Accuracy of ADC dynamic parameters measurement

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Abstract**-**Based on a model of a test signal generator, an accuracy estimator was designed and implemented. The proposed estimator uses computer simulations to obtain expected bias of dynamic parameters of an analog-to-digital converter (ADC). The usage of the method for estimation of ADC dynamic parameters measurement accuracy for the best sine-wave fit and the frequency domain analysis using the DFT will be presented in this paper.

I. Introduction

The imperfections of test signal generator and signal path from the generator to inputs of the tested ADC are main sources of errors in the testing and measurement of ADCs' dynamic parameters. Other parts of the testing chain are the ADC under test and relevant analytical framework that processes sampled data to obtain dynamic parameters of the tested ADC. The two latter parts are not relevant for bias evaluation – the ADC is considered to be a black box with parameters under investigation, and it is relatively simple to change the settings of analytical framework (compared to the possibilities of changing parameters of the test signal generator).

II. Proposed Approach

A. Test signal generator model

To reflect all usual sources of distortion in a sine-wave generator a model of the sine-wave signal generator was created. The model derivation begins with the formula describing a sine wave with DC offset (mean value) V in the time domain:

$$x(t) = V + A\sin(2\pi ft + \varphi) \tag{1}$$

Now, consider two main kinds of distortion to this ideal sinus signal: the parasitic amplitude modulation and additive noise:

$$m(n) = A_{\mathrm{mod}} \sin\left(2\frac{\pi n f_{\mathrm{mod}}}{f_{samp}}\right) \tag{2}$$

$$e(n) = A_{noise} e_{norm}(n) \tag{3}$$

where n is number of samples, A_{mod} is amplitude and f_{mod} is frequency of signal which causes parasitic amplitude modulation, A_{noise} is amplitude of noise distortion and $e_{norm}(n)$ is nth sample generated by a noise generator with normal distribution and standard deviation of 1.

The generic function combining both sources of distortion and the distorted mean value V can be defined as:

$$q_i(n, Q_{q_i}, \alpha_{q_i}, \beta_{q_i}, \gamma_{q_i}) = Q_{q_i} + \alpha_{q_i} e_{q_i}(n) + \beta_{q_i} \sin\left(2\frac{\pi n \gamma_{q_i}}{f_{samp}}\right) \quad (4)$$

where Q_{qi} is the distorted mean value, $e_{qi}(n)$ is the nth noise sample, α_{qi} is the amplitude of noise, β_{qi} is the amplitude of parasitic modulation and γ_{qi} is the frequency of parasitic amplitude modulation.

Using the function (4), the sine-wave generator with undesirable modifications of all three parameters (DC offset, amplitude and phase) can be modeled by the following formula:

$$\begin{aligned} &x(n, f, f_{vz}, DC, \alpha_{DC}, \beta_{DC}, \gamma_{DC}, A, \alpha_A, \beta_A, \gamma_A, \varphi, \alpha_\varphi, \beta_\varphi, \gamma_\varphi) = \\ &q_{DC}(n, DC, \alpha_{DC}, \beta_{DC}, \gamma_{DC}) + \\ &q_A(n, A, \alpha_A, \beta_A, \gamma_A) \sin\left(2\frac{\pi n f}{f_{samp}} + q_\varphi(n, \varphi, \alpha_\varphi, \beta_\varphi, \gamma_\varphi)\right) \end{aligned} \quad (5)$$

The proposed model allows simulation of the effects of noise, parasitic amplitude modulation, variations of the offset (DC level), and frequency of an ideal sine-wave.

B. Uncertainty analysis

Analysis of proposed model in terms of standard uncertainties was performed. Goal of analysis was to determine how results of test methods would be affected by each source of distortion present in signal generator model.
For uncertainty analysis signal generator model can be expanded into following form:

$$\begin{aligned} &x(n, f, f_{samp}, DC, \alpha_{DC}, \beta_{DC}, \gamma_{DC}, A, \alpha_A, \beta_A, \gamma_A, \varphi, \alpha_\varphi, \beta_\varphi, \gamma_\varphi) = \\ &\left[DC + \alpha_{DC} e_{DC}(n) + \beta_{DC} \sin\left(2\frac{\pi n \gamma_{DC}}{f_{vz}}\right)\right] + \\ &\left[A + \alpha_A e_A(n) + \beta_A \sin\left(2\frac{\pi n \gamma_A}{f_{samp}}\right)\right] \sin\left(2\frac{\pi n f}{f_{samp}} + \left[\varphi + \alpha_\varphi e_\varphi(n) + \beta_\varphi \sin\left(2\frac{\pi n \gamma_\varphi}{f_{samp}}\right)\right]\right) \end{aligned} \quad (6)$$

Analysis then can be separated in two parts: one for parasitic amplitude modulation, and one for additive noise influence. In following analysis use of ideal digitizer and coherent sampling is assumed.
For analysis of parasitic amplitude modulation let us suppose that all additive noise sources are equal to zero:

$$\begin{aligned} &x(n, f, f_{samp}, 0, 0, \beta_{DC}, \gamma_{DC}, A, 0, \beta_A, \gamma_A, 0, 0, \beta_\varphi, \gamma_\varphi) = \\ &\beta_{DC} \sin\left(2\frac{\pi n \gamma_{DC}}{f_{samp}}\right) + \\ &A \sin\left(2\frac{\pi n f}{f_{samp}} + \beta_\varphi \sin\left(2\frac{\pi n \gamma_\varphi}{f_{samp}}\right)\right) + \\ &\beta_A \sin\left(2\frac{\pi n \gamma_A}{f_{samp}}\right) \sin\left(2\frac{\pi n f}{f_{samp}} + \beta_\varphi \sin\left(2\frac{\pi n \gamma_\varphi}{f_{samp}}\right)\right) \end{aligned} \quad (7)$$

For test method using DFT, there are following spectral components present in amplitude frequency spectrum:

- one spectral component for DC offset distortion:

$$\beta_{DC} \sin\left(2\frac{\pi n \gamma_{DC}}{f_{samp}}\right) \quad (8)$$

- spectral components corresponding to phase modulation of carrier:

$$A\sin\left(2\frac{\pi nf}{f_{samp}}+\beta_\varphi \sin\left(2\frac{\pi n\gamma_\varphi}{f_{samp}}\right)\right) \tag{9}$$

- spectral components generated by parasitic amplitude and phase modulation of carrier:

$$\beta_A \sin\left(2\frac{\pi n\gamma_A}{f_{samp}}\right)\sin\left(2\frac{\pi nf}{f_{samp}}+\beta_\varphi \sin\left(2\frac{\pi n\gamma_\varphi}{f_{samp}}\right)\right) \tag{10}$$

These components will be reflected in SINAD as increase of distortion power as follows:

- DC component will increase distortion proportionally to β_{DC}.
- Phase modulation of carrier will introduce spectral components determined generally by Bessel function. However, if parameter β_φ is less than 0.2, phase modulation can be simplified to

$$A\sin\left(2\frac{\pi nf}{f_{samp}}\right)+A\frac{\beta_\varphi}{2}\cos\left(2\frac{\pi n(f+\gamma_\varphi)}{f_{samp}}\right)-A\frac{\beta_\varphi}{2}\cos\left(2\frac{\pi n(f-\gamma_\varphi)}{f_{samp}}\right) \tag{11}$$

  and frequency spectrum will contain two spectral components on frequencies $f + \gamma\varphi$ and $f - \gamma\varphi$ with magnitude $A\beta\varphi / 2$.
- By applying previous assumption, parasitic modulation of amplitude and phase of carrier will simplify to

$$\beta_A \sin\left(2\frac{\pi n\gamma_A}{f_{samp}}\right)\left[A\sin\left(2\frac{\pi nf}{f_{samp}}\right)+A\frac{\beta_\varphi}{2}\cos\left(2\frac{\pi n(f+\gamma_\varphi)}{f_{samp}}\right)-A\frac{\beta_\varphi}{2}\cos\left(2\frac{\pi n(f-\gamma_\varphi)}{f_{samp}}\right)\right] \tag{12}$$

  For $\beta_A << 1$ can cosine components be ignored, and previous formula can be reduced to

$$A\frac{\beta_A}{2}\cos\left(2\frac{\pi n(f+\gamma_A)}{f_{samp}}\right)-A\frac{\beta_A}{2}\cos\left(2\frac{\pi n(f-\gamma_A)}{f_{samp}}\right) \tag{13}$$

  Spectrum then will contain components on frequencies $f + \gamma_A$ a $f - \gamma_A$ with magnitude $A\beta_A / 2$.

Formula for SINAD calculation then can be updated to following form:

$$\mathrm{SINAD} = 10\log\frac{\mathrm{S}}{\mathrm{N}+\left(\mathrm{B}_{DC}+2\mathrm{B}_\varphi+2\mathrm{B}_A\right)} \tag{14}$$

$\beta_\varphi < 0,2$ and $\beta_A << 1$ (B_{DC}, B_φ and B_A are powers of corresponding spectral components).

For method using best sine-wave fit all assumptions and approximations used for DFT method can be used. Parasitic amplitude modulation then appears in subtraction of digitized and fit sine-wave residua as follows:

$$\begin{aligned}&\beta_{DC}\sin\left(2\frac{\pi n\gamma_{DC}}{f_{samp}}\right)+\\&A\frac{\beta_\varphi}{2}\cos\left(2\frac{\pi n(f+\gamma_\varphi)}{f_{samp}}\right)-A\frac{\beta_\varphi}{2}\cos\left(2\frac{\pi n(f-\gamma_\varphi)}{f_{samp}}\right)+\\&A\frac{\beta_A}{2}\cos\left(2\frac{\pi n(f+\gamma_A)}{f_{samp}}\right)-A\frac{\beta_A}{2}\cos\left(2\frac{\pi n(f-\gamma_A)}{f_{samp}}\right)\end{aligned} \tag{15}$$

RMS of residua then will be described by formula:

$$\begin{aligned}e_{rms}&=n_{quant}+\beta_{DC}\sqrt{\frac{1}{N}\sum_{n=1}^{N}\sin\left(2\frac{\pi n\gamma_{DC}}{f_{samp}}\right)^2}+\\&A\frac{\beta_\varphi}{2}\sqrt{\frac{1}{N}\sum_{n=1}^{N}\cos\left(2\frac{\pi n(f+\gamma_\varphi)}{f_{samp}}\right)^2}-A\frac{\beta_\varphi}{2}\sqrt{\frac{1}{N}\sum_{n=1}^{N}\cos\left(2\frac{\pi n(f-\gamma_\varphi)}{f_{samp}}\right)^2}+\\&A\frac{\beta_A}{2}\sqrt{\frac{1}{N}\sum_{n=1}^{N}\cos\left(2\frac{\pi n(f+\gamma_A)}{f_{samp}}\right)^2}-A\frac{\beta_A}{2}\sqrt{\frac{1}{N}\sum_{n=1}^{N}\cos\left(2\frac{\pi n(f-\gamma_A)}{f_{samp}}\right)^2}\end{aligned} \tag{16}$$

where n_{quant} is RMS value of quantization noise.

Standard uncertainty type B of SINAD can be evaluated from equation (14). For method using DFT (in compliance with methodology used for standard uncertainties evaluation), standard uncertainty type B is described by formula

$$u_B=\sqrt{\left(\sigma_{DC}{}^2+\sigma_A{}^2+\sigma_\varphi{}^2\right)\left(\frac{10}{N+\mathrm{B}_{DC}+2\mathrm{B}_A+2\mathrm{B}_\varphi}\right)^2} \tag{17}$$

where σ_{DC}, σ_φ a σ_A are standard deviations of B_{DC}, B_φ a B_A. Their values are determined by analysis of B_{DC}, B_φ a B_A values for different configurations of measurement (various numbers of samples, resolution of digitizer, etc.).

For method using best sine-wave first rewrite formula (16) to following form:

$$e_{rms}=n_{kvant}+\beta_{DC}a_{DC}+A\frac{\beta_\varphi}{2}a_\varphi+A\frac{\beta_A}{2}a_A \tag{18}$$

Then standard uncertainty type B of ENOB is defined by formula:

$$u_B=\sqrt{\frac{1}{e_{rms}{}^2}\left[(\sigma_{DC}a_{DC})^2+\left(\sigma_\varphi\frac{A}{2}a_\varphi\right)^2+\left(\sigma_A\frac{A}{2}a_A\right)^2\right]} \tag{19}$$

where σ_{DC}, σ_φ a σ_A are standard deviations of determination of parameters β_{DC} , β_φ a β_A.

Uncertainty caused by additive noise present in the model was left to statistical analysis, and should be evaluated as standard uncertainty type A, which is obtained as standard deviation of multiple measurements.

C. Simulations

The above mentioned model was used in extensive set of simulations, which covered reasonable span

of input parameters of model, to obtain detailed information about best sine-wave fit and frequency domain analysis test methods behavior.
Figure 1 shows representative result of simulations where sine-wave distortion sources influence the test signal. On the vertical axis is relative value of SINAD, horizontal axis represents combinations of sine-wave distortion sources corresponding with described model of the sine-wave generator. Increasing cumulative influence of different sources of distortions is clearly visible.

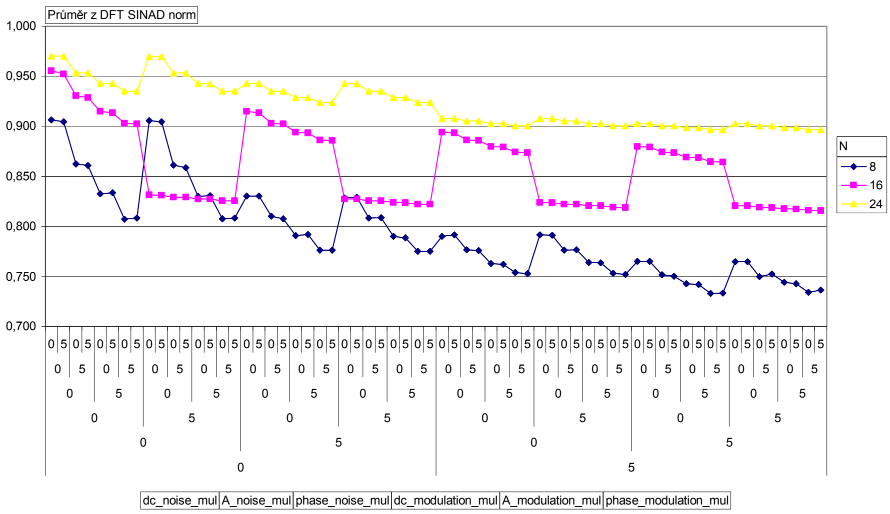

Figure 1. Relative value of SINAD obtained by analysis in frequency domain for selected resolutions (N=8, 16, 24 bits) and different distortion sources levels (number of samples 65535, 7-term Blackman & Harris time window, amplitude 98% of the full scale)

Simulations with parasitic amplitude modulation were also used to verify results of uncertainty analysis. Results of 16-bit digitizer behavior simulations of were compared to results given by formulas presented in uncertainty analysis (see table 1). Comparison shown, that estimations made by equation (14) are relevant for situations where values of model's parameters are not getting close to constraints given in uncertainty analysis. For wider range of parameters' values differences in results start to be significant.

Table 1. Comparison of simulated results and estimations given by uncertainty analysis. Resolution 16-bit.

parasitic amplitude modulation levels	$\beta_{DC}=\beta_A=\beta_\varphi=$ 0.100	$\beta_{DC}=\beta_A=\beta_\varphi=$ 0.500	$\beta_{DC}=\beta_A=\beta_\varphi=$ 1.000	β_{DC} = 0.5 β_A = 0.1 β_φ= 0.5
SINAD calculated	93.374	88.591	83.725	89.686
SINAD estimation	93.599	91.038	87.278	91.751
SINAD difference	0.225	2.448	3.553	2.065

C. The proposed estimator

Because results given by formulas obtained by model analysis were not accurate in wide enough range of input conditions, decision to create software bias estimator was taken. Estimator takes simulated data and by their analysis predicts how results of test method would be affected by signal generator parameters while digitized by ideal digitizer. It allows to decide whether measurement with particular signal source is or is not valid.
The estimator requires to perform comprehensive set of simulations which covers conditions in which generator should be used (generation of these data is automated, user sets parameters of simulations only). These data are loaded by estimator, and when parameters of signal source corresponding to parameters of described model are entered, estimator searches data to find how results were affected by

similar configuration in simulation. Because there are several ways how to get to desired results, all of these paths are followed and from average of simulated results bias estimation is calculated. Along with bias estimation its standard deviation is calculated.

Table 2 shows results of estimation of bias for particular configuration of signal generator model, and Table 3 presents several realizations of measurement for the same configuration of distortion sources (for 16-bit ADC). Values Δ SINAD DFT and Δ SINAD SWF represent differences between "ideal" SINAD and expected results of frequency domain analysis and best sine-wave fit, respectively, for given configuration of model's distortion sources (greater difference for DFT test is caused by test signal amplitude and time window influence). Also the standard deviation of bias estimation is calculated and displayed.

Table 2. Estimator output

Δ SINAD DFT [dB]		Δ SINAD SWF [dB]	
Bias estimation	Standard deviation of bias	Bias estimation	Standard deviation of bias
2.76	0.90	2.16	0.96

Table 3. Simulation results.

Realization	Δ SINAD DFT [dB]	Δ SINAD SWF [dB]
1	3.07	1.74
2	3.09	1.79
3	3.07	1.76
4	3.07	1.74
5	3.02	1.74
6	3.09	1.78
7	3.01	1.77
8	3.01	1.70
9	3.06	1.71
10	3.03	1.75

III. Conclusions

The model respecting the imperfections of the sine-wave test signal generator was presented. It was used to analyse behavior of test methods for evaluation of ADC dynamic parameters. Based on simulated data an estimator of accuracy of ADC dynamic parameters was designed and its predictions were verified.

References

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