Dither method for linearization of static characteristic of a class of converters

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Abstract- The characteristics of actual converters, including sensors, are – to smaller or larger extent – nonlinearity. Nonlinearity may result from the technologies applied to produce the converter, or from the type of processing to be realized by the converter. With respect to the mathematical model describing the characteristics, they can be characteristics with smooth nonlinearity or nonlinearity with discontinuities. The author presented the problem of linearization of the characteristic of converter with nonlinearity with discontinuity that appears in the small value area, i.e. with insensitivity area, applying a method utilizing the technique of added noise (dither). New potential of the converter with modified characteristic was shown: processing of input signals with values within the small value area, removal of discontinuities in output signal within the range of small values. The influence of the linearization method on converter accuracy was determined.

I. Introduction

There is a desired feature of converters, including sensors: the linearity of input-output characteristic. The characteristics of actual converters, including sensors, are – to smaller or larger extent – nonlinearity. If its value has an unaccepted influence on measurement accuracy, characteristics are linearized. If nonliearity appears, it means that sensitivity in the measurement range is variable. Measurement accuracy is then also variable, depending on the value measured.

Nonlinearity may appear in the whole range of input values (large scale nonlinearity), or only in certain sub-ranges for small, mean or large values (small scale nonlinearity). With respect to a mathematical model describing the characteristics, characteristics with nonlinearities can be divided in two types: characteristics with smooth nonlinearity or nonlinearity with discontinuities. Nonlinearities can be reduced with the use of system or program methods. The factors determining the selection of the method are first of all the technology applied to produce the converter and cost-performance analysis.

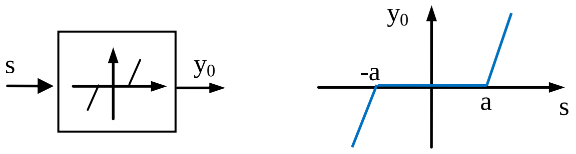
The paper presents the problem of linearization of the characteristic of converter representing nonlinearity with discontinuity that appears in the small value area (characteristic with insensitivity area). In general, this is not an easy task because of the type of nonlinearity and the local character of linearization. A method utilizing the technique of added noise (dither) is applicable in this purpose. This technique started to develop at the beginning of 60-ties. First applications concerned quantizing converters [1,2,3]. Up to now, the applications concerning quantization and AD converters have been most developed (hundreds of publications) [4,5,6,7]. Apart from a-d conversion, the dither linearization method found application: in nonlinear control systems with feedback (e.g. stability improvement) [8,9,10], in switching systems (e.g. sliding-mode control of power converters) [11], in converters of type MEMS (e.g. to improve SNR, especially at small accelerations) [12].

The linearization problem presented in the paper concerns converters operating in systems without feedback. Chapter II describes the rule of characteristic linearization with the dither method, the selection of dither signal and the selection of system eliminating any random component from the processing result (necessary element connected with the realization of the method). Chapter III shows new potential of the converter with the characteristic modified in that way: processing of input signals with values contained inside the insensitivity area, removal of nonlinearities in output signal, appearing in the range of small values; also the form of such characteristic was shown. Chapter IV determines the influence of linearization method on converter accuracy.

II. Linearization of characteristic with insensitivity area, with the use of dither signal

A converter with insensitivity area (IA) characteristic makes it impossible to process those values of signals that do not exceed the area range. The denotation and characteristic of such converter were shown in Fig. 1.

a) b) c)



$$y_0 = \begin{cases} k\cdot(s-a), & s>a \\ 0, & |s|\le a \\ k\cdot(s+a), & s<-a \end{cases}$$

Fig. 1. Converter with characteristic with insensitivity area:
a) symbol, b) input-output characteristic, c) processing equation

Consequently, in the output signal of the converter nonlinearities appear in the small value area, or the output signal does not appear at all if the values of input signal are smaller than the range of the insensitivity area. Both situations are illustrated in Fig. 2. In the case considered *k=1, a=0,2*.

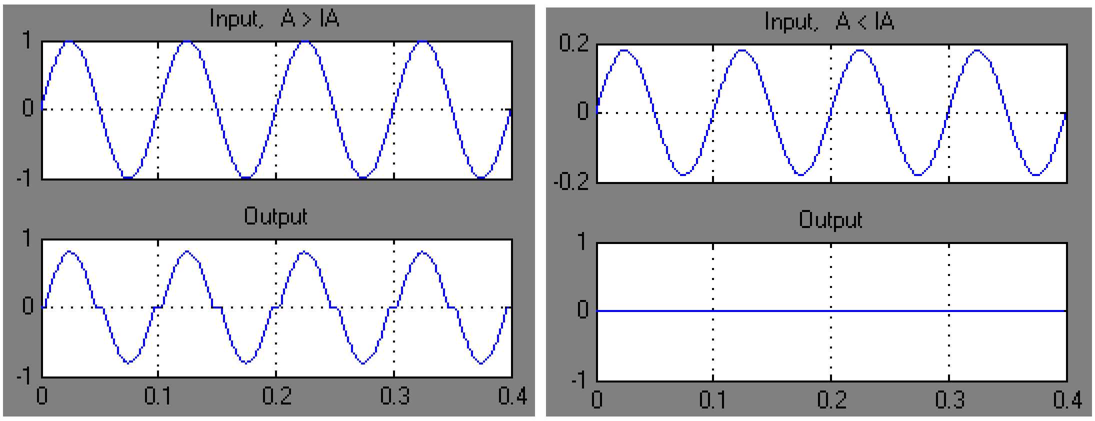


Fig. 2. Input and output signal of converter, -0,2≤ IA≤ 0,2, a) A=1, b) A=0,18

The appearance of nonlinearity in output signal is identical with a change (“enhancement”) of the spectrum of original i.e. with its nonlinear distortion. On the other hand, the inability of processing with small values is a limitation that forces us to apply solutions which usually cause substantial expansion of the measurement channel.

Characteristic linearization with the use of the method with dither signal consists in adding another signal (dither) to the input signal of converter, associative processing of both signals, and then averaging/filtration of the result. The idea of such linearization is presented in the diagram in Fig. 1.

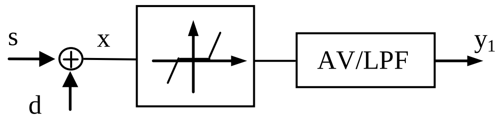


Fig. 3. Linearization of converter characteristic with dither signal

A. Choice of dither signal

The objective of the additional dither signal is the “taking out” of the values of the prime input signal, contained in the insensitivity area – “transferring” the values outside this area. Theoretically, every periodic signal (sinus, triangle, rectangle) having a suitable range of values and suitable frequency (considerably larger than the main signal frequency) or noise with a suitable variance, selected according to the size of insensitivity area, can play the role of dither [8,9,10]. Practically, however, nearly always it is a noise with rectangular or gaussian function of probability density, out of the following reasons:

- with respect to extremely complex structure, in the envelope of signal altered with such dither the form of prime signal is best preserved,
- with respect to many possibilities of further noise elimination (averaging, filtration, self-contained filtration).

► The considerations presented here relate to the case of applying dither being white noise with zero average value and with **rectangular** function of probability density.

If the converter has symmetric characteristic, the range of dither changes should be at least two times larger than the range of insensitivity area, i.e. $|D| \geq 2 \cdot IA$.

When dither *(d)* with span *D* is added to prime input signal *(s)* the values of which are partly or wholly contained inside the insensitivity area, the values of converter input signal *(s ± d)* begin to stick out of this area to the degree depending on the position of the values of prime input signal inside the area. Therefore, a signal appears at the converter output. The signal consists of random component coming from dither and slow-variable component from the prime input signal. As a result of averaging/filtration of output signal, the random component is eliminated, and the result obtained is equivalent to the processing of prime input signal in a converter with characteristic y_1 from Fig. 5.

If all the values of converter input signal *(s ± d)* are contained in the range of linear part of the characteristic, the result obtained is the same as in the case of processing signal *(s)* in converter without applying this method, because linearization has "local" character.

B. Random component elimination system

The purpose of AV/LPF system is to eliminate a random component from the converter output signal. It can be a system realizing the averaging (AV), or a typical low-pass filter (LPF). If dither was a narrow-band signal with the range located outside the converter band, then the elimination takes place as a result of self-contained filtration, and the role of elimination system is played by the converter itself.

The elimination can be realized with analog or digital method, depending on further purpose of the converter signal.

► In the present considerations an analog Bessel low-pass filter was used. It is a type of filter in which the best linearity of phase characteristic is obtained, compared to other types of filters of the same order.

III. Converter with modified characteristic

The application of the method described removes the nonlinearities in output signal; it also makes it possible to process input signals the values of which do not exceed the insensitivity area. Both cases are illustrated in Fig. 4.

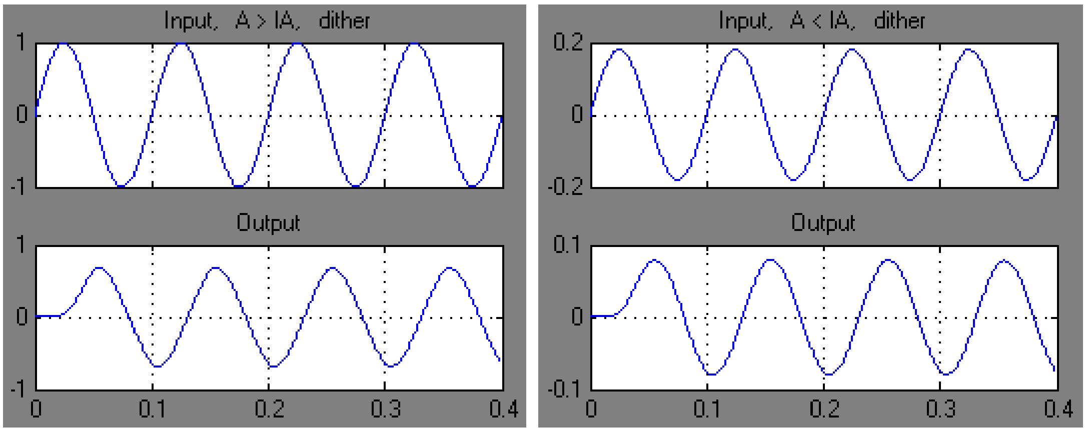


Fig. 4. Input and output signal of converter with dither, -0,2≤ IA ≤0,2, a) A=1, b) A=0,18

Differences between processing in systems from Fig. 1a and from Fig. 3 can be noticed when comparing corresponding pairs of signals: Fig. 2a and Fig. 4a as well as Fig. 2b and Fig. 4b. In the system from Fig. 3, the application of dither signal results in an effect equivalent to processing in converter with characteristic y_1, shown in Fig. 5.

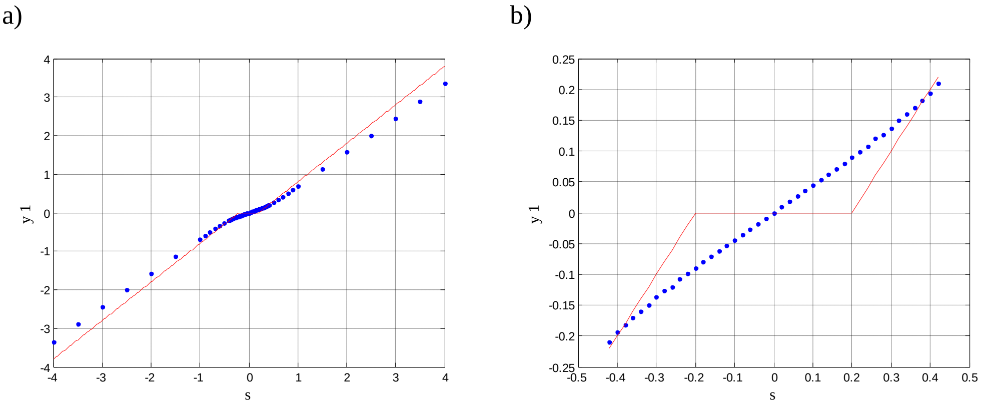


Fig. 5. Characteristic of converter with dither: a) global view b) local view

IV. Accuracy of converter with dither

Characteristic linearization with the use of the dither signal method requires two elements to be added to the converter: dither generator and AV/LPF system.

It is required from the generator to deliver a signal characterized by stable parameters. In case of generating noise dither, it is important that the zero average value and the stability of this value is ensured.

Averaging or filtration are linear operations. The AV/LPF system introduces amplitude error and phase error. The influence of the system on the amplitude reduction can be noticed when comparing the output signals – in Fig. 2a (without system) and 4a (with system). The influence of the system on the phase shift can be noticed when comparing the input and output signal in Fig. 4a or in Fig. 4b. The converter itself does not introduce phase shift – compare the input and output signal in Fig. 2a.

Without applying the linearization method, the degree of reduction of the input signal values not exceeding the insensitivity area is infinite, and the degree of the reduction of values coinciding with the linear part of characteristic depends on the slope of a straight line *(k)* (comp. Fig. 1) and it is constant.

After applying the linearization method, the degree of the reduction of signal values depends on the span of dither signal (variance) and the prime input signal value. This relation (for dither with rectangular function of probability density) was shown in Fig. 6a) for the input sinusoidal signal with amplitude $A=0{,}18<0{,}2=IA$, i.e. not exceeding the range of insensitivity area, and in Fig. 6b) for the input sinusoidal signal with amplitude $A=1>0{,}2=IA$. In case of amplitudes comparable with the width of insensitivity area, the reduction degree becomes finite (which was the aim of applying the method) but strongly dependent on the dither span. However, in case of amplitudes exceeding the width of insensitivity area several times, the relationship between the reduction degree and the dither span is much less.

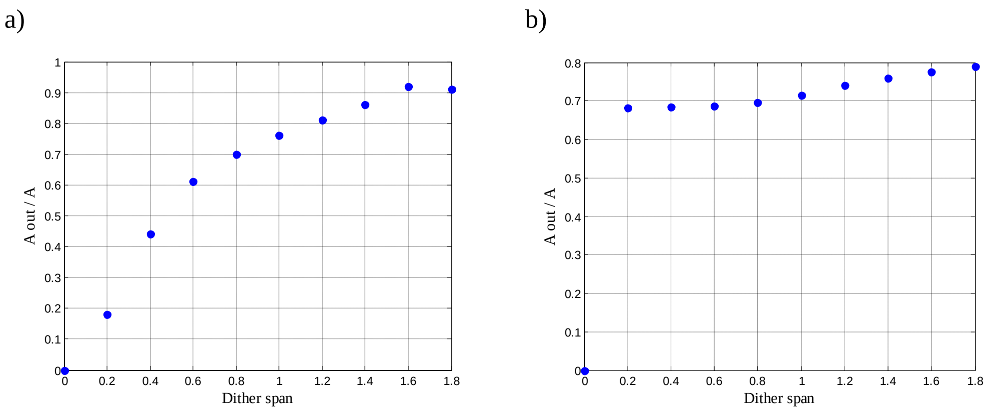


Fig. 6. Reduction degree of output signal amplitude of converter with dither a) A=0,18, b) A=1

V. Conclusions

The application of dither signal processing linearizes a converter characterized by insensitivity area. It effectively removes nonlinearities in output signal, caused by the area, i.e. removes nonlinear distortions of the signal processed. Using dither makes it also possible to process signals with values wholly contained in the area, which would be impossible in a converter without applying the method.
Side effects of applying the method are: amplitude reduction and phase shift of the processed signal, because of amplification error and phase error of the AV/LPF system, respectively. If we apply averaging (AV), it extends the time of processing.
In order to obtain the optimum result of linearization, a dither with suitable (in a given case) form of the probability density function and a variance adjusted to the size of the insensitivity area should be selected.

References

[1] Schuchman L., "Dither signals and their effect on quantization noise", *IEEE Trans. Commun. Technol.*, vol. 12, pp. 162-165, 1964.
[2] Jayant N., Rabiner L., "The application of dither to the quantization of speech signals", *Bell Syst. Techn. J.*,vol. 51, pp. 1293-1304, 1972.
[3] Gray R., Stockham T., "Dithered quantizers", *IEEE Trans. Inform. Theory*, vol. 39, pp. 805-812, 1993.
[4] Wagdy M., Goff M., "Linearizing average transfer characteristics of ideal ADC's via analog and digital dither", *IEEE Trans. Instrum. Meas.*, vol. 43, pp. 146-150, 1994.
[5] Carbone P., Petri D., "Performance of stochastic and deterministic dithered quantizers", *IEEE Trans. Instrum. Meas.*, vol. 49, pp. 337-340, 2000.
[6] Adamo F., Attivissimo F., Giaquinto N., Savino M., "Measuring the static characteristic of dithered A/D converters", *Measurement*, vol. 32, pp. 231-239, 2002.
[7] Attivissimo F., Giaquinto N., Lancolla A., Savino M., "Effects of midpoint linearization and nonsubtractive dithering in A/D convertrs", *Measurement*, vol. 40, pp. 537-544, 2007.
[8] Zames G., Shneydor N., "Dither in non-linear systems", *IEEE Trans. Autom. Contr.*, vol. 21, pp. 660-667, 1976.
[9] Iannelli L., Johansson K., Jönsson U., Vasca F., "Dither for smoothing relay feedback systems", *IEEE Trans. Circuis Syst. I, Fundam. Theiry Appl.*, vol. 50, pp. 1025-1035, 2003.
[10] Iannelli L., Johansson K., Jönsson U., Vasca F., "Averaging of nonsmooth systems using dither", *Automatica*, vol. 42, pp. 669-676, 2006.
[11] Frasca R., Iannelli L., Vasca F., "Dithered sliding-mode control for switched systems", *IEEE Trans. Circuis Syst. II, Express Briefs*, vol. 53, pp. 872-876, 2006.
[12] Kajita T., Moon U., Temes G., "A two-chip interface for a MEMS accelerometer", *IEEE Trans. Instrum. Meas.*, vol. 51, pp. 853-858, 2002.