Digital-image formation based on the inversion of microwave scattering data

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Abstract - Among the various imaging systems, the ones based on electromagnetic waves in near-field conditions are considered in this paper. In particular, two reconstruction approaches based on stochastic concepts are discussed. Starting from tomographic configurations, the integral equations of the inverse scattering problem are discretized and solved by using a genetic algorithm and a simulated annealing. The effects of the presence of the so-called nonradiating current are also taken into account.

Keywords - Imaging systems, microwaves, inversion techniques.

I. Introduction

"Microwave imaging" denotes a class of imaging techniques in which interrogating microwaves are used to inspect an unknown scenario [1]. Digital images of the scene are obtained by "inverting" measured data, which are represented by samples of the scattered field (i.e., the perturbation of the field due to the presence of the unknown objects).

In recent years, there has been a growing interest in microwave imaging since it represents a challenging problem with applications in several areas, including nondestructive evaluation and testing, medical diagnostic, subsurface introspection, etc.

One of the critical points related to the realization of efficient electromagnetic imaging systems is represented by the development of efficient reconstruction procedures. The image formation requires the "inversion" of nonlinear ill-posed operators, which are related to the inverse scattering problem.

The continuous model (the integral equations of the inverse scattering problem) is usually discretized by using a numerical method. In the present paper, we use the Richmond method [2], which assumes a partitioning of the test area in square subdomains and constant values of the unknowns. Moreover, the kernels of the integral equations are transformed into sets of coefficients written in terms of Bessel functions. Then, the problem is usually reduced to a global optimization problem.

Two inversion approaches are presented in this paper. In the first one, a genetic algorithm [2] is applied to minimize a functional defining the global optimization problem. In particular, a hybrid version is used, in which the classic evolutionary scheme is combined with a deterministic approach (a conjugate gradient method) in order to increase the convergence velocity. The stochastic minimization is applied to a functional constituted by two terms: 1) the “data term,” which is related to the difference between the measured scattered data and the data predicted by the procedure at any iteration; 2) the "state term," which is related to the so-called “state equation,” which imposes that the reconstructed object and the predicted electric field (inside the object) be consistent with the known incident field.

The deterministic procedure is a standard conjugate gradient. In this case, since the internal field is unknown, the problem is a minimization process involves a large number of unknowns and the application of the proposed hybrid code seems to be particularly suitable.

The second approach proposed is aimed at taking explicitly into account the limited information that can be obtained by considering only the "data equation." In particular, a minimum-norm solution is derived first. To this end, a singular value decomposition is applied. The minimum-norm solution does not take into account nonmeasurable induced currents [3]. These currents do not contribute to the scattered field and, consequently, cannot be retrieved starting by the "data equation." Successively, the distributions of nonmeasurable currents are derived by constructing another functional (essentially related to the "state equation").

II. Problem Formulation

In a classic tomographic configuration, the test area is successively illuminated by a set of incident TM-waves with the electric field polarized in the same direction of the cylinder axis, $\mathbf{E}^{v}_{inc}(\mathbf{r}) = E^{v}_{inc}(x,y)\mathbf{z}$, $v = 1,...,V$. Let us denote by $E^{o,v}_{scatt}(x,y)$ the scattered electric field collected (for any *v*) by a set of M probes located outside the investigation domain. A multi-illumination/multiview process is considered. The relationships among measured data, unknown parameters (namely, the dielectric parameters of the region under test, $\varepsilon_R(x,y)$ and $\sigma(x,y)$), and the internal electric field can be written as follows [2][5]:

$$E^{o,v}_{scatt} = \aleph^{v,m}\left\{\tau, E^{v}_{tot}\right\} \tag{1}$$

$$E^{v}_{inc} = E^{v}_{tot} - \Re^{v}\left\{\tau, E^{v}_{tot}\right\} \tag{2}$$

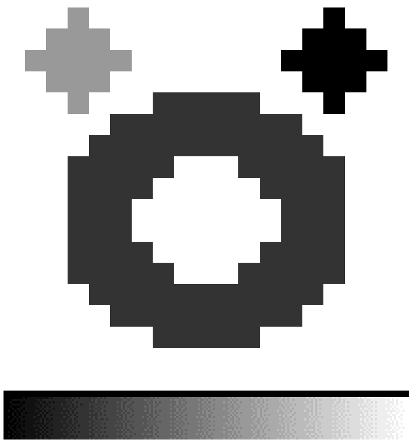
where $\aleph$ and $\Re$ are nonlinear operators defined in details in [5]. In equations (1) and (2), $\tau(x,y) = \varepsilon_R(x,y) - 1 - j\sigma(x,y)/\omega\varepsilon_0$ denotes the *object function*, which contains the information on the unknown dielectric properties. After discretization, the reconstruction problem is solved by minimizing a suitable cost function:

$$\psi\{\overline{\eta}\} = \left\| E^{o,v}_{scatt} - \aleph^{v,m}\{\overline{\eta}\} \right\|^2 + \left\| E^{v}_{inc} - E^{v}_{tot} + \Re^{v}\{\overline{\eta}\} \right\|^2 \tag{3}$$

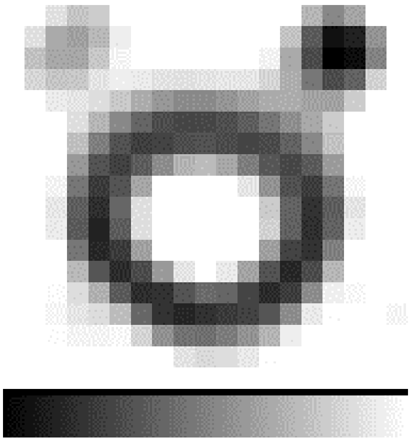
where $\overline{\eta}$ is the array of unknown quantities, which contains the values of the digitalized image of the *object function* in the transversal plane. Equation (3) is minimized by using an hybrid version of the genetic algorithm [2]-[4] or by considering a singular value decomposition and a simulated annealing [3]. In the last case, the objective is to overcome the main drawback of linearized imaging procedures, i.e., the inability to reconstruct some components of the equivalent current density induced in the objects under test (nonuniqueness). In particular, the aim is to explicitly account for that part of information that cannot be recovered from measurements. Actually, there are many reasons that may combine to make a current component non recoverable from measurements: in an inverse problem with complete measurement data, nonuniqueness is due to the so-called nonradiating currents, which give a null field outside their support [3]; more generally, in a discretized case with a limited measurement domain, one can refer to nonmeasurable currents, in the sense that the inverse mapping from the measurements to the object space covers only a subspace of the currents; therefore only those components that are inverse-mapped can be reconstructed. In particular, besides nonradiating currents, there may exist some components of the scattered field that are evanescent, so that, in practice, they do not contribute to the field at the measurement points. Furthermore, if the measurement points are not properly placed, one may have two or more measures carrying almost the same information, thus giving rise to instabilities in the solution. This is also one of the reasons that would make impractical the idea of using an imaging system with an arbitrarily large number of measurement points. In general, measurements can only provide a limited amount of information. The proposed approach is based on the reconstruction of the radiating (or, in general, the measurable) components of the equivalent current density, by a SVD of the discretized Green operator (equation (1)). Such components are then inserted into a nonlinear equation whose unknowns are the nonmeasurable components as well as the dielectric features of the body under test [3].

III. NUMERICAL EXAMPLES

The first numerical simulation concerns the reconstruction of the so-called "Osterreich configuration," which is an inhomogeneous 2D configuration widely used to test inversion algorithms for producing digital microwave images.



(a)



(b)

Fig. 1. Image of three separate scatterers (inhomogeneous configuration). Discretized domain: 19 x 19 subdomains. Number of views: V = 4. Investigation area: $\lambda \times \lambda$. Number of measurement points: M = 81. (a) Original and (b) reconstructed images.

In particular, the genetic algorithm has been used. The square investigation area (measuring $\lambda \times \lambda$, being λ the wavelength) is partitioned into 19×19 subdomains. Four views are used with 81 measurement points arranged on a arc of circumference (radius: 3/4 λ).

The other results are related to the second approach. In particular, Figure 2 reports a numerical test, which has been performed to evaluate the capabilities of the algorithm to reconstruct a dissipative homogeneous cylinder ($\lambda/4$ in diameter). The investigation area is discretized in 40×40 square pixels and 32 measurement points are used. Two values of the object function are considered: $\tau = 1.5\text{-}j0.25$ and $\tau = 1.5\text{-}j0.5$. In these cases it is mandatory to resort to a multiview arrangement: four orthogonal views are used, at the frequency of 10 GHz.

Finally, a two-layer cylinder is considered (Figure 3). The assumed configuration is the same as in the previous example (diameter: $\lambda/2$). Both multiview and multifrequency illumination can provide a substantial improvement in the image acquisition.

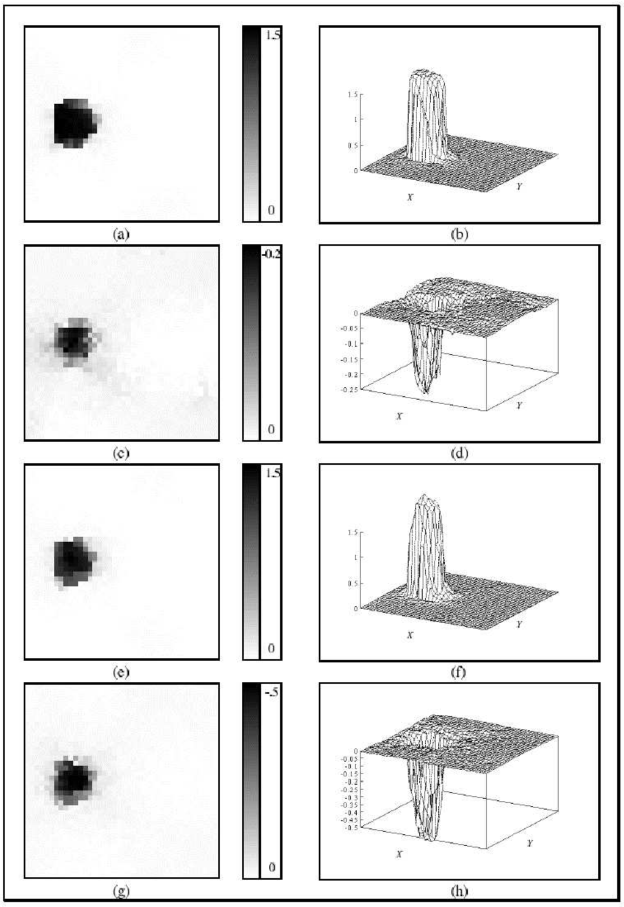


Figure 2. Reconstructed images of the cross section of a dissipative cylinder. From (a) to (d) τ = 1.5-j0.25; (a)-(b) real part; (c)-(d) imaginary part. From (e) to (h) τ = 1.5-j0.5; (a)-(b) real part; (c)-(d) imaginary part.

In particular, a multifrequency illumination can sensibly enhance the capability of the algorithm to discriminate among the various layers.

IV. CONCLUSIONS

Two approaches to microwave imaging have been reported. They are based on the stochastic inversion of measured scattered data. In both cases the inverse problem has been reduced to an optimization problem, which has been solved by considering two different strategies (genetic algorithm and SVD/simulated annealing).

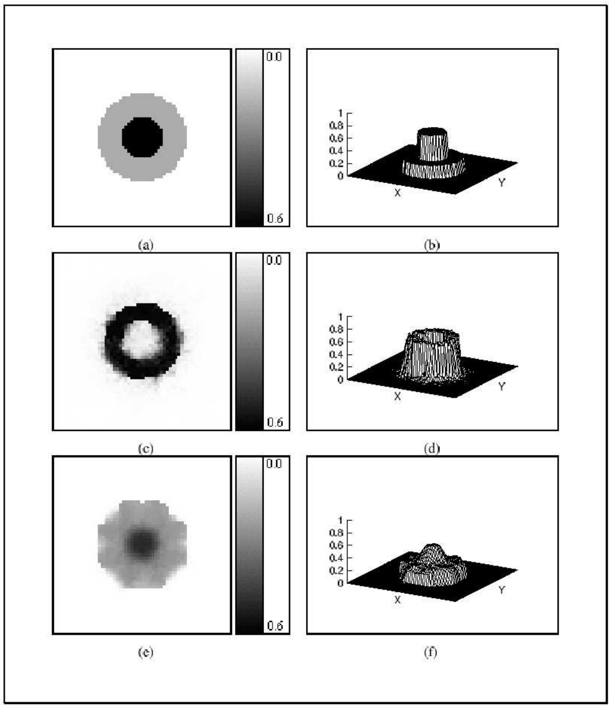


Fig. 3. Reconstructed images of a two-layer cylinder. Inner layer: τ = 0.6; external layer: τ = 0.3. (a)-(b) Ideal reconstruction. (c)-(d) Images obtained by 8 views and 1 frequency (10 GHz). (e)-(f) Images obtained by using 8 views and 3 frequencies (1.0, 6.0, and 14 GHz).

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