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FEB-based approach to the measurement of effective resolution of cyclic ADC[1]

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Abstract **-** The goal of the paper is presentation of a new approach to the measurement of effective resolution (effective number of bits - ENOB) of the cyclic A/D converters (CADCs). The core idea of the approach is direct measurement of ENOB using, as a numerical measure, the number of true bits before the first erroneous bit (FEB) position. The position of FEB is determined as the first non-zero bit in the binary presentations of conversion errors. The definition of ENOB based on FEB is introduced and discussed. The particularities of the proposed method are analysed in simulation experiments. There are presented typical evolutions of FEB distributions in sequential cycles of conversion. Values of ENOB obtained using FEB-based method are compared with results obtained using the conventional approach to ENOB assessment. The comparison is performed on example of analysis of influence of DNL and INL errors of internal A/D converter on ENOB of CADC. The proposed method of the ENOB measurement gives more adequate information about the actual ADC resolution and weakens the influence of the form of testing signals on the results of the ENOB measurement.

I. Introduction

Until now, there is no full concordance in the field of the methods of ADC resolution measurement. The main reason is the step-wise form of transition function which makes ADC strongly non-linear devices. The latter violates the principle of superposition, which is of principal meaning for the analysis of linear systems. For this reason, harmonic analysis is not an adequate tool for studying ADC performance. The results of measurement depend on the form of input signals, and the actual resolution of the converter may differ from the resolution given in the specification. This crucially complicates development of commonly accepted methods of adequate assessment of ADC resolution (further, effective number of bits - ENOB [1]), as well as of testing, analysis and comparison of the converters [2-4]. The always limited input range is a source of possible overloading of the converters that makes users apply much more accurate and expensive ADCs than necessary. For these reasons, development of adequate methods of ADC performance measurement remains an actual task of ADC design.

Additional complications appear in development of adequate methods of the cyclic ADCs (CADCs) performance assessment. Nowadays, CADCs are considered as “black-boxes”, and their performance is measured in the same way as that of the non-cyclic ADCs, using the same measures of quality (ENOB, SINAD, THD, INL, DNL, etc.) as presented in IEEE Standard 1241 [1].

The results of investigations ([5-9] and others) show that particularities of CADC work do not allow to assess its resolution and other characteristics using standard definitions and methods of ENOB assessment, which should be corrected taking into account the following factors:

1. Probability density function (PDF) of quantisation noise at the CADC output is not uniform. Low resolution of the coarse ($N_{ADC} = 1 \div 6$ bit) internal A/D converter (ADC_{In}) in CADC analogue part makes its quantisation noise ξ_k and the errors of conversion (CADC quantisation noise) significantly non-uniform. For the greater number of cycles, this PDF takes the Gaussian form [6,8].
2. Influence of testing signal characteristics (form, range, statistics and dynamics of changes) on the conversion quality practically disappears beginning with the second cycle of conversion. This is conditioned by the orthogonality of residual signals e_k routed to the input of amplifier of the analogue part of CADC (see Fig.1) in sequential cycles $k = 1,...,n$.
3. The unstudied dependence of growing, in sequential cycles, probability of CADC overloading on the internal noises acting in the analogue part of converters, as well as their influence on the number of potentially erroneous bits in the binary codes $\tilde{y}_k$ at ADC_{In} output (“observations”).

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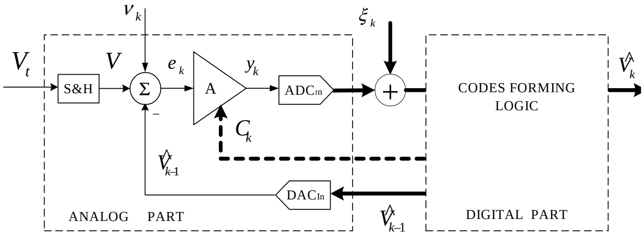


Fig. 1. General block-diagram of CADC

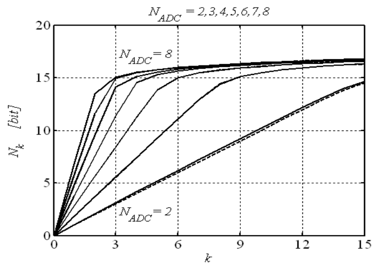


Fig. 2. ENOB as a function of the number of cycles under different ADC_{In} resolution [7]

The paper discusses the methods of adequate measurement of the ENOB of CADC, which take into account, at least in the main order, the above listed factors. Main attention is paid to further development of "FEB-approach" to the measurement of ENOB. Previous investigations [5-10] show it may provide the researchers and manufacturers of CADCs with the most accurate, complete and adequate information about the quality of conversion. Analytical backgrounds and results of simulation analysis of FEB-approach are presented. There is discussed a possibility to use this approach for evaluation of a direct dependence of ENOB on the differential and integral non-linearities (DNL and INL, respectively) of the internal ADC_{In}. These dependencies were investigated earlier [10] using non-direct (based on preceding measurement of root square error - RMS) measurement of ENOB briefly discussed in the next Section.

II. Mathematical description of CADC and RMS-based method of ENOB measurement

General block-diagram of CADC is shown in Fig. 1. The input signals $V_t = V(t)$ are considered as the stationary zero mean Gaussian random processes with the mean power not exceeding given value σ_0^2. Their spectral power density is assumed having zero values outside the frequency range [-*F*, *F*]. Sample-and-hold unit (S&H in Fig. 1) holds each sample $V^{(m)} = V(m/2F)$, ($m = 1,2,\dots,M$) at the first input of subtracting block Σ during the time $T = 1/2F$. Each sample $V^{(m)} = V(m/2F)$ is converted independently in $n = F_0/2F$ cycles (F_0 is the band-pass of the analogue part of the converter). The latter permits to reduce the analysis of CADC work to consideration of conversion of a single sample $V^{(m)} = V$. In this case, assuming CADC are ideal (no DNL, no INL), prior distribution and form of the actual or testing signals do not influence the quality of conversion.

Mathematical model of the digital part of each CADC can be presented by the recurrent equation:

$$\hat{V}_k = \hat{V}_{k-1} + L_k \tilde{y}_k; \qquad (k = 1,\dots,n), \tag{1}$$

where $\hat{V}_k$ is the code of the sample V in *k*-th cycle computed as a sum of the previous code $\hat{V}_{k-1}$ and digital observation $\tilde{y}_k$ formed by N_{ADC}-bit coarse pre-converter ADC_{In} in the analogue part of CADC.

Values of the gains L_k in (1) depend on the type of CADC. In turn, the work of analogue part of CADC can be satisfactorily well modelled by the piece-wise linear function [8]:

$$\tilde{y}_k = \begin{cases} C_k(V - \hat{V}_{k-1}^{DAC} + \nu_k) + \xi_k & \text{for } C_k\,|e_k| \le D; \\ D\,\mathrm{sgn}(V - \hat{V}_{k-1}^{DAC} + \nu_k) + \xi_k & \text{for } C_k\,|e_k| > D, \end{cases} \tag{2}$$

which is an approximation of commonly used ideal step-wise (static) transition function [1].

Value $\hat{V}_{k-1}^{DAC}$ in (2) represents the analogue equivalent of the estimate: $\hat{V}_{k-1} = E(V\,|\,\tilde{y}_1^{k-1})$ computed by digital part of CADC in previous cycle. Errors of D/A conversion can be included into the analogue noise ν_k, which is a sum of the noises of feedback chain, S&H block, subtractor Σ and possible external noise (below, we assume ν_k is a zero-mean white Gaussian noise with the variance σ_ν^2). Parameter D in (2) determines the boundaries [-*D*, *D*] of the full scale range (FSR) of ADC_{In}, and noise ξ_k is the quantization noise at the ADC_{In} output (its variance is denoted further as σ_ξ^2).

Main particularity of CADC work established in our researches was fast normalisation of the final conversion errors $\delta\hat{V} = V - \hat{V}_k$ for the greater number of cycles ([5-9] and other works). For this reason, as the main characteristic of CADC performance was used the empirical analogue of theoretical expression of ENOB $\hat{N}_k$ after k cycles of the sample:

$$\hat{N}_k = \frac{1}{2}\log_2 \frac{\sigma_\xi^2}{\hat{P}_k} = \log_2 \frac{FSR}{2\alpha\sqrt{\hat{P}_k}}, \quad \text{where} \quad \hat{P}_k = \frac{1}{M}\sum_{m=1}^{M}(V^{(m)} - \hat{V}_k^{(m)})^2 = (RMSnoise_k)^2 \quad (3)$$

The expression in the left side of (3) is a result of theoretical analysis [8, 9] and represents the amount of information about the value of input Gaussian signal in the output code, and $FSR = 2\alpha\sigma_0$ is the full scale (input) range of CADC. Value $\hat{P}_k$ in the right side of (3) is the empirical analogue of MSE of conversion errors $P_k = E[(V - \hat{V}_k)^2]$ after k cycles of conversion and $RMSnoise_k$ is the square root of MSE $\hat{P}_k$. The constant α is the saturation factor guarantying that the probability of CADC saturation never exceeds the acceptable value μ, (for instance, $\mu = 10^{-7}$; $\alpha = 5$). The single but principal difference between the definition of ENOB (3) and these given in [1, p (4.5.1.1)] is that MSE of quantisation errors in [1] is computed as $P_k = (\Delta_k / 2\sqrt{3})^2$ while in (3) and [5-10] it is assessed by the formula $P_k = (\Delta_k / 2\alpha)^2$, where Δ_k is the width of output quantisation interval. Typical runs of ENOB $\hat{N}_k$ versus number of k are shown in Fig. 2, ($\hat{N}_k$ computed according to (3)).

The investigations [5-9] show that for the Gaussian input signal formula (3) is adequate: a) at the first cycle of conversion and b) for the cycles near and after the ”threshold” point $n*$ determined by corresponding general relationships. For non-Gaussian and deterministic input signals, formula (4) is accurate for $k \geq n*$, and becomes only a rough evaluation of ENOB in the previous cycles. Necessity of more accurate estimation of ENOB at the most active initial “pre-threshold” interval [1, $n*$] led us to the concept of the first erroneous bit (FEB).

III. First erroneous bit (FEB) and ENOB measurements

Let the codes $\hat{V}_k$ of the samples V be computed according to (1) by N -bit processor. Then, each of them is determined with the error not greater than the accuracy $\Delta\hat{V}_k = LSB/2 = FSR/2^{N+1}$. In this case, continuous samples V can be replaced, without loss of accuracy of comparison with the estimates, by the set of 2^N discrete values, and both the samples and their estimates can be expressed by formulas:

$$V = \frac{FSR}{2^N}n - \frac{FSR}{2} + V_0, \qquad \hat{V}_k = \frac{FSR}{2^N}\hat{n}_k - \frac{FSR}{2} + \hat{V}_0, \quad (4)$$

where n, $\hat{n}_k$ are corresponding numbers of quantization levels, V_0 is the centre of the input range of CADC and it is assumed that $\hat{V}_0 = V_0$. Formulas (4) allow to express the relative error of conversion in the form:

$$\frac{err_k}{FSR} = \frac{|V - \hat{V}_k|}{FSR} = |n - \hat{n}_k| \frac{1}{2^N} . \quad (5)$$

The binary presentation of this error

$$\rho_k = \mathrm{mod}_2\left(\frac{err_k}{FSR}\right) = \mathrm{mod}_2\left(2^{-N}|n - \hat{n}_k|\right) \quad (6)$$

begins with a series of zeros which correspond to the “true”, coinciding bits in the sample and its estimate. First unity in the binary word (6) determines the number of position where first difference between the estimate and sample appears. This position determines the number of “first erroneous bit” (FEB$_k$) in the code of estimate of the sample (further denoted as n_k^F) and the current number of “true” bits (NOB$_k$).

Definition 1. Number of true bits in the estimate $\hat{V}_k$ is determined by formula: $\mathrm{NOB}_k = \mathrm{FEB}_k - 1$.

Having measured values of NOB$_k$, one may measure the ENOB$_k$ of CADC for each cycle of conversion (it is necessary to take into account that the values of the sample, its estimate and FEB are random). The simplest way to determine ENOB is as follows:

Definition 2. ENOB of the converter is the low boundary of the set of M values of NOB$_k$ obtained during conversion of M samples of the testing signal:

$$\mathrm{ENOB}_k = \min_{m=1,\dots,M}(\mathrm{NOB}_k^{(m)}) = \min_{m=1,\dots,M}[\hat{n}_k^F(m)] - 1 . \quad (7)$$

Formulas (4)-(7) show the method of measurement of the value of ENOB: one should convert M testing samples, find and collect the set of $\mathrm{NOB}_k^{(m)}$, ($m = 1,...,M$) using corresponding FEB and, finally, use formula (7). The testing samples can be generated digitally and routed to the input of CADC through N-bit D/A converter. Alternative testing method consists in applying the analogue signal to the inputs of CADC and of N-bit A/D converter with further comparison of the codes formed by each of them.

The advantage of the proposed method of ENOB measurement is its adequacy and simplicity – it is not connected with the form of the input signals and conditions of the experiment (at least for an ideal ADC) and can be easily implemented. It is also important that measurement of ENOB (7) does not require preliminary measurement of RMS or MSE which are not completely adequate characteristics of the conversion quality and, additionally, must undergo the non-linear (logarithmic) transformation.

Below, some results of simulation analysis of the proposed approach are presented. The experiments were carried out using the results of the current paper and the model of CADC described in [5-8] under the following parameters: $N_{ADC} = 4$, $N_{DAC} = 12$, variance of analogue noise $\sigma_v^2 = 6.25 \cdot 10^{-10}$, FSR ramp input signal in range [-1,1], M = 10000.

Typical distributions of frequency of the FEB appearance in different positions of the codes formed by CADC (here corresponding to initial (k = 1,2,3,4) cycles of conversion (from left to right)) are presented in Fig. 3. The plots have a complex form which depends on the number of conversion cycles.

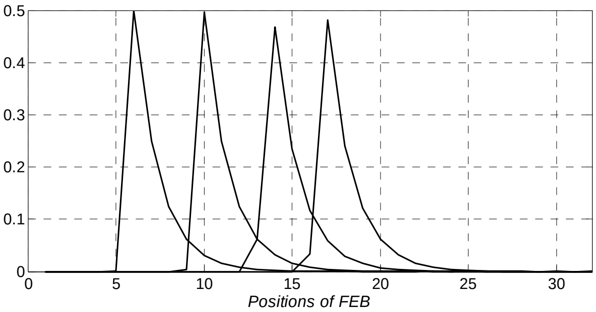


Fig. 3. Histogram of the FEB appearance in initial (k = 1,2,3,4) cycles of conversion.

In Fig. 4, changes of FEB distribution with the number of cycles are shown (intensity of grey in Fig. 4a refers to the more frequent FEB occurrences; grey area in Figs 4b,c corresponds to bit numbers for which number of FEB occurrences is greater than a given value, continuous line in Fig. 4c refers to ENOB assessed using non-direct method (3)).

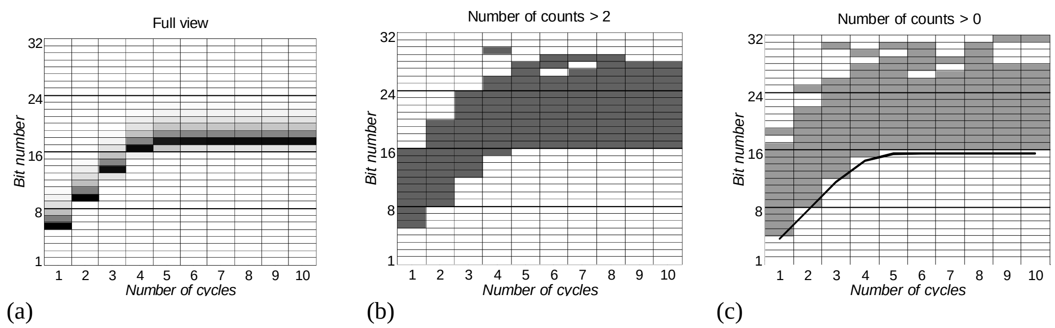
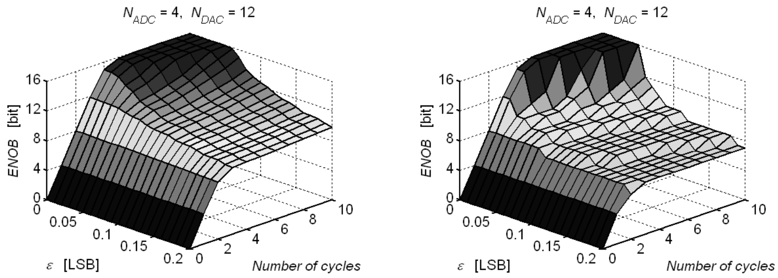


Fig. 4. Evolution of histogram of FEB appearance: a) view from the top; b), c) regions where FEB appeared at least three times and at least once, respectively.

The results of conventional and direct measurement of ENOB (using (3), and (7), respectively) under different values of DNL and INL of ADC_{In} are compared in Figs 5 and 6. DNL errors of ADC_{In} were modelled as independent random displacements of ADC_{In} quantization thresholds. The displacements of thresholds were uniformly distributed around nominal values of thresholds. The width of the interval $\varepsilon \cdot [-\Delta_{ADC}/2, \Delta_{ADC}/2]$ of possible displacements was set using the scale (intensity) coefficient ε. Value Δ_{ADC} describes the ADC_{In} quantization interval. INL errors of ADC_{In} were modelled, similarly as in

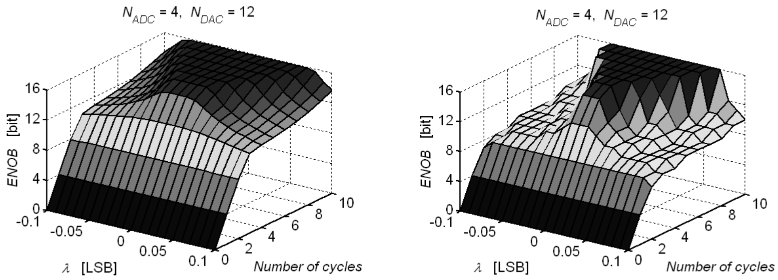
[12], by replacement of the linear approximation of ADC_{In} transfer function by a function of the $x^{1+\lambda}$ type where λ determines the intensity of INL errors:

$$f(x) = D\,\mathrm{sgn}(x)\left(\frac{|x|}{D}\right)^{1+\lambda}. \quad (8)$$



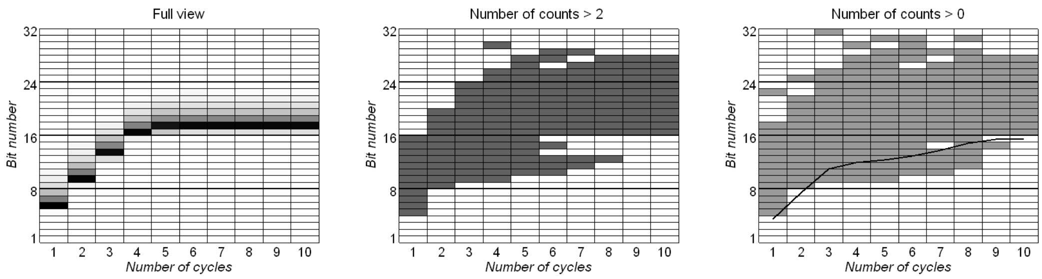
(a) (b)

Fig. 5. Influence of DNL of ADC_{In} on the ENOB measured using: a) conventional and b) FEB-based approach.



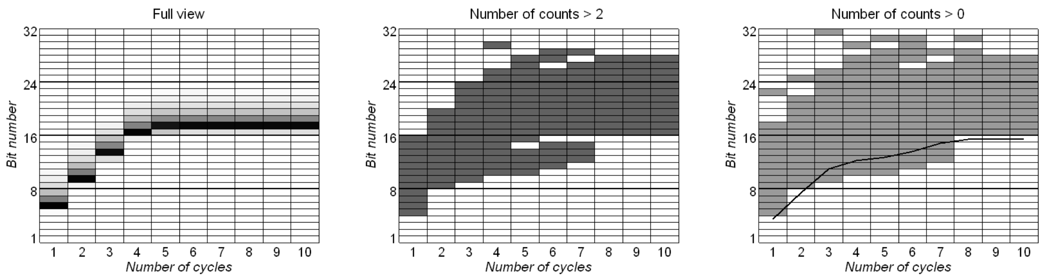
(a) (b)

Fig. 6. Influence of INL of ADC_{In} on the ENOB measured using: a) conventional and b) FEB-based approach.



(a) (b) (c)

Fig. 7. Evolution of histogram of FEB appearance in case of DNL errors for $\varepsilon = 0.05$: a) view from the top; b), c) regions where FEB appeared at least three times and at least once, respectively.



(a) (b) (c)

Fig. 8. Evolution of histogram of FEB appearance in case of INL errors for $\lambda = 0.05$: a) view from the top; b), c) regions where FEB appeared at least three times and at least once, respectively.

Changes of FEB distribution with the number of cycles caused by DNL and INL errors obtained for $\varepsilon = 0.05$ and $\lambda = 0.05$ [LSB], respectively, are presented in Figs 7 and 8. Continuous lines in Figs 7c and 8c refer to ENOB assessed using the conventional non-direct method (3). The results presented in Figs 7 and 8 coincide with the results in Figs 5 and 6. On the basis of these figures, we can conclude that the results of ENOB evaluation obtained using the direct method (4-7) are more rigorous than the ones obtained with the conventional method (3). ENOB assessed according to (3) is less sensitive to INL and DNL errors of the internal converter ADC_{In} of CADC, which during conversion of some samples lead to saturation of the internal converter and appearance of abnormal errors of conversion. The influence of these single abnormal errors is diminished by averaging the errors over a large amount when calculating RMS (3). This way we neither receive an adequate information about “normal” CADC performance quality nor of abnormal errors, because these two are mixed. Evaluation of ENOB according to (4-7) can be too rigorous in case of singular abnormal errors because it gives ENOB equal to the number of true bits in the worst estimate among all output codes. However, in place of (7) there can be used another, more adequate measure, but its formulation requires additional study of statistical characteristics of FEB and NOB.

IV. Conclusions

The results of investigations show that direct FEB-method of CADC ENOB measurement is a perspective and inherently more adequate tool for assessment and analysis of the cyclic ADC performance. Important feature of FEB-based approach is that it allows, in perspective, to separate regular performance characteristic and rare abnormal errors. Its further development may yield a universal, convenient and adequate method of ADC testing.

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