# Kautz-Volterra modelling of an analogue-to-digital converter using a stepped three-tone excitation

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***Abstract*** – In many test and measurement applications, the analogue-to-digital converter (ADC) is the limiting component. Using post-correction methods can improve the performance of the component as well as the over all measurement system. In this paper an ADC is characterised by a Kautz-Volterra (KV) model, which utilises a model-based post-correction of the ADC with general properties and a reasonable number of parameters. Results that are based on measurements on a high-speed 12-bit ADC, shows good results for a third order model.

## I. Introduction

The ADC is a key component in many applications, e.g. radio base stations and test&measurement instruments. In state of the art designed vector signal analyzers (VSAs) the ADC is the bottle neck and an improvement in ADC performance directly improves the VSA performance. The trend within in test&measurement of combining RF and digital signal processing has been prevalent for some time, it gives unique possibilities to create novel measurement technologies to reduce test costs and give important feedback to design, i.e. key issues for the users. These possibilities are directly dependent on the VSA performance, i.e. the ADC performance. As a consequence of this the requirements on the ADC are increasing. Post processing of measured data is commonly used for signal conditioning as well as to facilitate new and flexible measurement methods. An ADC suffers from a (weak) nonlinear and dynamic behaviour but these imperfections can be adjusted for afterwards by using some kind of post-correction method [1]. The use of model-based post-correction requires accurate models of the ADC.

Post-correction of ADCs is difficult since it involves the inverse of a nonlinear dynamic transfer function. ADCs have typically short memory, typically one sample. Inverse models have, however, longer memory effects [2]. A Volterra model, which can be used for any causal stable nonlinear system with fading memory, could in principle be used [3, 4]. The high number of parameters and the computational complexity make a full Volterra model unsuitable in practice. Models that are special cases of the Volterra model such as Hammerstein and Weiner models are, thus, used [5]. These models may give good results when used in post-correction algorithms, although they do not have the general properties of a Volterra model. In this paper we report the use of a Kautz-Volterra (KV) model to characterize an ADC. The model uses orthonormal basis functions, so-called Kautz functions [6]. The KV model has the same general properties as the Volterra model, but the number of parameters can be significantly lower. A Volterra model is based on FIR filters; a KV model uses IIR filters and can in practice be used for systems with longer memory effects [7]. Kautz-Volterra models have been successfully used for modelling power amplifiers [8, 9] and for digital pre-distortion [9, 10], but have to our knowledge, previously not been used for ADCs.

An experimental problem when validating post-correction algorithms for ADCs is that it is difficult to generate spectrally pure signals. This is in particular problematic when post-correction including nonlinear memory effect is used. When identifying a nonlinear dynamic system it is of course an advantage if a properly designed noise signal can be used, since it excites all frequencies (fundamentals, sums and difference frequencies) and all amplitudes. It is, however, difficult to generate distortion free noise signals. Therefore, a three tone signals is used, since the nonlinear order of the ADC can be considered to be at most three. That is, the nonlinearity of the ADC is estimated to be a polynomial of order three. By using a number of three tone sequences we excite several fundamental, sum and difference products or many points in the volumes of the Volterra kernels [11]. The model that we use is a parametric one, and it can (hopefully) interpolate between the points in the two and three dimensional frequency space of the Volterra kernels.

In this work we combine advanced methods for generating spectrally pure (i.e. distortion free) multi-tone signals (three-tone signals) with KV models for the ADC transfer function. The motivating force is that the direct model, with reference to previously described properties, can be used for post-correction of the ADC in future applications, when the memory effects of the linear term are negligible [9].

## II. Theory

It has been shown in [12] that the input-output relation of any time-invariant nonlinear system with fading memory can be well approximated by a finite Volterra series representation to any precision. Volterra filters are simple to use and have nice properties. For example, they are linear in the parameters and hence standard and well-behaved parameter estimations techniques can be applied. However, the need of intensive computational schemes somehow limited their practical use. This is mainly due to the slow convergence huge number of coefficients that must be estimated in order to calculate the kernels. The output $y_V(n)$ of a discrete-time time-invariant truncated $O$-th order and memory length $M$ Volterra model with input sequence $u(n)$ is

$$\begin{aligned} y_V(n) &= \sum_{m=0}^{M} h_1(m)u(n-m) \\ &+ \sum_{m_1=0}^{M}\sum_{m_2=m_1}^{M} h_2(m_1,m_2)u(n-m_1)u(n-m_1)+\cdots \\ &+ \sum_{m_1=0}^{M}\cdots\sum_{m_O=m_{O-1}}^{M} h_O(m_1,\ldots,m_O)u(n-m_1)\cdots u(n-m_1) \end{aligned} \tag{1}$$

where $h_i(m_1,\ldots,m_i)$ are the $i$-th order time domain Volterra kernels. The $i$-th order frequency domain Volterra kernel $H_i(z)$ is obtained by the Laplace transform of $h_i(m_1,\ldots,m_i)$. Thus, the $H_1(z)$ is a linear FIR filter, $H_2(z)$ is a quadratic FIR filter, *et cetera*. Previous work as [7] interprets (1) as a filter bank of $M$+1 linear filters (filter $i$ is defined as $G_i(z)=z^{-(i-1)}$ ) followed by a $M$+1 input, static nonlinear Volterra system. In order to reduce the number of coefficients caused by high model order $O$ and memory length $M$ the Kautz-Volterra model pre-filters the input signal $u(n)$ with orthonormal IIR filters, where the poles are chosen to *a priori* knowledge of the system properties. By judicious choice of the filters, a model with good accuracy can be achieved with a small number of basis functions. The KV model of order $O$ is defined as

$$\begin{aligned} y_{KV}(n) &= \sum_{l=1}^{N_1} b_l x_{1,l}(n) + \sum_{l_1=1}^{N_2}\sum_{l_2=l_1}^{N_2} b_{l_1 l_2} x_{2,l_1}(n) x_{2,l_2}(n)+\cdots \\ &+ \sum_{l_1=1}^{N_O}\sum_{l_2=l_1}^{N_O}\cdots\sum_{l_O=l_{O-1}}^{N_O} b_{l_1 l_2\cdots l_O} x_{O,l_1}(n)x_{O,l_2}(n)\cdots x_{O,l_O}(n) \end{aligned} \tag{2}$$

where $N_1$ is the number of basis functions for order 1, *et cetera*, and $x_{i,l}(n)$ is the $i$-th nonlinear order output of the filter $G_{i,l}$ exited by input signal $u(n)$.

$$\begin{aligned} G_{i,l}(z) &= \sqrt{\left[1-\left(\xi_l\xi_l^*\right)^2\right]\left[1-\delta_l^2\right]}\frac{z}{\left(z-\xi_l\right)\left(z-\xi_l^*\right)}\prod_{\substack{l>1\\q=1}}^{l-1}\frac{\left(1-\xi_q z\right)\left(1-\xi_q^* z\right)}{\left(z-\xi_q\right)\left(z-\xi_q^*\right)}, \quad l \text{ odd} \\ G_{i,l}(z) &= \sqrt{1-\left(\xi_l\xi_l^*\right)^2}\frac{z\left(z-\delta_l\right)}{\left(z-\xi_l\right)\left(z-\xi_l^*\right)}\prod_{\substack{l>1\\q=1}}^{l-1}\frac{\left(1-\xi_q z\right)\left(1-\xi_q^* z\right)}{\left(z-\xi_q\right)\left(z-\xi_q^*\right)}, \quad l \text{ even} \\ l &= 1,2,\ldots,N \\ \delta_l &= \left(\xi_l+\xi_l^*\right)/\left(1+\xi_l\xi_l^*\right) \end{aligned} \tag{3}$$

where $\xi_q$ and $\xi^*_q$ are the complex-conjugate poles-pairs of order $i$.

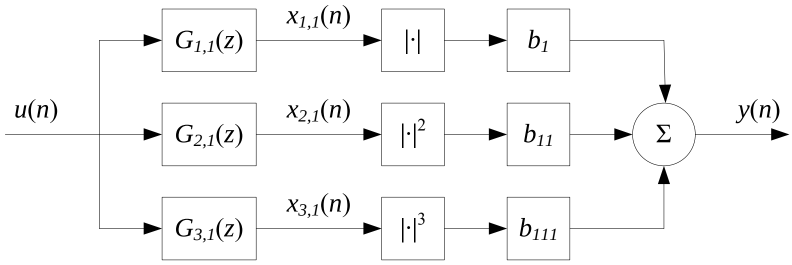


Figure 1: A 3rd order KV-model with one basis function for each order describing the input-output relation of the ADC under test.

We use the notation $\mathbf{N}=[N_1\ N_2\ N_3]$, for the number of basis functions of the respective order in the experiments and denote the poles $\xi=[p_1, p_2, p_3]$. The KV model could be used with several poles per order, see (3); we used one complex pole-pair per nonlinear order. If all the poles are at origin, i.e. $\xi=[0\ 0\ 0]$, the KV model becomes the Volterra model, and if also $\mathbf{N}=[1\ 1\ 1]$, it becomes a polynomial model [9]. For the ADC under test a third order model is used. The 3rd order KV-model with one basis function for each order is shown in Figure 1.

If the poles in $\xi$ are known, the constants $b_l\ldots$ can be identified from $u(n)$ and the experimental output $y(n)$, using standard techniques for system identification, i.e. minimising the mean square error of the experimental and model outputs, see e.g. [13, 14]. The optimum poles, in a mean square error sense, were found as follows. We first determined $p_1$, with the quadratic and cubic (second and last) terms in (1) put to zero; we keep $p_1$ fix and then determine $p_2$ with only the cubic term equal to zero, and finally $p_3$ is determined with $p_1$ and $p_2$ fixed.

Three-tone signals are used as input signals, $u(n)$, for identification. As validation signal a three tone signal that was not used for identification was used.

## III. Experiments

The test set-up (which is in detail described in [15]) used a state-of-the-art signal generator together with specially designed signal conditioners ensured that the test signals are generated and distributed to the ADC input. For a three-tone scan test scenario, band pass filters are used to suppress spurs and noise from the test signal. However, the filters attenuate the signals; therefore it was amplified (with an ultra low distortion amplifier) to obtain sufficient drive level (-0.5dBFS). Inside the filter bandwidth the noise floor is far above the level of distortion products from the ADC due to the intermodulation products caused in the output stage of the arbitrary signal generator and the other components in the signal chain.

To overcome this limitation on measurement the 3rd order IM products predistortion was used to obtain spectrally pure three-tone signals for the measurement. Generally, output signal from the generator contains unwanted components (harmonics, IM products and spurs). The idea behind pre-distortion is to add signals to the wanted signals so that the output from the generator will be distortion-free. The implemented method is further described in [16] and is based on iterative algorithms and spectrum analyzer measurements.

The test device was a 12-bit pipeline ADC (AD9430). Filters used were SAW–filter with 40 MHz bandwidth and 70 MHz centre frequency. All frequencies have been chosen according to coherent sampling [17]. The sampling frequency was close to 175 MHz (174997504 Hz) and the signal frequency range was 63.6 - 74.7 MHz (see Table 1)

Table 1: Frequencies used.

| Requested Frequency [MHz] | Used Frequency [Hz] |
|---|---|
| 63.6 | 63583993 |
| 65.8 | 65827003 |
| 67.7 | 67685497 |
| 70.0 | 69992593 |
| 72.0 | 71957897 |
| 74.2 | 74200907 |

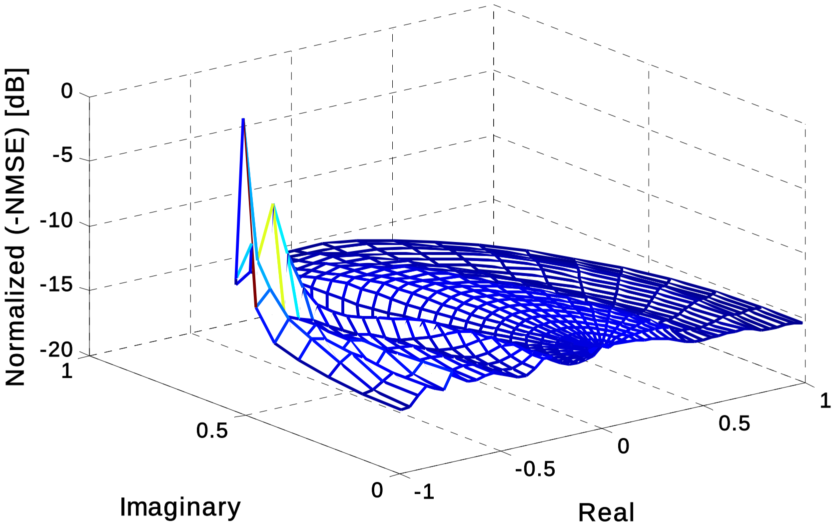


Figure 2: Show variations in the NMSE due to the position of the third-order pole placement. The optimal placement of the pole was found at –0.7±0.7j and gave an NMSE of –50.3 dB (here normalized to 0). The identification of the KV model was performed based on a three-tone signal at fixed frequencies.

## IV. Results

The results are based on twenty independent sets of three-tone signals as input signals; that is all possible combinations of the six frequencies given in Table 1. The identification is based on a set of 8192 samples. The validation of the model is based on a set of 4096 new samples. Six hundred twenty five (625) different pole-pair placements in the complex plane were evaluated for each set of three-tone signals. The pole placements that gave the lowest normalised mean square error (NMSE) are considered to be optimal pole placements.

For the linear and second order non-linearity the dynamic effect was negligible, but for the third order nonlinearity the improvements was significant. The optimal pole placements were found close to the unit circle. In Figure 2, the pole placement for a single set of three-tone stimulus is shown. The optimal placement of the pole was found at $-0.7\pm0.7j$, which imply a resonance frequency at 65.6 MHz. In [18], similar results was achieved when the same ADC was characterized by single-tone with a step-wise increased frequency as stimuli.

In Figure 3, the pole placements for all twenty sets of data are shown. The optimal pole-pairs are centred in a limited sector in the complex plane. Thus, the optimal pole placement can be considered to be frequency independent in the used band-width.

A choice of the ordinary Volterra model, i.e. choosing $\xi$=[0 0 0], would have resulted in a more complex model structure compared to the proposed structure; due to a higher memory length.

## V. Conclusions

Post-processing of measured data is commonly used for signal conditioning which also facilitate that imperfections in the ADC can be adjusted, by using model-based post-correction methods. In this paper a Kautz-Volterra model of order three is used to characterize an ADC. The properties of a KV model facilitate the use of an inverse model with a reasonable number of parameters.

The KV model is obtained from 20 three-tone measurements in a frequency range of 63.6 - 74.7 MHz. The dynamic behaviour is mainly due to the third order nonlinearity, while the linear and second order non-linearity dynamic effect was negligible. The optimal 3rd order pole placement is frequency independent in the used frequency range. Moreover, the obtained optimal pole placement results in frequency properties for the KV model that are in accordance with earlier obtained results for the ADC under test.

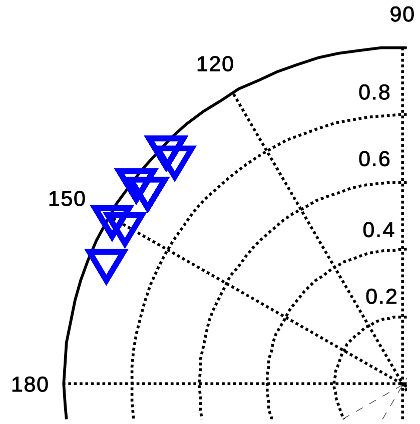


Figure 3: The optimal 3rd -order pole placements for the KV model are showed. All poles are complex conjugated pole-pairs. The figure shows only the poles with a positive imaginary part.

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