

# An experimental study aimed at analysing horizontal quantization and time-base jitter effects in waveforms generated by means of DACs

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**Abstract**-This work analyzes the imperfections that characterize waveforms generated by means of DACs. The attention is mainly paid to horizontal quantization, which is due to the discrete nature of the waveform to be played. Also the effects of time-base jitter, which occur in DAC functioning, are investigated when combined to horizontal quantization. Further remarks are related to other minor effects such as those caused by vertical quantization, which is due to the limited resolution of any DAC.

## I. Introduction

Waveform generation by means of digital-to-analogue converters (DACs) is the basic generation technique adopted by arbitrary waveform generators (AWG) and digital modulators. Sometimes, this technique represents the unique solution to play in a controlled manner signals characterized by ringing, spikes, glitches, etc., as well as signals attained through complex modulation schemes.

The operation mode exploited by AWGs and digital modulators that use an internal DAC is simple in principle, and characterized by great versatility and straightforwardness. In fact, the internal DAC, operating as zero-order-hold circuit, gives in output a piece-constant waveform that approximates the desired waveform. The approximation grade is finely improved imposing a generation frequency much greater than the Nyquist frequency. In addition, since the waveform produced by the DAC is characterized by a discrete series of abrupt voltage steps, an all-purpose low-pass filter is adopted to smooth the aforementioned steps while creating the final waveform.

The major criticisms that can be addressed to this operation mode are:

- the use of a generation frequency that is much greater than the minimum one required involves a suboptimal use of the generator;
- since the smoothing filter definitively determines the precision related to the parameters of the output waveform, the user should be capable of configuring it according to the features of the particular waveform that intends to play; at the contrary, he has only limited configuration choices;
- it is very difficult specifying the precision related to the parameters of the generated waveform.

At the state of art, however, no alternative operation mode offering superior or at least comparable performance has been proposed. Also, all drawbacks that characterize waveforms generated by means of DACs, especially in the presence of critical choices of the generation frequency, have still not been thoroughly investigated. Nonetheless, the comprehension of the nature and features of these drawbacks is essential to either improve the performance of generators relying on the described operation mode, or develop new generators based on different approaches.

This work analyzes phenomena, such as horizontal quantization, time-base jitter, and vertical quantization, which spoil waveforms produced by DAC, and limit the performance of the generator. The attention is mainly paid to horizontal quantization, which is due to the discrete nature of the waveform and becomes the chief degradation factor whenever the adopted generation frequency is not sufficiently greater than the Nyquist frequency.

In particular, a model for the analysis of horizontal quantization and time-base jitter effects in the frequency domain is first presented. Then, an experimental approach is exploited to assess the proposed model, investigate the effects of vertical quantization, which is due to the limited resolution of any DAC, and examine DDS mechanism that originate time-base jitter. Concluding remarks suggest guidelines to extend the applicability of the proposed model to the analysis of non sinusoidal waveforms.

## II. Waveform generation

*The role of the DAC:* There are two fundamental approaches in waveform generation through digital to analogue conversion, and two correspondent kinds of generator. Both kinds utilize DACs that work as zero-order-hold circuits to convert the input waveform in analogue form. The input waveform is described by a series of data points that are downloaded and arranged sequentially into the local memory of the DAC. To address data points a suitable address register is utilized.

The generator of the first kind operates throughout the following actions:

- a clock signal, the rate of which can be chosen by the user, is exploited to trigger the DAC;
- anytime the DAC is triggered, the address register is incremented to withdraw a new data point from the memory, and the output voltage steps up or down to a new voltage level.

Thus, data points in the DAC memory are sequentially scrolled. The time duration that characterizes the output waveform depends on the chosen clock rate, also referred to as generation frequency.

The generator of the second kind, which is based on direct digital synthesis (DDS), proceeds differently. In particular, DDS theory of operation utilizes a long register, named accumulator, which is separated in two smaller parts: the first one, made up of the P most significant bits, is utilized as address register, the second one, containing the remaining Q bits, is a residual register. The generator operates throughout the following actions:

- the DAC is triggered at a fixed clock rate, typically the maximum one;
- the value in the accumulator is incremented by a certain amount at any clock pulse: for a given generation frequency fg, and a fixed clock rate fck, the incremental value is the integer part of the value:

$$\Delta\theta = 2^Q / (f_{ck} / f_g) \quad (1)$$

- when the accumulated increments produce a carry bit that, in turn, increments the value contained in the P most significant bits, the output voltage steps up or down to a new voltage level.

Thus, data points loaded in memory are not scrolled at any trigger event, but only when a certain amount, not necessarily constant, of clock pulses has been accumulated. When high scrolling speeds are required, the value  $\Delta\theta$  is greater than 2Q and some points in the DAC memory are skipped. The time duration is regulated by the number of clock pulses needed to scroll each data point.

*The role of the output filter:* Concerning smoothing filters, for continuous sine waves or narrow band signals, elliptical filters characterized by nearly flat passband and sharp cutoff should be adopted, while, in the presence of wideband signals, linear phase filters are the best, since elliptical filters exhibit severe ringing.

Actually, AWGs permit to switch between a few available filters, designed for general applicability, while digital modulators are just equipped with a single filter, characterized by a fixed frequency response, even if they can generate waveforms whose bandwidth varies according to the acknowledged baud-rate. The choice of a generation frequency much greater than the bandwidth of the signal to be played, is, thus, necessary to make effective the use of the all-purpose output filter.

### III. Horizontal quantization

This section is exclusively focused on the analysis of horizontal quantization effects, thus, the generation period  $T_g$  is here a constant parameter, and no time-base jitter effects are considered.

Let  $x(n)$  be a digital signal to be produced in analogue form by means of a digital-to-analogue converter (DAC) that plays as zero-order-hold circuit. The obtained analogue signal  $y_x(t)$  is piece-constant and can be represented by:

$$y_x(t) = \sum_{n=-\infty}^{\infty} x(n)g(t - nT_g) \quad (2)$$

where  $g(t)$  is a rectangular pulse of unitary amplitude and time duration equal to  $T_g$ . For a deterministic signal  $y_x(t)$ , the spectrum  $Y_x(f)$  is given by the Fourier transform:

$$Y_x(f) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(n)g(t - nT_g) \cdot e^{-j2\pi ft} dt = \sum_{n=-\infty}^{\infty} x(n) \int_{nT_g}^{(n+1)T_g} e^{-j2\pi ft} dt = T_g \operatorname{sinc}(T_g f) \cdot e^{-j\pi T_g f} \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j2\pi n T_g f} \quad (3)$$

In the presence of a sinusoidal digital input  $x(n)$ :

$$x(n) = \sin(2\pi f_0 n T_g) \quad (4)$$

characterized by a nominal frequency  $f_0$  and a generation period  $T_g$ , from the general expression given in (3), it is obtained:

$$Y_x(f) = T_g \operatorname{sinc}(T_g f) \cdot e^{-j\pi T_g f} \sum_{n=-\infty}^{\infty} \frac{e^{j2\pi n(f-f_0)T_g} - e^{-j2\pi n(f+f_0)T_g}}{2j} \quad (5)$$

Using the identity:

$$\sum_{n=-\infty}^{\infty} T_g e^{j2\pi n f_g} = \sum_{k=-\infty}^{\infty} \delta(f - k f_g) \quad (6)$$

where  $\delta(\cdot)$  represents the Dirac function, and  $f_g$  is the generation frequency, the expression in (5) can be rewritten as:

$$Y_x(f) = \frac{1}{2} \operatorname{sinc}\left(\frac{f}{f_g}\right) \cdot e^{-j2\pi \frac{f}{f_g}} \cdot e^{-j\frac{\pi}{2}} \sum_{k=-\infty}^{\infty} [\delta(f - (k f_g + f_0)) - \delta(f - (k f_g - f_0))] \quad (7)$$

From expression (7) it results that the spectrum of a sine wave generated by means of a zero-order-hold circuit includes several spectral components, in addition to the expected one at  $f_0$ , residing at frequencies  $k f_g \pm f_0$ . These components are due to horizontal quantization, that turns to be relevant whenever the adopted generation frequency is not sufficiently greater than the bandwidth of the signal, and no suitable output filter is available.

#### IV. Combined effects of horizontal quantization and time-base jitter

As it is shown in the literature [3], the spectrum  $Y_{x,j}(f)$  of a signal  $x(t)$  generated in the presence of time-base jitter is given by:

$$Y_{x,j}(f) = \sum_{m=-\infty}^{\infty} A(f) Y_x\left(f - \frac{m}{R} f_j\right) \quad (8)$$

in which,  $A(f)$  is a weighting function, and  $Y_x(f)$  is the spectrum of the signal in the absence of time-base jitter; concerning the parameters in (8),  $f_j$  is the fundamental frequency of the time-base jitter,  $R$  is the smallest integer that makes  $R f_{ck} = S f_j$ , where  $f_{ck}$  is the clock rate, and  $R$  and  $S$  are integer and prime each other. The weighting function  $A(f)$  has a complicated expression, thus, approximated formulas are often taken into consideration to obtain straightforward estimations (details related to the expression of the weighting function  $A(f)$  and its straightforward approximations can be found in [3], which presents a synthetic study of jitter effects in digital-to-analogue conversion). If periodic jitter affects the time-base, spurious components are expected aside each relevant component present in the spectrum of the waveform.

#### V. Experimental study

The experimental study has been conducted by means of the automatic test equipment, which has been configured in different modes according to the exigencies. Before describing in detail the tests carried out to investigate the effects modelled in the previous sections, and highlight other minor effects not included in the theoretical analysis, however, some notes concerning the choice of the generation frequency, which also include results of simulations, as well as settings that can originate jitter in DDS based generators are given.

##### A. Notes on generation frequency and vertical quantization

The choice of a generation frequency equal to an integer multiple of the nominal frequency of the desired sinusoidal waveform produces a deterministic sequence of quantization errors, characterized by the same period of the sine wave. Sine waves corrupted by random-like quantization errors that justify the name, commonly used, of quantization noise, can be generated if the ratio between the chosen generation frequency and the nominal frequency is an irrational number. Anyway, the choice of two frequency values that share a very low maximum common divisor or are definitely prime each other (in other terms, the generation period and the nominal cycle of the sine wave have a very large minimum common multiple), is sufficient to have quantization errors that resemble noise, provided that the extent of the observed time interval is inferior to the aforementioned minimum common multiple.

As an example, the results of simulations referred to a sine wave generated by means of an 8 bit resolution DAC, and characterized by a nominal frequency,  $f_0$ , equal to 200 kHz, are shown in Figure 1. In particular a grid of four diagrams is presented: the diagrams on the first column are related to a signal characterized by a generation frequency  $f_g = 5 f_0$ , those on the second column to a signal characterized by a ratio between the generation frequency and nominal frequency equal to 1973/379; diagrams in each column show, respectively, a portion of the quantization error, and the spectrum of the signal in the frequency interval ranging from 0 to 500 kHz. The frequency interval considered does not show the effects of horizontal quantization, which are external to this bandwidth, but, opportunely highlights the additional effects due to quantization error. In particular, in the first case, the quantization error is periodic with the same period of the sine wave; its spectral components reside at frequency  $f_0$  and all its harmonics. In the second case, the spectral contents of the quantization error, spreading all along the frequency axis, is responsible of the increased floor, and, there is no choice for a uniform sample rate that substantially changes the power spectral density displacement.

In all experiments the nominal frequency and the generation frequency of the output sine wave have always been chosen in order to avoid harmonic spectral components due to vertical quantization.

## B. Notes on timing jitter

Concerning time-base jitter, a distinction between the different kinds of generator presented in Section II, has to be done. For generators of the first kind, which generally utilize a time-base circuit that uses a clock and a frequency divider based on the use of PLL circuits, the inherent time-base jitter exhibits a deterministic component, which is always predominant on the random contribution, and it is usually described in terms of a fundamental frequency [3].

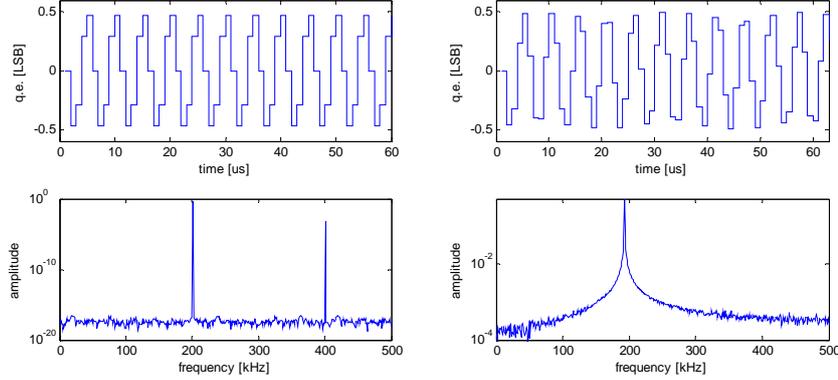


Figure 1. Diagrams on the first column are related to a signal characterized by a generation frequency  $f_g = 5f_0$ , those on the second column to a signal characterized by a ratio between generation frequency and nominal frequency equal to 1973/379; diagrams in each column show both a portion of the quantization error, and the spectrum of the signal in the frequency interval ranging from 0 to 500 kHz.

Differently, if the generator is based on DDS operations, and utilizes a stable fixed frequency clock, jitter is negligible if the chosen generation frequency is coherent with the clock rate. But, the very DDS mechanism causes non negligible jitter anytime settings involving non coherence between generation frequency and clock rate are needed. This jitter usually contains a constant term, which is responsible of a frequency offset in the output sine wave, and a periodic term, generally described just in terms of a fundamental frequency. The frequency  $f_j$  of the fundamental component of the jitter injected by the DDS mechanism depends on the clock rate  $f_{ck}$  adopted by the generator, the desired generation frequency  $f_g$ , and the length of the residual register in terms of number of bits  $Q$ . In particular, due to rounding operations, the exact calculation has to be performed solving the system of equation:

$$\begin{cases} f_j = \frac{f_g}{V} \\ U \lfloor \Delta\theta \rfloor = V2^Q \end{cases} \quad (9)$$

where  $\Delta\theta$  is expressed by (1),  $\lfloor \cdot \rfloor$  represents the integer part operator, and  $U$  and  $V$  are the lowest integer values for which the (9) is satisfied.

## C. Tests on signals affected by horizontal quantization

The experiments focused on horizontal quantization effects have been performed configuring the automated test equipment as shown in Figure 2 a), thus operating with AWG, spectrum analyzer, and remote control unit. In particular the adopted AWG is a generator of the first kind according to the classification given in Section II.

Horizontal quantization effects have been investigated on a number of sine waves characterized by a nominal frequency modestly lower than the generation frequency. Tests have been performed for many different combinations of nominal frequency of sine wave, generation frequency, and number of bits of resolution.

As an example, Figure 3 shows the spectrum of a sine wave characterized by 2477 complete wave cycles in 16384 data points, as measured by spectrum analyzer. In this example the sine wave has been played imposing a generation frequency equal to 1.6 MHz, to have an output sine wave characterized by a nominal frequency equal to 241.895 kHz. The settings of the spectrum analyzer related to Figure 3 involve a frequency span from 100 kHz up to 8 MHz, which allows a complete view of many spectral components produced by horizontal quantization. The adopted resolution bandwidth has been 1 kHz. Furthermore, in all tests the use of a big attenuation factor, equal to 50 dB, has been necessary to avoid second order and third order intermodulation terms, which the non-linear behaviour of the mixer internal to the spectrum analyzer could produce. In Figure 3 a weak spurious component characterized by a frequency equal to 4.3 MHz also appears: this component is present in all traces related to different signals and settings produced

by the same generator; it represents a spurious from an external mechanism or source. Specific settings of the spectrum analyzer have been required to accurately measure the frequency of every spectral component due to horizontal quantization. Measurements have confirmed that the frequencies of all spectral components agree with the value obtained from equation (7). Moreover, if the resolution of the DAC is lowered from 12 to 6 bits, no relevant modifications are observed in the spectrum of the output waveform. The power of the overall imperfections affecting the generated sine wave remains, in fact, mainly concentrated in the spectral components due to horizontal quantization.

#### D. Tests on signals affected by horizontal quantization and time-base jitter

The experiments focused on combined effects of horizontal quantization and time-base jitter have been executed by means of the automatic test equipment shown in Figure 2 b). The waveform under test is provided by AWG; an auxiliary function generator has been utilized to supply a clock signal affected by a controlled jitter.

As an example of typical results achieved during the tests, Figure 4 shows the spectrum of a sine wave characterized by a nominal frequency equal to 30.7 kHz, a non-constant generation frequency characterized by an average value equal to 200 kHz and affected by a jitter implying sinusoidal frequency deviations with peak value equal to 1 kHz. The nominal frequency  $f_j$  of the imposed jitter is 10 kHz, thus the value of  $R$  to be adopted in (8) is unitary, while  $S$  is equal to 20. The main spectral components due to jitter are expected at  $\pm f_j$  hertz offset from any component present in the spectrum. In Figure 4 the frequency span ranges from 20 kHz up to 260 kHz. It allows the detection of the first two spectral components due to horizontal quantization, which reside at 169.3 kHz and 230.7 kHz. In Figure 4 also the effects due to jitter are clearly visible aside the two spectral components due to horizontal quantization; specifically, they are represented by the lower spectral components that reside at 159.3 kHz and 179.3 kHz, as well as 220.7 kHz and 240.7 kHz, all characterized by an absolute amplitude slightly superior to -40 dBm. As observed in tests described in subsection C, no appreciable changes are revealed in the spectrum of the output waveform if the resolution of the DAC is lowered from 12 down to 6 bits.

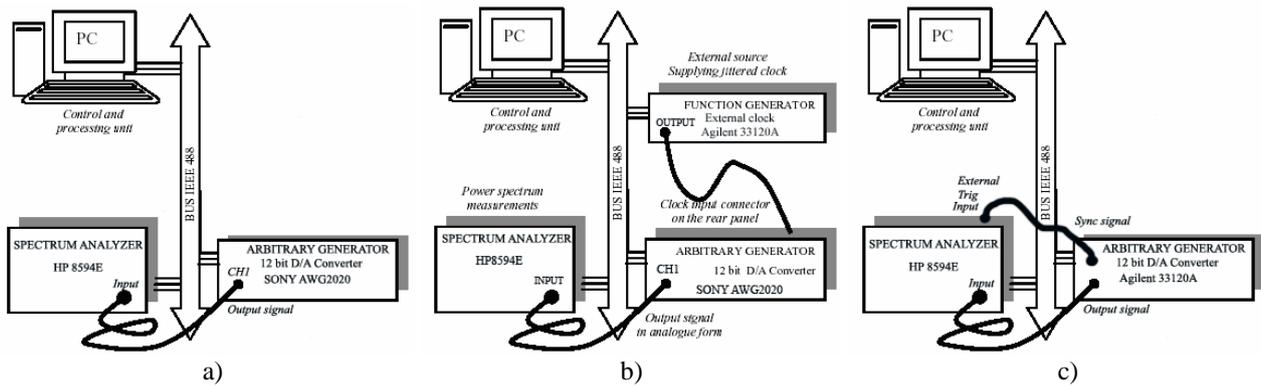


Figure 2. Different automated test equipment configurations arranged to carry out the tests related to horizontal quantization and time-base jitter effects.

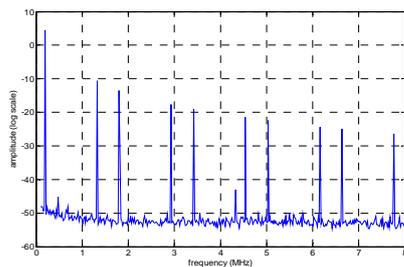


Figure 3. Spectrum of a sine wave characterized by 2477 complete wave cycles in 16384 data points. The sine wave has been played imposing a generation frequency equal to 1.6 MHz, to have a nominal frequency equal to 241.895 kHz.

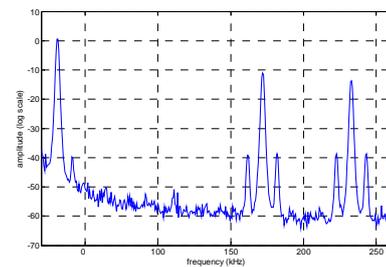


Figure 4. Effects due to jitter are clearly visible aside the two spectral components due to horizontal quantization placed at 169.3 kHz and 230.7 kHz: they are represented by the lower spectral components characterized by an absolute amplitude slightly superior to -40 dBm.

Further experiments have been executed by means of the automatic test equipment shown in Figure 2 c). A DDS-based generator has been utilized to generate the output signal. It allows the choice of the rate at which repeatedly generate the finite-length time record loaded in the DAC memory. Moreover, it uses an internal clock at a fixed frequency equal to 40 MHz, and a 48-bit accumulator register with  $P = 14$  and  $Q = 34$ . The experiments have been performed injecting a controlled jitter by choosing a nominal frequency non coherent with the clock rate for the output sine wave.

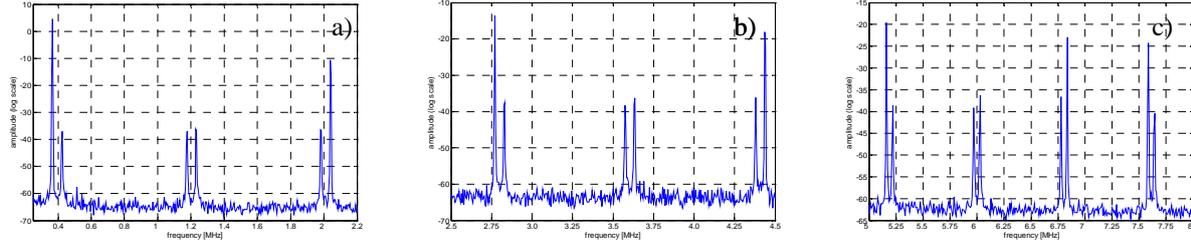


Figure 5. Portion of the spectrum of a signal affected by horizontal quantization and time-base jitter effects. The frequency span ranges from 0.25 MHz to 2.2 MHz in a), 2.5 MHz to 4.5 MHz in b), and 5 MHz to 8 MHz in c).

As an example the case of a sine wave characterized by 2477 complete wave cycles in 16000 data points, is illustrated. The generation frequency  $f_g$  is chosen equal to 2.4 MHz, and it is attained imposing the rate at which repeatedly generating the finite-length record containing 16000 points to  $f_g/16000$ , that is 150 times per second. The nominal frequency of the output signal  $f_0$  is 371.55 kHz. The jitter due to DDS is almost sinusoidal, characterized by a fundamental frequency,  $f_j$ , equal to 800 kHz; in fact, the values of  $U$  and  $V$  satisfying the system given in (9) are, respectively, equal to 50 and 3, thus,  $f_j$  is exactly three times smaller than the generation frequency.

Spectral components due to jitter are expected at frequencies  $kf_g \pm f_0 \pm mf_j$ ,  $k$  and  $m$  integer numbers. The results are shown in Figure 5 a), Figure 5 b), and Figure 5 c). All spectral components below -35 dB shown in the aforementioned figures are due to jitter, while the higher ones are caused by horizontal quantization. In particular, Figure 5 a) shows the frequency band between 250 kHz and 2.2 MHz, Figure 5 b) shows the frequency band between 2.5 MHz and 4.5 MHz, and figure 5 c) shows the frequency band between 5 MHz and 8 MHz. Table I highlights the frequency values of each spectral component, predetermined by the proposed model, likewise detected by means of the spectrum analyzer.

Table I Frequency values in MHz of the spectral components observed in Figure 5 a), Figure 5 b) and Figure 5 c).

frequency span 250 kHz – 2 MHz		frequency span 2.5 MHz – 4.5 MHz		frequency span 5 MHz – 8 MHz	
due to horizontal quantization [MHz]	due to time-base jitter [MHz]	due to horizontal quantization [MHz]	due to time-base jitter [MHz]	due to horizontal quantization [MHz]	due to time-base jitter [MHz]
$f_g - f_0 = 2.13$	$-f_0 + f_j = 0.43$ $f_0 + f_j = 1.17$ $f_g - f_0 - f_j = 1.23$ $f_g + f_0 - f_j = 1.97$	$f_g + f_0 = 2.77$ $2f_g - f_0 = 4.43$	$f_g - f_0 + f_j = 2.83$ $f_g + f_0 + f_j = 3.57$ $2f_g - f_0 - f_j = 3.63$ $2f_g + f_0 - f_j = 4.37$	$2f_g + f_0 = 5.17$ $3f_g - f_0 = 6.83$ $3f_g + f_0 = 7.57$	$2f_g - f_0 + f_j = 5.23$ $2f_g + f_0 + f_j = 5.97$ $3f_g - f_0 - f_j = 6.03$ $3f_g + f_0 - f_j = 6.77$ $3f_g - f_0 + f_j = 7.63$

## VI. Conclusions

The paper has presented the results of an experimental study focused on the imperfections affecting waveforms generated by means of DACs. Also a suitable model for the analysis of horizontal quantization and time-base jitter effects has been presented and assessed. It is worth noting that, in a first attempt aimed at including in the analysis modulated signals, ranging from narrow band to digitally modulated signals characterized by a wide occupied bandwidth, the proposed model can still be referred to. In this case, the carrier frequency of the modulated signal plays the role of the nominal frequency of the sine wave. In particular, several attenuated replicas of the spectrum of the modulated signal are expected centred at frequencies  $kf_g \pm f_c$ .

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