

Approaches to the ADC transfer function modelling

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Abstract- This paper describes several approaches to model ADC nonlinearities. It mentions and describes power series approximation, Chebyshev polynomials, Bessel functions and Hammerstein block structure.

I. Introduction

The performance of current signal processing tasks is limited by the analogue front end and analogue-to-digital converter's (ADC) non-idealities. It is therefore an essential task to test and model the crucial parameters of analogue-to-digital converters.

An analogue-to-digital converter's performance is not, as commonly known, ideal. It behaves not only nonlinearly but also suffers from dependency on the test signal. There exist many different opinions in describing nonlinearities, for example dividing the nonlinearities into two groups, static and dynamic. Static part is described by, for example, a polynomial or a function, while the dynamic part depends on the parameters of the input signal, for example, the frequency or amplitude. Another approach classifies nonlinearities according to where in an ADC they arise.

The curve of the INL specifies how the overall transfer function deviates from the ideal staircase function. The curve is not a smooth function, it usually contains sharp transitions. The IEEE standard [1] proposes several ways of measuring the INL of a real converter. One of the methods uses time consuming statistical approach that requires a huge number of samples obtained from the ADC output. The procedure to obtain the number of needed samples is dependent on the confidence level and is described in [1]. The number of needed samples would be, for example, round 8 million for a 14 bit ADC and a confidence level of 99% [2].

The nonlinearity is inherently described by the Integral Non-Linearity (*INL*) – a parameter that is commonly given in manufacturer's datasheets – and plots the overall shape of the transfer function of an analogue-to-digital converter (ADC).

In the following text, the static model means a polynomial expression of nonlinearity, while the dynamic is dependent on the input signal. It can also be understood that taking only the static portion of the non-linearity means considering only input frequencies much lower than the sampling frequency. This paper attempts to provide an overview on mostly used types of models.

II. Approximation of the INL functions

One of the most simple, although used, possibilities to model the *INL* curve is based on polynomial expression. In the following text the power series representation, Chebyshev polynomials and Bessel functions will be described. Generally, the influence of the nonlinearity can be described as

$$y(t) = f[x(t)] = h[x(t)] + g[x(t)], \quad (1)$$

where $f[.]$ represents passing of the signal through an ADC, $h[.]$ linear function and $g[.]$ nonlinear function. In this section, the following input signal is for simplicity considered

$$x(t) = \cos(\omega t). \quad (2)$$

A. Polynomial approximation

Power series approximation

If we have a linear only system we would observe no higher harmonics in the output spectrum when a single sine wave is input. If harmonics are observed the system is nonlinear and contains static nonlinearity given by the output-to-input relation (the transfer function). An ADC with only static nonlinearity is nearly an ideal case. The static nonlinearity (1) applied to an input signal can be expressed

by a power series expansion

$$y(t) = x(t) + \sum_{i=2}^n a_i x^i(t), \quad (3)$$

where y is the ADC output, x is the input and a_i are the coefficients of the static nonlinearity. If the input signal was (2) then the output of the static nonlinearity would yield

$$y(t) = x(t) + \sum_{i=2}^n a_i \cos^i(\omega t), \quad (4)$$

The powers of a cosine can be calculated by (5) [5]

$$\begin{aligned} \cos^{2n} \alpha &= \frac{1}{2^{2n}} \left[\binom{2n}{n} + 2 \sum_{k=0}^{n-1} \binom{2n}{k} \cos(2\alpha(n-k)) \right] \\ \cos^{2n-1} \alpha &= \frac{1}{2^{2n-2}} \sum_{k=0}^{n-1} \binom{2n-1}{k} \cos(\alpha(2n-2k-1)). \end{aligned} \quad (5)$$

The output – expressed as (4) – contains only higher cosine terms, therefore no phase shift. All higher cosine terms representing higher harmonic components of the input signal are in-phase with the input signal. General expression of the output spectrum from known polynomial coefficients is

$$Y_k = \sum_{n=0}^m \frac{(2n+k)!}{2^{2n+k-1} n!(n+k)!} a_{2n+k} X_1^{2n+k}, \quad (5)$$

where X_1 is the power of the input signal, $m = (N-k)/2$ for $N-k$ even and $m = (N-k-1)/2$ for $N-k$ odd, where N is the highest power considered.

The disadvantage of the simple power series approach is that the powers of the input are not orthogonal; therefore an error in estimation of one coefficient will affect other coefficients.

Chebyshev polynomials approximation

As mentioned in the introduction, the number of samples in case of classical statistical approach of obtaining the *INL* achieves huge numbers. This number can significantly be reduced when the approximation by Chebyshev polynomials is used. The price paid for the speed is that this approximation only presents the low frequency of the nonlinearity curve.

The Chebyshev polynomials of the first kind $T_n(x)$ are defined as

$$T_n(\cos(x)) = \cos(nx) \quad (6)$$

and can be calculated recursively as

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x). \quad (7)$$

The benefit from the Chebyshev polynomials is their orthogonality on the interval $[-1;1]$. If the output forms

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t), \quad (8)$$

where the a_n are coefficients of the Fourier series representation of the output function and are formed by the static nonlinearity directly. Then, using the a_n coefficients the nonlinearity can be modelled as

$$g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n T_n(x). \quad (9)$$

The coefficients can directly be obtained from the FFT of the output when a harmonic signal (2) is input.

The authors of [3,4] developed two versions of so called Chebyshev test. The former one was obtained from the FFT but was limited to coherent sampling, while the later one uses the parametric spectral estimation method (multi-harmonic sine estimation). Then the a_n coefficients can be found from the spectrum after a simple scaling. Generally spoken, the *INL* is a function with lots of sharp transitions. This approximation, however, only reconstruct the smooth part of the *INL* as is not capable of finding details in the *INL* curve.

The authors of [3,4] sum up, that from the modelled nonlinearity they needed only 8 thousand samples regardless the ADC resolution. The future effort should be aimed to approximate the *INL* by the Chebyshev polynomials of the second kind that are believed to handle also out-of-phase spectrum components that are assigned to hysteresis.

B. Bessel function approximation

Another approach to the polynomials is to use Bessel functions to approximate the curve of the *INL*. Unlike the Chebyshev polynomials the Bessel functions are capable to approximate also the smooth portion of the *INL* curve. The method proposed in [2] models the *INL* curve more precisely and thus can also provide some dynamic parameters such as Spurious Free Dynamic Range (SFDR) or Total Harmonic Distortion (*THD*). The method is also based on Fourier series expansion

$$y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)], \quad (10)$$

where the coefficient a_n and b_n are found using the classical expressions. In real processing the evaluation is carried out by FFT and

$$\begin{aligned} \cos[\alpha \cos(p)] &= J_0(\alpha) + 2 \sum_{h=0}^{\infty} J_{2h}(\alpha) (-1)^h \cos(2hp) \\ \sin[\alpha \cos(p)] &= 2 \sum_{h=0}^{\infty} J_{2h+1}(\alpha) (-1)^h \cos((2h+1)p) \end{aligned} \quad (11)$$

where $J_h(\alpha)$ is the Bessel function of the first kind with order h . Then, the output of the ADC can be rewritten as

$$y(n) = x(n) + \sum_{h \geq 0} S_h^t \cos(h\varphi_n), \quad (12)$$

where S_h^t is the theoretical amplitude of the h^{th} harmonic component. S_h^t can be expressed as a sum of Bessel functions so that even components of the *INL* function (represented by even a_n coefficients) produce even harmonic components while odd ones are associated with odd harmonic components.

C. Hammerstein block structure

The previous model described only static nonlinearity. When dynamic behaviour is needed a dynamic model has to be used. One of the possible choices is so called Hammerstein structure, where a static nonlinearity is followed by a linear filter as we can see in the Figure 1.

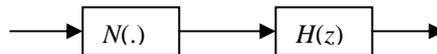


Figure 1. Block model of a Hammerstein system

The authors of [6] show, that when different powers of nonlinearity are filtered by different filters – such a model is called a Parallel Hammerstein model. This type of a model can also be efficiently used for modelling. The model then consists of multiple dynamics parts.

III. Conclusions

In the paper a selection of approximation of the transfer function of an analogue-to-digital converter was described and compared. The models considered were a power series approximation, Chebyshev polynomials, Bessel functions, Hammerstein block structure and the approximation based Volterra kernels. Another modelling technique is based on Volterra kernels usage. This model has the advantage that it can also model weak memory effects in nonlinear system. This model is not described in this paper.

References

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