EXPERIMENTAL VALIDATION OF A GENERAL-PURPOSE NON-LINEAR DYNAMIC MODEL FOR S/H-ADC DEVICES

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Abstract – *Experimental results are provided in this paper in order to validate the Discrete-Time Convolution approach, recently proposed for the modelling of both static and dynamic non-linearities in S/H-ADC devices. The comparison between actual device response and model predictions is provided, showing a very good agreement under operation conditions which are not suitably described by a conventional model, based on the characterisation of the static non-linearity only. A brief discussion about the possibility of embedding into the model almost any recent empirical technique for the characterisation of static sources of uncertainty within the device is proposed as well.*

Keywords: Analogue-to-Digital Converter, Sample-Hold, Non-Linear Dynamic Model, Volterra Series.

1. INTRODUCTION

The characterisation and modelling of digital data acquisition devices consisting of a Sample/Hold circuit cascaded with an Analogue-to-Digital Converter (S/H-ADCs in the following) are usually performed through two independent steps. The S/H component is separately modelled as an analogue circuit: to this aim different classical approaches can be followed, such as the characterisation of each lumped element that appears in the circuit design (passive and active devices within the switches, amplifiers and/or attenuators in the input conditioning circuitry, etc.) or the description, from a higher system standpoint, of all the sub-blocks in which the S/H can be divided. Many practical methods are based on a combination of the two techniques mentioned. Whatever the chosen modelling approach may be, the wide family of parameters that are needed can be obtained through direct measurements and/or data-sheet specifications provided by the device manufacturer. As far as the ADC is concerned, it is usually described through a model which is based, in many cases, on the measurement of its *static* input/output characteristic: offset and gain errors, along with integral and differential non-linearities are the typical experimental data exploited for model characterisation purposes. Even when some *dynamic* effects associated with the device behaviour are taken into account, they are usually related to linear components within the ADC architecture, and described by means of linear, frequency-domain operators (for example the transfer functions of the sub-blocks which model, at a system level, the linear components): finally, if the device presents residual non-linear effects, these are included in the model by means of non-linear algebraic relationships. A much more detailed survey on the ADC modelling approaches that have been recently proposed in the literature can be found in [1].

When the S/H-ADC is operated at high sampling frequencies (due to the nature of the application, as happens for instance for flash-converters), or in any case when an input signal is applied, whose higher spectral components are located near the boundary of the device operation digital bandwidth, a model based on static empirical data (eventually completed with information about the response of linear dynamic sub-blocks) could not be suitable for the prediction of *non-linear dynamic* effects, which can strongly affect the device behaviour under such operating conditions.

In order to overcome these limitations, a behavioural ("black-box") non-linear dynamic model has been recently proposed [2]-[4], which is capable of predicting the non-linear dynamics of S/H-ADC devices independently of the internal architecture and technology adopted. The model represents the application to digital acquisition devices of a very general approach, proposed for the behavioural modelling of a wide family of non-linear dynamic systems through a modified formulation of the Volterra series [5,6]. Besides preserving the same generality and theoretical validity of the classical approach[1], the modified series presents interesting convergence properties, which allow to successfully apply it by considering a *reduced* number of kernels, if mild hypotheses on the system are satisfied.

2. THE NON-LINEAR DYNAMIC MODEL

The essential details about the model analytical formulation are briefly recalled in the following.

The actual digital data acquisition device with input signal $s_I(t)$ can be described (see Fig. 1) as an *ideal*

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[1] It can be shown that the modified kernels can be derived directly from the Volterra ones, without any approximation.

S/H-ADC which samples and converts to digital the output $y(t)$ of a non-linear dynamic system: this block is introduced in order to take into account both *static* and *dynamic* non-idealities associated with the S/H-ADC behaviour. Following the modelling approach based on the modified Volterra series (see in particular [6]), the non-linear system can be functionally represented as shown within the dashed line in Fig. 2: the purely-linear system (first element of the cascade) describes the memory effects introduced by the input signal conditioning circuitry and the sample/hold process, and can be simply characterised by means of its transfer function. The second block, instead, which is controlled by the linear transformation $s(t)$, generates the output $y(t)$ as the sum of two contributions:

$$y(t) = y^{(S)}(t) + y^{(D)}(t) \qquad (1)$$

where

$$y^{(S)}(t) = z_0[s(t)] = y_0 + \sum_{r=1}^{\infty} \frac{1}{r!} a_r s^r(t) \qquad (2)$$

is an algebraic function with respect to $s(t)$ and coincides with the zero-order term of the modified series (a_r being the coefficients of its polynomial expansion, which can be theoretically computed by integrating the corresponding-order Volterra kernel [2,5,6]), while

$$y^{(D)}(t) = \int_{-T_A}^{T_B} w[s(t),\tau]\cdot[s(t-\tau)-s(t)]d\tau \cong$$

$$\cong \sum_{\substack{p=-P_A \\ p\neq 0}}^{P_B} [s(t-p\Delta\tau)-s(t)] \sum_{n=1}^{N} \beta_{pn} s^n(t) \qquad (3)$$

is a purely-dynamic, non-linear *functional* with respect to $e(t,\tau) = s(t-\tau) - s(t)$ and represents the first-order term of the modified Volterra series, $w[s(t),\tau]$ being the first-order kernel and $[t - T_B, t + T_A]$ the memory time of the non-linear system[2]. In Eq. (3) the contribution $y^{(D)}(t)$ has been further expressed through a *discretisation* of the integration domain (i.e. the memory time) into $P = (P_A + P_B)$ elementary time intervals of equal width $\Delta\tau$ and each term $w[s(t), p\Delta\tau]\Delta\tau$ (deriving from the evaluation of the first-order kernel at the p-th $\Delta\tau$ interval) has been expanded into a N-order homogeneous polynomial with coefficients β_{pn}. The modified series terms $y^{(S)}(t)$ and $y^{(D)}(t)$ can be seen, from a system standpoint, as the output of a non-linear algebraic (i.e. without memory) network and a non-linear purely-dynamic system with *short* memory, respectively. Since the contribution $y^{(D)}(t)$ is purely-dynamic, the function $z_0[\cdot]$ in Eq. (2) represents the static characteristic (before the quantization) of the S/H-ADC device. The Discrete-Time Convolution Model (DTCM), which is functionally described in Fig. 2, characterises the input/output relationship of the S/H-ADC, by taking into account both static and *dynamic* non-linear effects in the device behaviour.

3. EXPERIMENTAL VALIDATION

The procedure devoted to the experimental characterisation of DTCM parameters has been described in details in [4]. It consists of simple quasi-independent measurement steps, which exploit conventional instrumentation and reliable numerical algorithms for the processing of the empirical data obtained. A commercial digital acquisition board, based on a successive-approximation 12-bit ADC, has been fully characterised through the DTCM approach and the model obtained implemented in the framework of different circuit analysis CAD tools (see [3] for details). Preliminary simulation results, carried out by means of analyses performed both in time- and frequency-domain, have shown the model capabilities of pointing out the non-linear purely-dynamic contribution to the error at the device output. In particular the coefficients β_{pn}, which fully characterise the purely-dynamic non-linear functional of Eq. (3), have been proposed as a novel set of parameters for the specification of non-linear dynamic effects in S/H-ADC devices and, from a more general standpoint, within the data acquisition channels of sampling instrumentation: Fig. 3 shows the magnitude of a typical $\underline{\beta}$ matrix for a commercial device with P_A=P_B=3, N=5, $\Delta\tau$ =20 ns.

In order to experimentally validate the proposed approach and test the accuracy achievable by the DTCM model in the prediction of device behaviour, particularly when dynamic non-linear phenomena introduce important contributions of error at the output of the S/H-ADC, comparisons have been performed between model predictions and actual device response, the latter operated under different input conditions. The signals applied to the input of the DUT have been characterised by means of a reference acquisition channel, much more accurate in terms of analogue bandwidth, sampling frequency, resolution and, in general, linearity. The corresponding simulations have been performed by importing the information obtained through the reference channel and using them in order to correctly synthesise the excitations at the input of the model implemented.

A simple but significant test is the application of a sinusoidal input test signal, at a frequency close to the device large-signal analogue bandwidth. Under these conditions, the S/H-ADC is still correctly operated (no specification about the input bandwidth is broken), but the non-linear dynamics can be considered already excited. The time-domain error sequence response of a 250 kS/s 12-bit device, predicted by the corresponding DTCM model when a nominal 4 V amplitude sinusoidal input at f_0=78 kHz is applied, is shown in Fig. 4 (time-

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[2] In the model formulation the memory time extension $T_M = T_A + T_B$ is supposed *short* with respect to the input signal typical period. This hypothesis is essential for the truncation of the modified series to the first-order, and is largely satisfied for S/H-ADC devices (see for example the discussion in [4]).

equivalent sampling technique). The amplitude of the error introduced by the static block of output $y^{(S)}$ (below) can be compared with the purely-dynamic non-linear error represented by the sequence $y^{(D)}$ (above). The influence of non-linear dynamics can be further appreciate by considering that the static error also includes offset and gain non-idealities, which do not contribute to the output distortion and can be easily compensated. In such operation conditions, the corresponding output spectrum, evaluated by applying accurate FFT algorithms to the time-equivalent sample sequence at the output of the device, can be considered, both for the actual device and the DTCM model. The time-equivalent sampling technique has been exploited in order to extend the digital bandwidth of the board and allow the computation of several harmonics. Fig. 5 shows the results of the spectral analysis. Distortion harmonics deriving from the analysis on the corrupted data at the output of the actual S/H-ADC are compared with the DTCM predictions, showing a very good agreement up to the fifth-order. In the same plot, the predictions derived from the output of the static network of Fig. 2 *alone* are depicted as well. As can be noticed, the static model is not capable of predicting the overall distortion, since it does not take into account any non-linear dynamic effect, which are instead considered by the complete DTCM implementation.

It is important to notice that the cascade consisting of the dynamic linear block and the algebraic network with output $y^{(S)}(t)$ represents by itself a consistent characterisation of the device, which can be considered sufficient for the prediction of the S/H-ADC behaviour when slow dynamics are applied to its input. In other words, the DTCM approach allows to “embed” any accurate device model which is based on the empirical characterisation of the static characteristic (in particular by measuring the integral and differential non-linearities). The most appropriate technique can be adopted, between those offered by the recent literature, in order to perform this step. Afterwards, the identification of the modified Volterra series first-order contribution allows to include the effects of non-linear dynamics within the system in the complete formulation of the DTCM model. These effects are taken into account only in presence of high-frequency and high-amplitude input signals, while the model “returns” to its static nature when the dynamics involved are mild or under small-signal operation.

In order to better understand this important feature of the DTCM model, other results are presented in the following. In particular, a two-tone input signal (f_1 = 79.5 kHz, f_2 = 80.5 kHz, V_1=V_2= 2 V (nominally)) has been synthesised by means of a general-purpose waveform generator and applied to the input of the DUT. The non-ideality of the generator has resulted into distortion contributions to its output at $2f_1 - f_2$ and $2f_2 - f_1$ (*third-order intermodulation products*), which have been accurately characterized by means of the reference channel, along with the main tones. Such a test signal represents a very narrow-band critical excitation, consisting of both large- and small-amplitude spectral components, the small ones being placed in correspondence with possible distortion terms introduced by the S/H-ADC non-linearity. Fig. 6 shows the in-band spectra at the output of the actual device and of the DTCM model. Again, the agreement is very good, but in this case the overall prediction does not differ in a significant way from the static one (i.e. the static contribution $y^{(S)}$ at the output of the model is sufficient to describe the device behaviour within the input signal base-band). This is due to the fact that the intermodulation distortion is a very mild effect in the S/H-ADC device behaviour and is negligible, under the operation considered, with respect to the impurity of the input signal source chosen, whose intermodulation products have been taken into account during the simulation as desired input contributions (i.e. *all* the four tones have been inserted at the input of the model implemented). As can be deduced, the static model embedded is accurate enough to estimate the in-band device response, while the contribution $y^{(D)}$ introduces intermodulation products which are correctly very low and do not influence the output, even though the non-linear dynamics described by this term are quite important at this input frequency, since they generate the most relevant contribution to the harmonic distortion (see Fig. 5). The distortion prediction provided by the overall DTCM model does not corrupt small-signal components that appear in the input signal (the response to which being thus describable by an accurate static model) and are placed in critical regions of the spectrum, where a purely-dynamic non-linear block could potentially introduce wrong spectral estimates.

4. CONCLUSIONS

Experimental tests have been carried out in order to validate the Discrete-Time Convolution Model for digital data acquisition channels of sampling instrumentation. The proposed model is capable of describing both static and dynamic non-linear effects within the S/H-ADC cascade behaviour. Examples have been provided, which show the good agreement between model predictions and actual device response, under operation conditions which can not be suitably described by a conventional model, based on the empirical characterisation of device static sources of uncertainty. Nevertheless, an accurate characterisation of the static integral non-linearity can be “embedded” within the DTCM model, allowing the choice of the preferred technique between those proposed in the recent literature. From this standpoint, the non-linear dynamic contribution at the output of the DTCM formulation does not reduce the accuracy achievable by the static characterisation, when the non-linear dynamics in the device behaviour are negligible with respect to the static distortion response.

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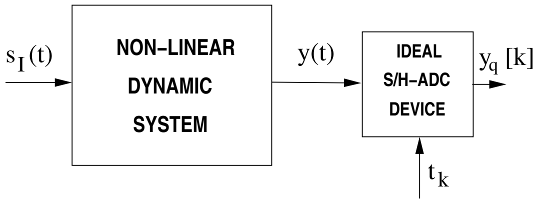

Fig. 1 – Functional description of the S/H-ADC for the characterization of non-linear (both static and dynamic) effects.

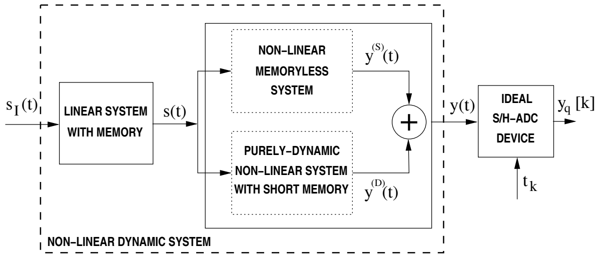

Fig. 2 – Sub-blocks into which the system of Fig. 1 can be further expanded, following the proposed modelling approach.

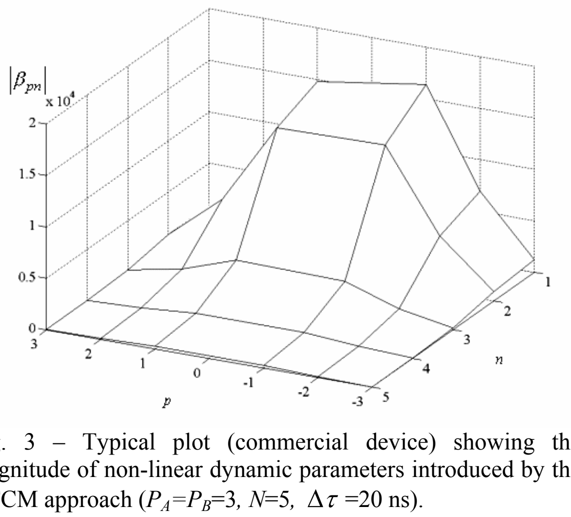

Fig. 3 – Typical plot (commercial device) showing the magnitude of non-linear dynamic parameters introduced by the DTCM approach (P_A=P_B=3, N=5, $\Delta\tau$ =20 ns).

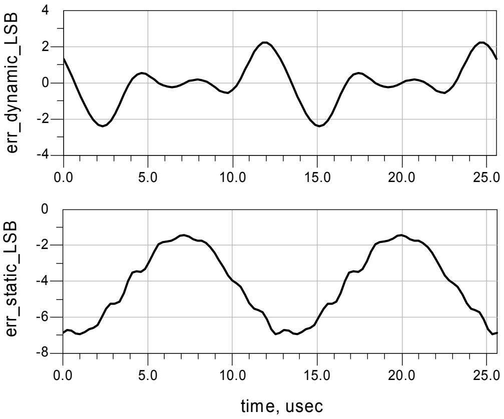

Fig. 4 – Dynamic (non-linear) and static (overall) error contributions to the output of the S/H-ADC device model (sinusoidal input at f_0= 78 kHz, 4 V nominal amplitude).

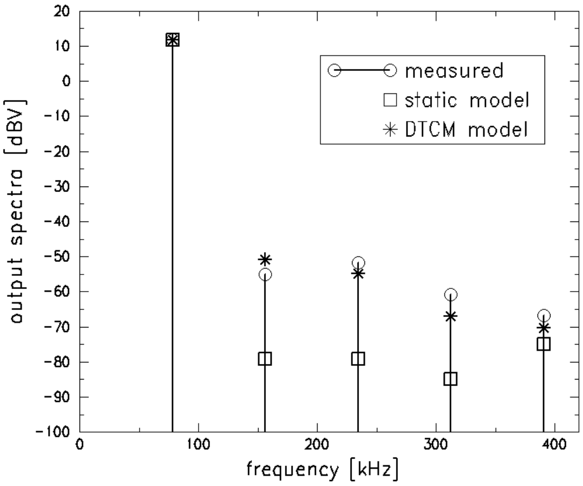

Fig. 5 –Spectra at the output of the actual device and predicted by the DTCM model. Sinusoidal input: f_0= 78 kHz, 4 V nominal amplitude.

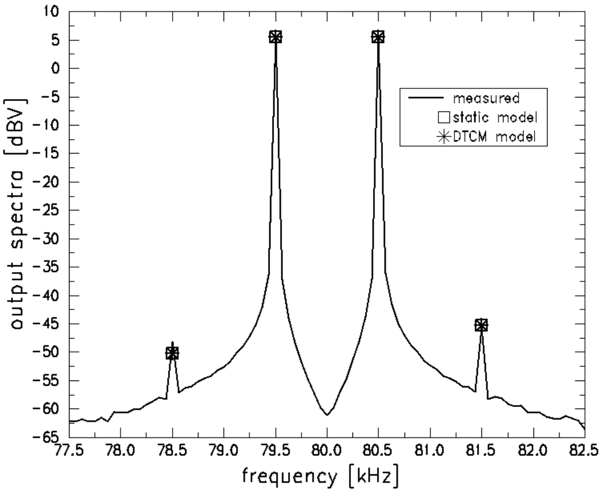

Fig. 6 – In-band spectra at the output of the actual device and predicted by the DTCM model, when a two-tone intermodulating signal is applied at the input (f_1= 79.5 kHz, f_2= 80.5 kHz, 2 V nominal amplitudes).