Distortion Analysis of Analog-to-Digital Converter by IDFT and Least-Squares fitting algorithm

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Abstract **-** This paper compares two basic procedures used in estimations of the significant components in the residual spectrum in the ADC dynamic testing. The properties of the weighted DFT interpolations for the frequency, amplitude, and phase estimations in order to reduce the leakage effect of the fundamental component in the investigation of the residual spectrum is described. The non-parametric interpolation algorithms retain all benefits of DFT approaches and improve the estimation accuracy as a function of a number of the signal cycles in the estimation interval. The comparison of the three-point estimation algorithms to the least-squares four-parameter sine-fit estimation described in the IEEE standard 1241 shows the estimation effectiveness.

I. Introduction

The main issue in dynamic testing of the Analog-to-Digital Converters (ADC) and ADC-based devices is determining the deviation of the characteristic of the actual device from that of an ideal ADC. Dynamic testing of an ADC means to excite the ADC with a pure sine wave, look for the sine wave which best fits the output samples, and analyze the difference [1]. Since we are dealing with periodic signals, the integral frequency transformation with $e^{-j2\pi ft}$ is in principle the best approximation to periodicity in the signal and analysis of the frequency spectrum gives opportunity to see systematic periodicities in the presence of the reduced random noise by integration. The actual ADC output has following components: a DC offset spectral line at zero frequency, a spectral line of the stimulating signal, harmonics of the input sine-wave due to analog circuitry nonlinear distortion and integral non-linearity of ADC, non-harmonic spurious tones due to interferences, and broadband noise due to differential non-linearity, analog circuitry noise, etc.

The problem of evaluating the spectral performance of a given ADC reduces to the parameters estimation of each spectral component (frequency, amplitude, and phase) in the presence of the noise. Estimation methods can be classified in parametric [2,3] and non-parametric ones [4,5]. Parametric methods are model-based and have very good selectivity and statistical efficiency, but require computationally intensive algorithms and very good 'model agreement' with a real multi-component signal [2,6]. For this reason, such methods are unsuitable for many ADC testing problems. Better approach is with the non-parametric methods, which estimate the spectral parameters of interest by evaluating first the discrete Fourier transform (DFT) of the ADC output and then the parameters of the particular component [7]. The normal situation for testing ADC is the non-coherent or quasi-coherent sampling [8] and in the sampled signal can be also spurious components. When failing to observe an integer number of periods of even a single tone, the tone energy is spread over the whole frequency axis, and the leaking from neighboring components can significantly bias estimations of the component parameters. Among most common techniques to compensate for the leakage effect [9], the interpolated DFT techniques [5] enable an investigation and an estimation of a single component with reducing coherent dependency. This work presents and explains a procedure that derives the estimation of the significant components in the residual spectrum in ADC dynamic testing using interpolated DFT procedure in comparison with the four-parameter least-squares fitting (LSF) algorithm.

II. Estimation of component parameters

A. Least-squares fitting algorithm

A sampled periodic band limited analog signal $g(t)$ can be written as follows:

$$g(k\Delta t)_N = w(k)g(k\Delta t)_\infty = \sum_{m=0}^{M} A_m \sin(2\pi f_m k\Delta t + \varphi_m), \qquad k = 0, ..., N-1 \tag{1}$$

where A_m, f_m, and φ_m are the amplitude, frequency and phase relevant to the measured component of the signal, respectively. To follow the procedure of the four-parameter least-squares fitting algorithm [1] the multi-component signal should be expressed in form

$$g(k\Delta t)_N = \sum_{m=0}^{M} \left(B_m \cos(2\pi f_m k\Delta t) + C_m \sin(2\pi f_m k\Delta t) \right) \tag{2}$$

where, for one component only, the expression is reduced on four unknown parameters: three for the investigated component f_m, B_m, C_m and one for the DC component B_0 ($f_0 = 0$). The procedure needs a few iterations [1] for final estimation of the basic parameters f_m, $A_m = \sqrt{B_m^2 + C_m^2}$, and $\varphi_m = \tan^{-1}(-C_m/B_m)$.

B. Interpolated Discrete Fourier Transform

The DFT of signal on N sampled points (1) at the spectral line i is given by:

$$G(i) = -\frac{j}{2} \sum_{m=0}^{M} A_m \left[W(i - \theta_m) e^{j\varphi_m} - W(i + \theta_m) e^{-j\varphi_m} \right] \tag{3}$$

where θ_m is the component frequency related to base frequency resolution depending on the window span $\Delta f = 1/N\Delta t$ and can be written in two parts:

$$\theta_m = \frac{f_m}{\Delta f} = i_m + \delta_m \qquad -0.5 < \delta_m \leq 0.5\,, \tag{4}$$

where δ_m is the displacement term due to the non-coherency.

It has been shown [10] that the best estimation results in reducing long leakage effects gives the three-point estimation using the Hanning window. In the estimations the three largest local DFT coefficients $|G(i_m - 1)|$, $|G(i_m)|$, and $|G(i_m + 1)|$ are used for frequency:

$$_3\delta_m \cong 2 \frac{|G(i_m+1)| - |G(i_m-1)|}{|G(i_m-1)| + 2|G(i_m)| + |G(i_m+1)|}\,, \tag{5}$$

amplitude:

$$_3A_m \cong \frac{\pi\delta_m}{\sin(\pi\delta_m)} \frac{(1-\delta_m^2)(4-\delta_m^2)}{3} \cdot \left[|G(i_m-1)| + 2|G(i_m)| + |G(i_m+1)| \right] \tag{6}$$

and phase:

$$_3\varphi_m = \frac{(1-\delta_m)\varphi_{i_m-1} + 4\varphi_{i_m} + (1+\delta_m)\varphi_{i_m+1}}{6} - \frac{2a\delta_m}{3} + \frac{\pi}{2} \tag{7}$$

where phase parts of the DFT $\varphi_{i_m+s} = \arg[G(i_m + s)]$, $s = -1, 0, 1$ and $a = \pi(N-1)/N$ are used.

III. Uncertainty of the Component Estimations

The price for the effective leakage reduction is in increasing of the estimation uncertainties related to the unbiased Cramér-Rao bounds (Figs. 1,2,3) fixed by the Signal-to-Noise-Ratio for particular component $\mathrm{SNR}_m = A_m^2/(2\sigma_t^2)$ corrupted by a white noise with standard uncertainty $\sigma_t = A_m/2^b \cdot 1/\sqrt{3}$ where the effective number of bits ($\mathrm{ENOB} = b$) of the A/D conversion is high enough $b \geq 3$ [10].

$$\sigma_{\delta_m} \geq \frac{\sqrt{3}}{\pi} \frac{1}{\sqrt{\mathrm{SNR}}} \frac{1}{\sqrt{N}} = \sigma_{\mathrm{CRB},f}\,, \tag{8}$$

$$\sigma_{A_m} \geq \frac{1}{\sqrt{\mathrm{SNR}}} \frac{1}{\sqrt{N}} = \sigma_{\mathrm{CRB},A}\,, \tag{9}$$

$$\sigma_{\varphi_m} \geq 2 \frac{1}{\sqrt{\mathrm{SNR}}} \frac{1}{\sqrt{N}} = \sigma_{\mathrm{CRB},\varphi} \tag{10}$$

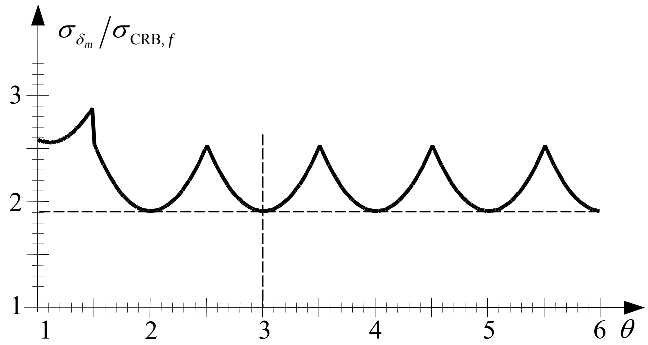

Fig. 1. Standard uncertainty of the three-point displacement estimation related to the CR bound (8)

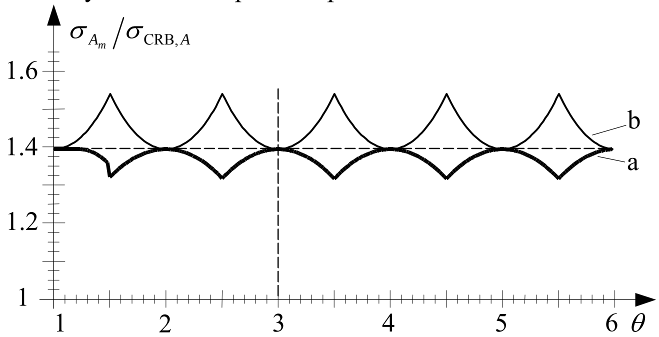

Fig. 2. Standard uncertainty of the amplitude three-point estimation related to the CRB (9) (a: θ is estimated, b: θ is known)

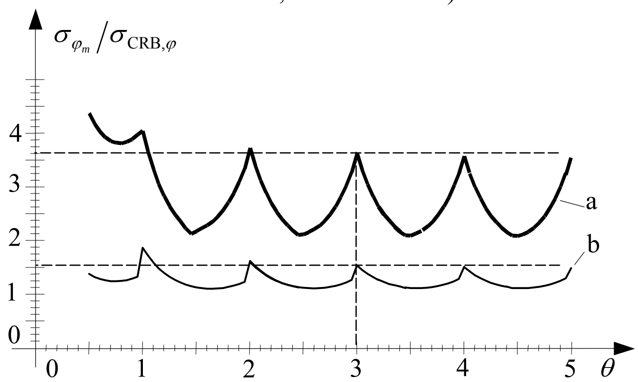

Fig. 3. Ratios of the uncertainties of the phase three-point estimations related the CRB (10) (a: θ is estimated, b: θ is known).

Figs. 2,3 show that the largest uncertainties are in the vicinities of the frequency integers, whenever we have successive estimations (estimations: $f \to A$ or $f \to \varphi$). To get a figure of how good the estimations are, we checked the errors of estimations for one sine component in the signal with double scan ($A_m = 1$; $N = 1024$; $1.5 < \theta \le 6$, $\Delta\theta = 0.01$ and $-\pi/2 \le \varphi \le \pi/2$, $\Delta\varphi = \pi/90$) with added suitable noise to the sine signal at each frequency and phase angle. In estimations of the component, first the three fundamental parameters were estimated, and then a theoretical sine signal was generated on the basis of them and also its DFT (rectangular window). The difference of spectra shows the goodness of estimation. The largest deviations are around the largest DFT components ($\ldots, i_m - 1, i_m, i_m + 1, \ldots$). To avoid the short leakage effect of the rectangular window, the relative maximal values of errors at the given relative frequency were searched for the amplitude DFT coefficient at index $i = i_m + 2$ (Figs. 4, 5):

$$\left|e(i_m + 2)\right|_{\max} = \left|\frac{\left|G(i_m + 2)\right|_{\text{est+noise}} - \left|G(i_m + 2)\right|_{\text{true (noiselees)}}}{A_m}\right|_{\max} \tag{11}$$

The number of simulation points has been reduced due to the exploded computer load when we compare the three-point interpolated DFT algorithms to the least-squares four-parameter sine-fit estimations (ten iterations for each estimation according to [1]). The rectangular distributed noise was added 40 times at each value of the phase at the specific frequency and the largest error was searched. We can see, that after the systematic leakage contribution drops under the noise floor (time-domain noise is reduced for about $-30\,\mathrm{dB} \approx 20\log\left(1/\sqrt{1024}\right)$) the behavior of errors from Fig. 2 changes in a manner from Figs. 4 and 5. The maximal values of errors of the four-parameter sine-fit estimations changes values for about three times depend on the position of the frequency ($e_{4\mathrm{p}}(\delta \approx 0)/e_{4\mathrm{p}}(\delta \approx 0.5) \approx 3$), but they are lower then the maximal values of errors of the three-point estimation for about 1.4 times (Fig. 5: $\hat{e}_{\mathrm{IDFT}}(\delta \approx 0.5)/\hat{e}_{4\mathrm{p}}(\delta \approx 0.5) \approx 1.3$ and $\hat{e}_{\mathrm{IDFT}}(\delta \approx 0)/\hat{e}_{4\mathrm{p}}(\delta \approx 0) \approx 1.5$). The behaviors of the maximal errors have the shapes, which are very close to the shapes of the theoretical standard deviations of the amplitude estimations in Fig. 2.

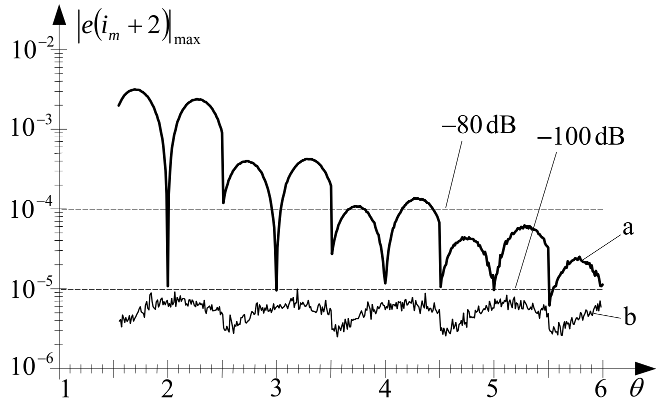

Fig. 4. Relative maximal errors of the DFT at spectral line $i_m + 2$: a – 3-point IDFT, b – four-parameter sine-fit estimation; $b(\mathrm{ADC}) = 12$, $\sigma_t = 1/2^{12}\sqrt{3} = 1.41 \cdot 10^{-4}$, and $\mathrm{SNR} \approx 77\,\mathrm{dB}$

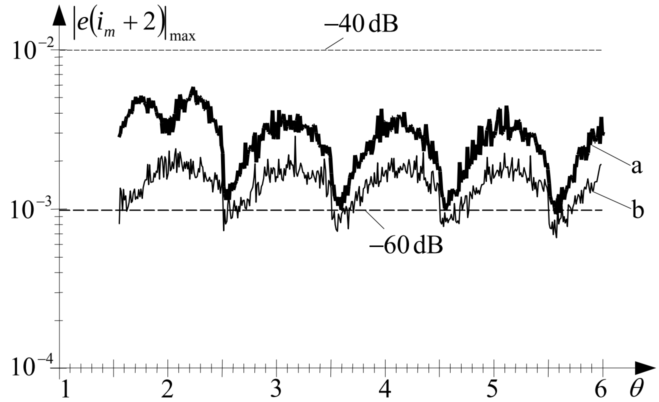

Fig. 5. Relative maximal errors of the DFT at spectral line $i_m + 2$: a– 3-point IDFT, b – four-parameter sine-fit estimation; $b(\mathrm{ADC}) = 4$, $\sigma(t) = 1/2^4\sqrt{3} = 3.608 \cdot 10^{-2}$, and $\mathrm{SNR} \approx 29\,\mathrm{dB}$

IV. Experimental results

An example of real measurement system: sampling DVM (HP3458: $f_\mathrm{S} = 100\mathrm{kHz}$, $U_{\mathrm{range}} = 1\mathrm{V}$) and arbitrary waveform generator (HP3245A)) has been used. The original sample with $N = 1024$ points of the stimulating sine function $g(t) = 1 \cdot \sin(2\pi \cdot 5.9 \cdot t/T_\mathrm{M} + 0.35)$ has been taken in $T_\mathrm{M} = 10.24\,\mathrm{ms}$ with $\theta_m = 5.898$. For experimental evaluation of algorithms, a section of the sample containing a number of samples that is as near an integer value of periods as possible is chosen, what is usually done in measurement practice (reduced sample: $N_1 = \mathrm{round}\left[N \cdot \mathrm{intPart}(\theta_\mathrm{m})/\theta_\mathrm{m}\right] = 868$). Since the number of data points in the reduced sample is usually not a power of two any more, it is necessary to use a standard DFT rather than a more effective FFT algorithm. At this step, parameters of the fundamental component have to be estimated and the estimated excitation sine has to be subtracted from the signal (Fig. 6). Estimated sine on the reduced sample has $\theta_m = 4.9996$, $A_m = 1.0002$, $\varphi_m = 0.35031$.

With the interpolated DFT algorithms is possible to investigate also the residual components from Fig. 6a that is not the case with the four-parameter sine-fit estimation. To show the effectiveness of the three-point estimation we have made examinations of the second harmonic component (Fig. 6c), the third harmonic component (Fig. 6d), and the non-harmonic component (Fig. 6e). The second harmonic has the amplitude DFT value $|G(i_m = 10)| = 2.181 \cdot 10^{-4}$ and estimated values as follows: frequency $\theta_{2H} = 10.02$ and amplitude $A_{2H}(\theta_{2H}) = 2.262 \cdot 10^{-4}$. The third harmonic has the amplitude DFT value $|G(i_m = 15)| = 6.142 \cdot 10^{-5}$ and estimated values: $\theta_{3H} = 14.81$ and $A_{3H}(\theta_{3H}) = 6.394 \cdot 10^{-5}$. It is possible to estimate also the interharmonic component around $i_m = 18$: $|G(i_m = 18)| = 2.937 \cdot 10^{-5}$, $\theta_e = 18.22$, and $A_e = 3.543 \cdot 10^{-5}$. All estimated amplitudes differ from the amplitude DFT coefficients: the larger is the distance of the frequency to the integer value the larger is the difference. The component estimation is even more important at higher frequencies since these components don't coincide with the spectral lines. The results could be better by subtraction the largest residual components before the estimations of the smaller ones. The least-squares four-parameter sine-fit estimations completely fail in all three cases since the amplitudes of the components are on the level of the time domain noise $A_{noise} = \sqrt{3} \cdot \sigma_t \approx 2.44 \cdot 10^{-4}$ ($\leftrightarrow$ the second harmonic component: $A_{2H} = 2.262 \cdot 10^{-4}$) or even lover (the third harmonic and the non-harmonic components).

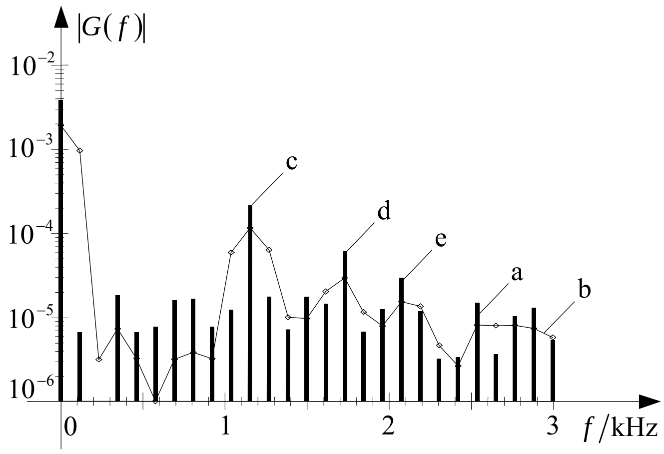

Fig. 6. The DFT of the residual after subtraction of the fundamental component leakage on the reduced sample: a. with the rectangular window; b. with the Hanning window; c. the second harmonic component; d. the third harmonic component; e. the non-harmonic component

V. Conclusions

The paper compares two basic procedures used in estimations of the significant components in the residual spectrum in an ADC dynamic testing. The comparison of the three-point estimation algorithm to the least-squares four-parameter sine-fit estimation described in the IEEE standard 1241 shows the time effectiveness (no iterations) and the reduction of the computational effort, which is very important with a huge amount of the sampling points in the test methods for the A/D converters. The three-point estimation can be used also for the harmonic components estimations under the time domain noise floor, if we have enough sampling points, where the four-parameter sine-fit estimation often fails.

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