EVALUATING MEASUREMENT UNCERTAINTY IN A/D CONVERTERS WITH AND WITHOUT DITHER

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Abstract – ***The paper gives formulae for evaluating the uncertainty of measurements (both "direct" and "indirect", i.e. functions of measurements) performed through an ADC-based device (like an oscilloscope, a plug-in DAQ board, etc.). The way of modeling the errors and writing the specifications is discussed. The mathematically complex but nowadays common technique of dithering is also analyzed. The results are intended as a contribution towards a standard way of writing and using ADC specifications.***

Keywords: measurement uncertainty, measurement error, A/D converters.

1. Introduction

Instruments based on an analog-to-digital converter (ADC) and subsequent processing of the digitized data are, of course, very powerful and flexible, and nowadays they are probably the most popular solution for many typical measurement needs. Of widespread use are, for example, data acquisition boards providing easy connection with personal computers and sophisticated built-in technologies like auto-calibration and dithering.

Working out this kind of measurement systems is often simple and straightforward, but certainly not is evaluating the uncertainty of the obtained measurements. The main problem lies in the *interpretation* and *use* of the ADC metrological specifications, which are written in varied and ambiguous ways, and are often misunderstood (or at least understood in different ways) also by "specialists". As a consequence, the authors are not sure that everybody calculates the measurement uncertainty of a given commercial ADC-based instrument in the same way. It must be said that international standards, both more general about the problem of uncertainty evaluation (e.g. the ISO Guide [1]), and more specific about ADCs and their errors (e.g. the IEEE Standards [2], [3]) are of very little help in solving this problem.

The aim of the present paper is contributing to reach a standard way of writing and using specifications of ADC-based instruments. To this purpose, the authors consider a simple, but often accurate enough, ADC error model (Section 2), define the ADC uncertainty specifications, based on the model, in a univocal and operational way (Section 3), and evaluate the uncertainty of a very general class of measurements in the same operational way (Section 4). Many of the given formulae do not involve theoretical novelties about uncertainty evaluation or ADC characterization; nevertheless, they are not obvious and cannot be found in any Guide, Standard or textbook known to the authors. Besides, new formulae are presented for the case (mathematically difficult [4], [5], but nowadays very common in practice [6]) of ADC *with dither*.

2. ADC error model

An ideal ADC is described by a perfectly linear quantization characteristic, $quant(x)$, like the one represented in Fig. 1.

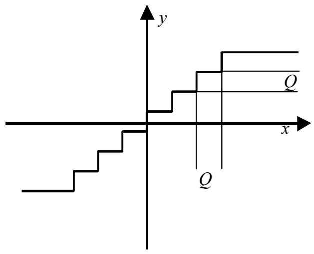

Fig. 1. – The ideal quantization $quant(x)$.

A real-world ADC, obviously, differs quite a lot from this ideal device. The error model considered in this paper is pretty simple (Fig. 2).

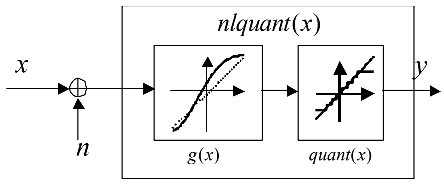

Fig. 2 – ADC error model

The analog voltage *x* at the input is added to a noise voltage *n*, and the result $x+n$ goes through the function $nlquant(x)$. This is the well-known nonlinear quantization staircase, which can be seen as a $quant(x)$ cascaded with a continuous nonlinear function $g(x)$, as illustrated in Fig. 3. The $g(x)$ function can be furtherly decomposed in the sum

$$g(x) = Gx + O + inl(x) = x + \Delta Gx + O + inl(x) \quad (1)$$

being ΔG the *gain error*, O the *offset error*, $inl(x)$ the *integral nonlinearity error*.

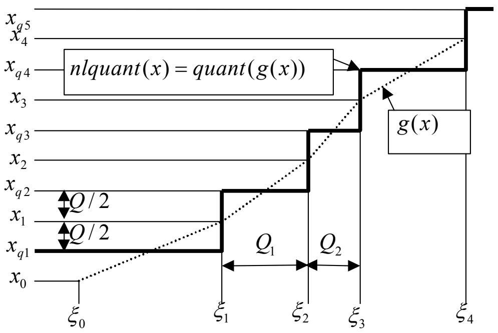

Fig. 3. – The nonlinear quantization staircase $nlquant(x)$ *, as a cascade of the functions* $g(x)$ *and* $quant(x)$ *.* ξ_k *are the actual threshold levels,* x_k *the ideal threshold levels,* x_{qk} *the quantized outputs,* Q_k *the actual quantization steps. The function* $g(x)$ *connects the points with coordinates* (ξ_k, x_k) *.*

The described error model is very simple and does not take into account the following important phenomena:

- linear dynamic transformations of the input signal (like those due to the finite bandwidth of the instrument);
- nonlinear dynamic transformations of the input signal (which cause a typical THD increase at higher input frequencies);
- timebase errors (both systematic, like a fixed error in the sampling frequency, and random, like the aperture uncertainty or the jitter in the sample clock signal).

The model can be confidently used, however, in the common case of input signal with *slow variations* with respect to the aperture time and the bandwidth of the ADC. Even with this model, in fact, the problem of uncertainty evaluation can be not completely trivial.

3. Uncertainty specifications

The sole exactly known parameter of a real ADC is the quantization step Q (the difference between consecutive digital output levels), which characterizes the ideal quantization block. The other error parameters, i.e. the actual values of n, ΔG , O, and $inl(x)$ are instead unknown. The manufacturer cannot know and provide these values, but can assure something about their *distributions*.

It is worth to put in some words about the "distributions" of fixed parameters like ΔG and O. The ISO Guide [1] employs the notion of "subjective probability", which allows different "subjects" to prefer different figures, on the basis of different belief about the quality of the hardware. We try to define these distributions in a more operational and "engineer-like" way.

The "distribution of ΔG " is, in the following, the one obtained by considering *all the ADCs of the same kind from the same manufacturer, correctly maintained and employed*. This means that the distribution (that the manufacturer can obtain from actual tests on the converters, from knowledge about the tolerances of the manufacturing process, or both) yields the probability that the measurements will be prone to a gain error in a given range; or even more operationally, the percentage of ADCs with gain errors in a given range. Of course, if all the manufacturers write in the same way the specifications, the distribution can be referred to *the population of all the ADCs with the same metrological specifications*.

We define in the same way the distributions of the other fixed parameters, i.e. O and $inl(x)$, with the quite reasonable additional assumption that the latter does not depend meaningfully on x. As regards the noise n, we can simply employ the usual notion of distribution of a stationary and ergodic process.

As a final needed hypothesis, we can suppose all the distributions to be zero-mean and *independent each other*. This is quite reasonable considering the nature of the considered error parameters: knowing the actual value of the gain error ΔG affecting a measurement can hardly provide information about the actual values assumed by O , $inl(x)$ or n in the same measurement.

Having clearly defined the involved quantities, we can give now the *set of uncertainty specifications* we will use in the following. We assume to know first of all the *worst-case uncertainties* U_G , U_O , U_{inl} , defined by (when applicable):

$$|\Delta G| \le U_G ,\ |O| \le U_O ,\ |inl(x)| \le U_{inl} , \tag{2}$$

and secondly the *standard uncertainties* given by

$$\sigma_G = \sqrt{E[\Delta G^2]} \qquad \sigma_O = \sqrt{E[O^2]} ,$$
$$\sigma_{inl} = \sqrt{E[inl(x)^2]} \quad \sigma_n = \sqrt{E[n^2]} . \tag{3}$$

The noise is assumed to be Gaussian and therefore without a finite worst-case uncertainty. When the distributions of ΔG , O, and $inl(x)$ can (or must) be considered uniform in $\pm U_G$, $\pm U_O$, $\pm U_{inl}$, the relevant standard uncertainties are of course

$$\sigma_G = U_G / \sqrt{3} ,\ \sigma_O = U_O / \sqrt{3} ,\ \sigma_{inl} = U_{inl} / \sqrt{3} . \tag{4}$$

We introduce, finally, the symbols

$$U_q = Q/2; \quad \sigma_q = Q/\sqrt{12} \tag{5}$$

which will be used in the following and are, under proper assumptions, the worst-case and the standard uncertainty relevant to the ideal quantization error.

4. Formulae for evaluating uncertainty of measurements

In order to provide formulae for the evaluation of uncertainty, we adopt an operational viewpoint, in the same spirit of the definitions of the uncertainty parameters in Section 3.

Consider a particular measurement result *y*, and the distribution of the measurement error $e = y - x$, evaluated for *all the possible measurements yielding the result y*. "All the possible measurements" refers to the set of "all the ADCs with the same metrological specifications" (section 3), and to all the input voltages *x* which can give the outcome *y*. It must be noted that this is exactly the situation of an ADC user who obtains the measurement *y* without having any prior information about the measurand or the converter – apart from the provided specifications.

We consider the following kinds of measurement:

A) Measurements with no (or negligible) noise
1) single or "direct" measurement y
2) "indirect" measurement $y = g(y_1, \ldots, y_n)$

B) Measurements with noise, but without averaging
1) single or "direct" measurement y
2) "indirect" measurement $y = g(y_1, \ldots, y_n)$

C) Measurements with noise, and with averaging
1) single or "direct" measurement y
2) "indirect" measurement $y = g(y_1, \ldots, y_n)$

For all the measurements listed above, we give formulae for the following quantities, related to the distribution of the measurement error:

$$US(y) = \sqrt{E[e^2]} \qquad (6)$$

$$U(y) = \max(|e|) \qquad \text{(when applicable)} \qquad (7)$$

naming the first "standard uncertainty" (according to the familiar terminology of the ISO Guide) and the second "worst case uncertainty" (this is simply an expanded uncertainty with 100% confidence level). Of course the latter is applicable only to case *A* (no noise).

Having clearly defined the involved quantities, deriving the uncertainty for the group *A* and *B* is pretty straightforward. As well as the uncertainty parameters defined in section 3, formulae relevant to indirect measurements $g(y_1, \ldots, y_n)$ requires the usual "sensitivity coefficients"

$$k_i = \partial g(y_1, \ldots, y_n) / \partial y_i \qquad (8)$$

Firstly we write down the formulae for the cases *A* and *B*. The error affecting a single measurement y_i is

$$e_i \cong n_i + \Delta G y_i + O + inl(x_i + n_i) + eq(g(x_i + n_i)) \qquad (9)$$

where x_i is the measurand and n_i is the value assumed by the noise in the measurement (we have approximated $\Delta G(x_i + n) \cong \Delta G y_i$, neglecting a second-order term). From simple calculations we obtain:

A1)

$$US(y) \cong \sqrt{\sigma_G^2 y^2 + \sigma_O^2 + \sigma_{inl}^2 + \sigma_q^2} \qquad (10)$$

$$U(y) \cong U_G\,|y| + U_O + U_{inl} + U_q \qquad (11)$$

A2)

$$US(y) \cong \sqrt{\sigma_G^2 \left(\sum_{i=1}^{n} k_i y_i\right)^2 + \sigma_O^2 \left(\sum_{i=1}^{n} k_i\right)^2 + \left(\sigma_{inl}^2 + \sigma_q^2\right) \sum_{i=1}^{n} k_i^2} \qquad (12)$$

$$U(y) \cong U_G \left|\sum_{i=1}^{n} k_i y_i\right| + U_O \left|\sum_{i=1}^{n} k_i\right| + \left(U_{inl} + U_q\right) \sum_{i=1}^{n} |k_i| \qquad (13)$$

B1)

$$US(y) \cong \sqrt{\sigma_G^2 y^2 + \sigma_O^2 + \sigma_{inl}^2 + \sigma_q^2 + \sigma_n^2} \qquad (14)$$

B2)

$$US(y) \cong \sqrt{\sigma_G^2 \left(\sum_{i=1}^{n} k_i y_i\right)^2 + \sigma_O^2 \left(\sum_{i=1}^{n} k_i\right)^2 + \left(\sigma_{inl}^2 + \sigma_q^2 + \sigma_n^2\right) \sum_{i=1}^{n} k_i^2} \qquad (15)$$

It should be highlighted that the formulae relevant to indirect measurements ("combined" uncertainties) take into account the correlation between errors in a very simple and elegant way, without explicit evaluation of correlation coefficients. The multipliers of σ_G^2 and σ_O^2 quantify, in a very expressive way, the role of gain and offset errors in a given measurement: it is immediately recognized, for example, that a measurement of the kind $y = y_2 - y_1$ is exempt from offset errors, one of the kind $y = y_2 / y_1$ is exempt from gain errors, and one of the kind $y = (y_4 - y_3)/(y_2 - y_1)$ from both. The formulae for case B (which includes noise, but without averaging) are a trivial extension of those for case A.

Case C is a quite different matter, because averaging results in a *non-subtractive dithering* of the converter. In other words, averages not only reduce the effect of noise, but also *increase the ADC resolution*. In case C1) we suppose that the result *y* is obtained by averaging *m* measurements with the same input voltage *x*; in case C2), similarly, each term y_i is obtained by averaging m_i measurements with the same input voltage x_i, and then the final result $y = g(y_1, \ldots, y_n)$ is evaluated.

The measurement uncertainties for these two cases are given by the approximate formulae below, reported without a detailed proof for the sake of conciseness. They are obtained by considering that averaging m_i

measurements: (i) divides by m_i^2 the noise and the *random portion* of the quantization error; (ii) leaves unchanged the *deterministic portion* of the quantization error, which is derived by the convolution of $quant(x)-x$ with the probability density function of the noise. Here are the formulae:

C1)

$$US(y) \cong \sqrt{\sigma_G^2 y^2 + \sigma_O^2 + \sigma_{inl}^2 + \sigma_{qd}^2 + \frac{\sigma_n^2 + \sigma_q^2 - \sigma_{qd}^2}{m^2}} \quad (16)$$

C2)

$$US(y) \cong \left[\sigma_G^2 \left(\sum_{i=1}^{n} k_i y_i \right)^2 + \sigma_O^2 \left(\sum_{i=1}^{n} k_i \right)^2 + \right.$$

$$\left. + \left(\sigma_{inl}^2 + \sigma_{qd}^2 \right) \sum_{i=1}^{n} k_i^2 + \left(\sigma_n^2 + \sigma_q^2 - \sigma_{qd}^2 \right) \sum_{i=1}^{n} \frac{k_i^2}{m_i^2} \right]^{1/2} \quad (17)$$

In both equations the new symbol σ_{qd}^2 denotes the deterministic portion of the quantization error. It can be shown that this term depends only upon the ratio σ_n / Q, i.e., on the rms value of the noise expressed in LSB units. We have evaluated numerically this term, obtaining the curve depicted in Fig. 4.

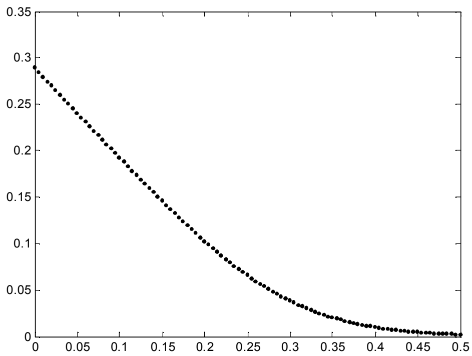

Fig. 4. – Plot of the residual deterministic quantization error σ_{qd} as a function of the input noise σ_n (both are in LSB units).

The curve shows that the deterministic quantization error is (obviously) $\sigma_{qd} = 1/\sqrt{12} = 0.2886$ LSB at $\sigma_n = 0$, and that it is (less obviously) practically nullified for $\sigma_n \geq 0.5$ LSB. This is the usual choice of dither level in DAQ boards, and the curve shows that this is indeed an optimal choice, which fully randomizes the quantization error by adding only the strictly necessary noise.

5. Conclusions

The paper presents formulae for evaluating the uncertainty of measurements performed by means of ADCs affected by gain error, offset error, nonlinearity error, and noise, and distinguishing three cases: negligible noise, noise and no averaging, noise and averaging (dither).

In the first two cases deriving the uncertainty is a simple exercise. For these cases the original contribution does not consist so much in the formulae themselves, but in the adopted scheme of work. The authors, indeed, made an effort to define (and consequently use) in an operational way the uncertainty specifications, using well-known concepts of the ADC technology, and avoiding as much as possible the use of subjective figures or assumptions. Besides being of practical use, the obtained equations take into account the correlation between errors in a nice form, that makes intuitive the different role of gain, offset, and nonlinearity errors in an “indirect” measurement.

The third analyzed case is a bit more difficult. The authors have derived new approximate equations to evaluate the uncertainty, introducing the concept of *residual deterministic portion of the quantization error*.

Another implicit result of the work is that a way of writing and interpreting the uncertainty specifications of ADCs – at least as regards *static errors and amplitude noise* – is suggested. It appears clearly counterproductive (and maybe misleading) to “aggregate” gain, offset, nonlinearity, and quantization errors in one or two figures [7], [8]. Likewise, it appears of little use to specify the amount of quantization + noise uncertainty for a given number of averages, like e.g. in [6].

In the authors’ view, it could be worth considering the inclusion of the provided formulae (and maybe also of the underlying conceptual scheme) in future editions of a pertinent Standard, like [1], [2], [3], or [9].

References

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