Experimental verification of different models of the ADC transfer function

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Abstract- The performance of current devices is mostly limited by the analogue front-end and analogue-to-digital converter's (ADC) imperfections. ADC performance is not, as commonly known, ideal. One of the most important parameters is the nonlinearity, which if it is known, can be corrected in the output data. The performance of three different approximations of ADC nonlinearity (common polynomials, Chebyshev polynomials and Fourier series) and achieved results concerning accuracy of approximations, noise sensitivity and nonlinearity correction are presented in the paper.

I.   Introduction

There are many different approaches of describing nonlinearities of an ADC and their dependences – particularly the dependence on input signal parameters. Even though the nonlinearity shows strong dependency on the signal frequency, considering signal frequencies significantly lower than ADC sampling frequency, the frequency dependency of the curve of the ADC nonlinearity can be mostly neglected. In the case of low frequency input signal the ADC nonlinearity can be described as a function of signal input level only.

The ADC nonlinearity is inherently described by the Integral Non-Linearity $INL(n)$, so the difference of ADC output and input as the function of the input level. However, very often only a single number for the INL is presented in manufacturer's datasheets and it states the maximum value of the $INL(n)$ curve. Generally, the nonlinearity causes a distortion in the digitized signal, which can be expressed in the frequency domain by the THD (Total Harmonic Distortion) parameter. This is also a single value parameter, but the frequency spectrum can provide similar information as the curve of the $INL(n)$ in the code domain. However, the $INL(n)$ is more advantageous in the case when a correction of the ADC transfer function is demanded.

The curve of the $INL(n)$ can be split into the low code frequency component (LCF), that is responsible for harmonic distortion at lower harmonic components, and the high frequency component (HCF) that is responsible for high harmonic components in the output frequency spectrum. To approximate the rough curve of the $INL(n)$ and therefore the LCF, the approximation of the HCF is not needed, although an exact break-point between those two ones does not exist [1,2,3,4].

Histogram test method for the $INL(n)$ curve determination described in IEEE 1241 Standard [5] demands a huge number of samples in a record to achieve reasonable confidence levels; however, both LCF and HCF parts of the $INL(n)$ can be computed. All the approximations mentioned in this paper describe only the LCF of the $INL(n)$ and they demand significantly lower number of samples as they work with output (recorded) signal spectrum. The considered methods of ADC non-linearity approximation calculate the coefficients from the frequency domain. Their sensitivity to disturbances was analyzed in previous papers [6] and [7]. The approximations accuracy and their coefficients' sensitivity to noise is analyzed in this paper.

Three types of approximation – common polynomials [1], Chebyshev polynomials [8] and Fourier series [9] – were examined, their properties were simulated and nonlinearity correction was evaluated.

In the following text, the accuracy of approximations is analyzed in chapter III and noise sensitivity in chapter IV. For that purpose the $INL^{real}(n)$ of a real 14bit converter NI PXI 5122 [10] was measured and used. Based on a histogram measurement this real $INL^{real}(n)$ provides both LCF and HCF parts of the $INL(n)$ although it spans the input range from about 80 % only but it is still sufficient for such analyses.

All analysis were done by simulations using simulated output signal $y(n)$ according to (1). A coherently generated pure input sine wave $x(n)$ without offset was converted through previously measured $INL^{real}(n)$ (in (1) this nonlinear function is represented by $INL[*]$). Also noise source $e(n)$ can be added for the noise analysis – see Fig. 1.

$$y(n) = x(n) + INL\big[x(n)\big] + e(n) \qquad (1)$$

Subsequently, the frequency spectrum of $y(n)$ was calculated and used as input data for approximations of the $INL^{approx}(n)$.
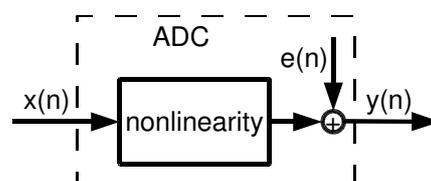

Figure 1. Noise model considered in simulations

II. Approximation of $INL(n)$

A. Common polynomials

In the case of common polynomials [1] the $INL(n)$ is approximated by

$$INL(n) = \sum_{h=0}^{H_m} a_h x^h(n) \qquad (2)$$

where a_i are the coefficients of the nonlinearity up to the maximum order H_m, which is the highest harmonic component, index h in summation starts from zero because all even order nonlinearity coefficients induce a DC level as well as all higher odd nonlinearity coefficients induce a component of the first order. For larger orders of approximation (number of coefficients higher then two hundreds approximately) common polynomials fail to approximate the $INL(n)$ curve.

B. Chebyshev polynomials

In the case of Chebyshev polynomials [8] of the first kind $T_h(\cos(x)) = \cos(hx)$, the $INL(n)$ is approximated by

$$INL(n) = \frac{c_0}{2} + \sum_{h=2}^{H_{max}} c_h T_h(n) \qquad (3)$$

where a_i are the coefficients of the nonlinearity up to the maximum order H_m, (same as above). Orthogonality on the interval $[-1; 1]$ is a basic and important characteristic of Chebyshev polynomials. The summation coefficient starts from two, because due to the orthogonality the induction as mentioned at common polynomials does not occur and no constant nor linear term would be needed – assuming the terminal based $INL(n)$. Chebyshev polynomials provide rather precise approximation starting from less then a hundred of coefficients.

C. Fourier series

In the case of Fourier series [9] the $INL(n)$ is approximated by

$$INL(n) = \frac{a_0}{2} + \sum_{k=0}^{2^N-1} \left[a_k \cos(\frac{2\pi}{2^N} nk) + b_k \sin(\frac{2\pi}{2^N} nk) \right] \qquad (4)$$

where a_k and b_k of a known $INL(n)$ can be found using the well-known expressions

$$a_k = \frac{1}{2^N} \sum_{n=0}^{2^N-1} INL(n) \cos(\frac{2\pi}{N} nk), k \in \{0,1,...,2^N-1\}$$

$$b_k = \frac{1}{2^N} \sum_{n=0}^{2^N-1} INL(n) \sin(\frac{2\pi}{N} nk), k \in \{1,...,2^N-1\} \qquad (5)$$

where $INL(n)$ is the Integral Nonlinearity at the code n, 2^N-1 is the number of transition levels of an N bit ADC and k is the index of a coefficient. Similarly to Chebyshev coefficients, Fourier series also provide reasonably good approximation starting from around a hundred of coefficients. The $INL(n)$, from which a_k and b_k are calculated, is considered periodical; from this reason it is better to use the terminal-based $INL(n)$ [5].

III. Accuracy evaluation of $INL(n)$ approximations

The accuracy of approximations was evaluated by comparing approximated $INL^{approx}(n)$ calculated from the spectrum of $y(n)$ (see(1)) with the previously measured (real) $INL^{real}(n)$ of the same ADC. For numerical evaluation several characteristics were considered, among which the Mean Square Error MSE (6) and the absolute value of maximum error E_{max} (7) showed to be most appropriate.

$$MSE = \frac{1}{N} \sum_{n=0}^{N-1} \left(INL^{real}(n) - INL^{approx}(n) \right)^2 \qquad (6)$$

$$E_{max} = \max \left| INL^{real}(n) - INL^{approx}(n) \right| \qquad (7)$$

In (6) and (7) the $INL^{approx}(n)$ is the approximated curve of a real $INL^{real}(n)$, N is the length of the $INL(n)$ curve and is the same as the number of transition levels ($2^{Nbit}-1$), $Nbit$ is the nominal number of bits of the tested ADC. The results with respect to the number of coefficients (100, 200 and 2000 coefficients) are shown in Table 1.

Table 1. Mean square error MSE and maximum error E_{max} for different numbers of coefficients, no noise added.

	100 coefficients		200 coefficients		2000 coefficients	
	MSE (LSB^2)	E_{max} (LSB)	MSE (LSB^2)	E_{max} (LSB)	MSE (LSB^2)	E_{max} (LSB)
Common polynomials	0.08	1.53	0.65	3.39	0.65	3.39
Chebyshev polynomials	0.05	1.39	0.04	1.20	0.01	0.50
Fourier series	0.05	1.56 (1.32)[*)]	0.04	1.49 (1.01)[*)]	0.01	0.50

For small degrees of nonlinearity (small number of coefficients) – in our case 100 coefficients (see Fig. 2a) – all approximations successfully fit the nonlinearity and the values of MSE and E_{max} achieve comparable levels. When the number of estimated coefficients (the order of nonlinearity) reaches roughly 170, the approximation by common polynomials fails and it provides only a straight line as the result (see Fig. 2b). The value of MSE and E_{max} parameters (considering 200 coefficients) for Chebyschev polynomials and Fourier series achieve similar values but parameters for common polynomials are rather higher. For the largest number of coefficients (see Fig. 2c – simulated for 2000 coefficients) the MSE and E_{max} are further improving but all the same they stay at comparable values for Chebyshev polynomials and Fourier series.

a) 100 coefficients
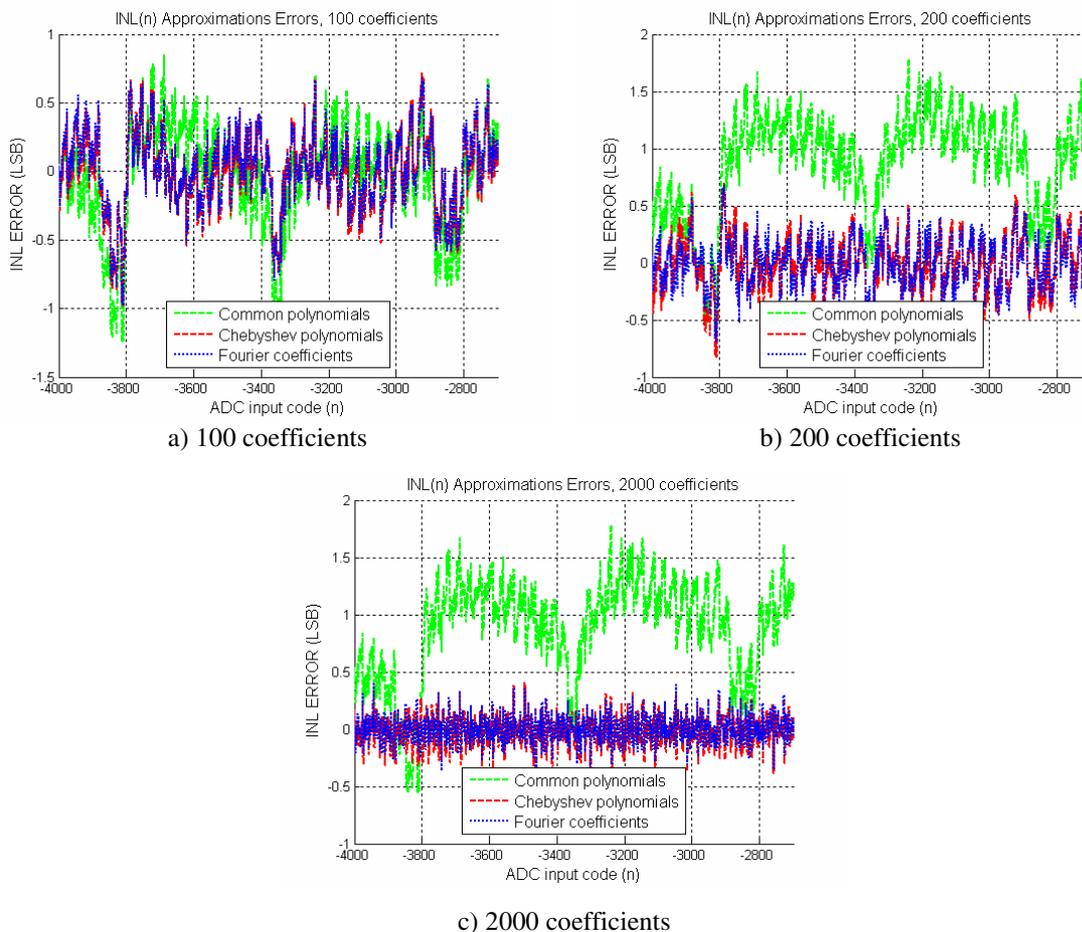

b) 200 coefficients

c) 2000 coefficients

Figure 2. Approximation results for different number of coefficients, difference (error) is displayed

It is important to mention that in the case of Fourier series the most error-prone part of the approximated $INL(n)$ are the ends of the $INL^{approx}(n)$ curve. It is probably caused by non-strictly continuous periodical extension in higher derivations of the $INL^{approx}(n)$ curve. If these parts are omitted from the calculation, the MSE and E_{max} are even smaller (this is denoted by an asterisk [*)] in Tab. 1) for 100 and 200 coefficients. In the case of 2000 parameters the number of coefficients is already sufficient to approximate also these higher derivations discontinuance in the $INL^{approx}(n)$ curve.

In general, performance of Chebyshev coefficients and the Fourier coefficients is comparable, although the complexity of Chebyshev polynomials is smaller and they are also easier to implement.

IV. Noise sensitivity evaluation of $INL(n)$ approximations

It is also crucial to know how evaluated approximations are sensitive to noise in the recorded data and how the estimation of nonlinearity coefficients is influenced by noise. The model structure used for this analysis was described above in Fig. 1, where $x(n)$ is the input signal, $y(n)$ is the output signal, which is the output code of an ADC, and $e(n)$ is the noise added to the converted signal. Wideband noise with normal distribution defined by its variance σ^2 was considered.

The procedure of noise sensitivity evaluation (simulated in Matlab) consists of adding noise $e(n)$ to the converted input signal and varying the variance of noise σ^2. Then, the coefficients for approximation are calculated from the complex spectrum of the output signal y(n). As the last step, the approximated $INL^{approx}(n)$ is created from the calculated coefficients (under noisy condition) and compared to the real $INL^{real}(n)$. Achieved results are presented in Fig. 3 and 4.

In the case of common polynomials approximation the noise performance can be evaluated only for 100 coefficients (see Fig. 3a, 4a). The maximum error is numerically between Chebyshev polynomials (lowest) and Fouries series (worse). Concerning approximations using 200 coefficients, the noise sensitivity of approximation applying Chebyshev polynomials and Fourier series are comparable (see Fig. 3b, 4b). By greater number of coefficients (which is needless for a practical application), the noise sensitivity increases for the both types of approximations (see Fig. 3c, 4c).

## V. Nonlinearity correction based on $INL(n)$ approximations

By having the coefficients and therefore the $INL(n)$ curve, the nonlinearity can be compensated. For this a transfer function TF has to be calculated by adding a straight line to the $INL^{approx}(n)$. If the differences between two adjacent codes n in the $INL(n)$ is not bigger that 0.5 LSB the transfer function TF will be monotonical and its inverse can be found. The inverse then serves as the look-up-table (LUT) according to which the output data is recalculated. The frequency spectra of the output signal with and without correction are showed in Fig. 5a, 5b.

VI. Further work

All analyses carried out in the paper employ coherent sampling that can be in practise, unfortunately, very difficult to achieve. In case of non-coherent sampling that represents the most probably situation in the real measurement, leakage occurs and windowing is unavoidable. Windowing cause more precise estimation of higher harmonic components but their phases estimation is less precise. The influence of the non-coherency and window types should be evaluated.

V. Conclusions

Accuracy and noise sensitivity of the three types of approximations (common polynomials, Chebyshev polynomials and Fourier series) of the $INL(n)$ curve were analysed. The input data for simulations (frequency spectra) were obtained using simulated output signal $y(n)$. The approximated curve of $INL(n)$ was calculated using this data and subsequently it was compared to the measured (real) $INL(n)$ of a real 14bit ADC.
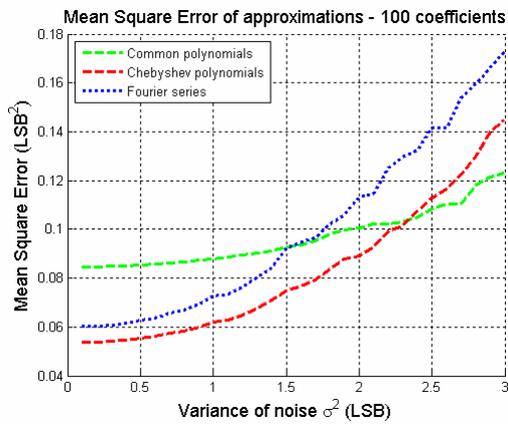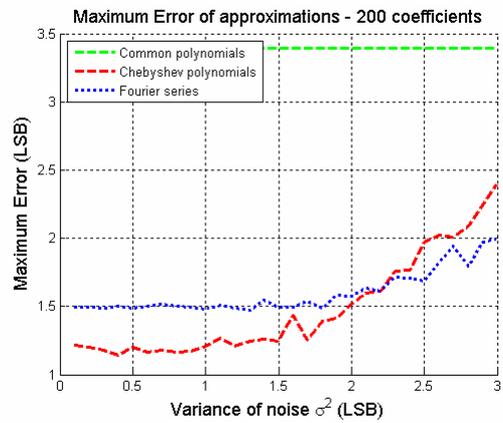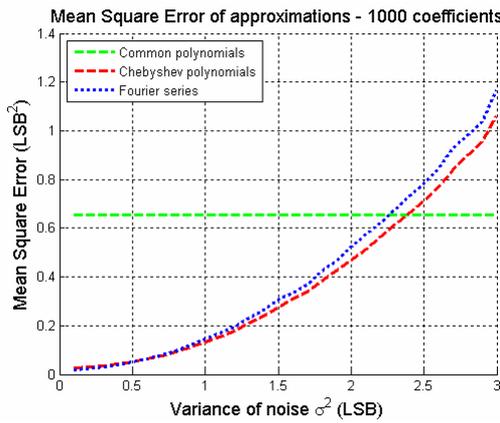
a) 100 coefficients
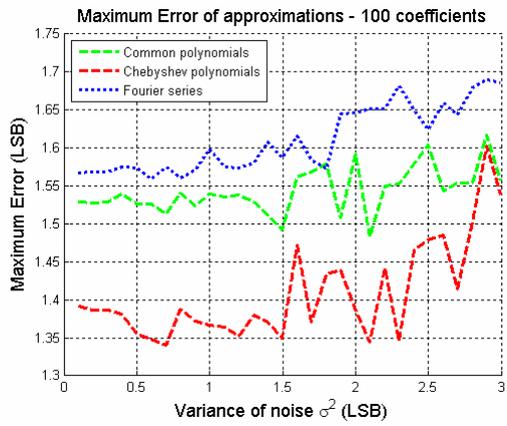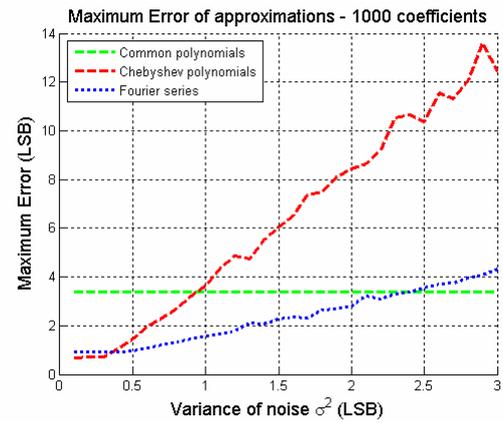

a) 100 coefficients
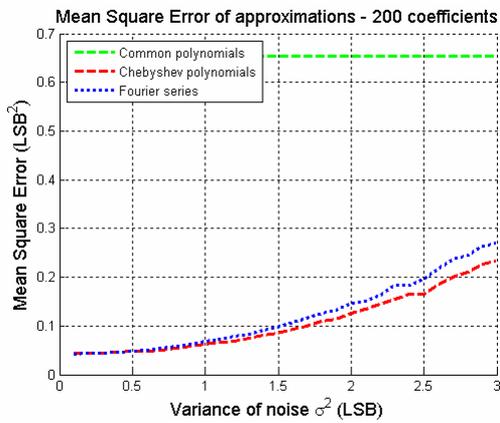
b) 200 coefficients

b) 200 coefficients

c) 1000 coefficients

c) 1000 coefficients

Figure 3. Noise influence to approximation

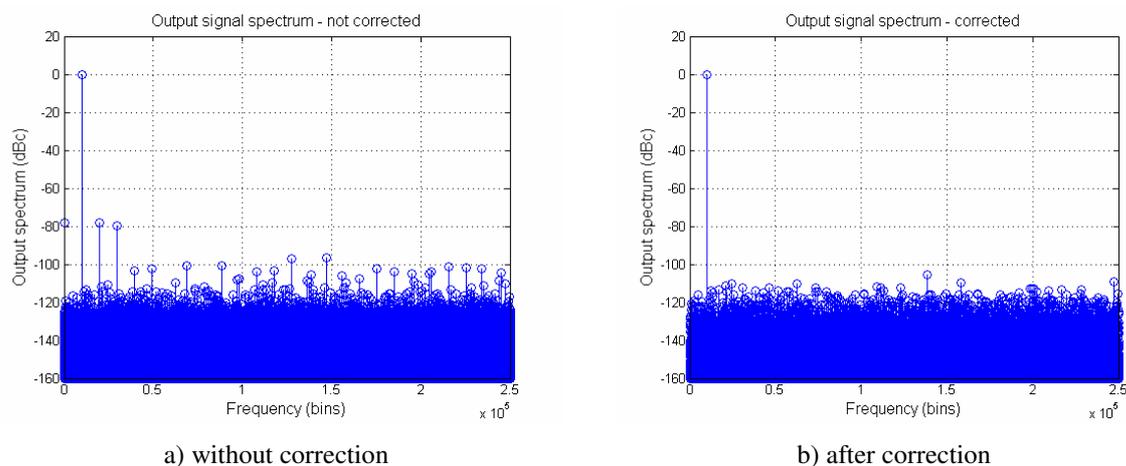Figure 4. Noise influence to approximation

a) without correction b) after correction

Figure 5. Spectrum of output signal *y*(*n*)

The accuracy evaluation showed that for small orders of used approximation, all types of approximations perform comparably. For higher number of estimated coefficients (roughly few hundreds) the approximation by common polynomials fails. For large number of coefficients Chebyshev polynomials and Fourier series are further improving and perform similarly. In the case of Fourier series and when the *INL*(*n*) from that the Fourier coefficients are calculated is not perfectly repeatable (periodicity is important), the Fourier series approximation features some oscillations at the beginning and end of the *INL*(*n*) curve.

The noise sensitivity evaluation showed that an increase of the maximum error is similar for all approximations in the case of the low number of coefficients. It further showed that more coefficients are not always better because a rise of the number of coefficients escalates noise sensitivity in all examined cases.

The correction method was tested using the proposed Chebyshev polynomials approximations as well. It was verified that the nonlinearity in absolute values larger than 1 LSB can be corrected. In the tested case, the improvement in the spectrum was around 30 dB of higher harmonic components suppression.

References

[1] L. Michaeli, P. Michalko, J. Saliga, "Fast Gesting of ADC Using Unified Error Model", *The 17th IMEKO world congress*, pp. 534–537, Dubrovnik, Croatia, 2003.

[2] A. C. Serra, M. F. da Silva, P. Ramos, R. C. Martins, L. Michaeli, J. Saliga, "Combined Spectral and Histogram Analysis for Fast ADC Testing", *IEEE Transactions on Instrumentation and Measurement*, vol. 54, No. 4, August 2005.

[3] N. Björsell, P. Händel, "Achievable ADC Performance by Postcorrection Utilizing Dynamic Modeling of the Integral Nonlinearity", *EURASIP Journal on Advances in Signal Processing*, vol. 2008, Article ID 497187, 10 pages, 2008. doi:10.1155/2008/497187

[4] N. H. Saada, R. S. Guindi, A. E. Salama, "A New Approach for Modeling the Nonlinearity of Analog to Digital Converters Based on Spectral Components", *Behavioral Modeling and Simulation Workshop, Proceedings of the 2006 IEEE International*, September 2006, pp. 120–125. ISBN: 0-7803-9742-8

[5] *IEEE 1241-2000 Standard for Analog to Digital Converters*. The Institute of Electrical and Electronics Engineers, 2001.

[6] P. Suchanek, V. Haasz: Approaches to the ADC transfer function modeling. *The 15th IMEKO TC4 Symposium*, Iasi, Romania, September, 2007.

[7] P. Suchanek, D. Slepicka, V. Haasz: Models of the ADC transfer function—sensitivity to noise. *I²MTC 2008 – IEEE International Instrumentation and Measurement Technology Conference*, Victoria, Canada, May 2008, pp. 583–587.

[8] F. Attivissimo, N. Giaquinto, I. Kale, "INL reconstruction of A/D converters via parametric spectral estimation", Transactions on Instrumentation and Measurement, vol. 53, No. 4 , pp. 940-946, August 2004.

[9] J. M. Janik, V. Fresnaud, "A Spectral Approach to Estimate the INL of A/D Converter", to appear in IEEE Transactions on Instrumentation and Measurement.

[10]   National Instruments, http://www.ni.com