal. From these it was shown how to measure linear and nonlinear errors through a power series or orthogonal representation of the distorted signal. We have also shown how this representation could be used as an horizontal metric for adequacy of the amplitude distribution in the histogram method and how to derive the PSD of the stimulus. One important application, the correction of nonlinearities in the stimulus signal using the histogram method, was presented, together with experimental results. This application configures the possibility of using cheaper and more easily obtainable generators as well as the use of triangular waves which have been consistently put aside due to the difficulty in generating them with the required linearity.

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