17th Symposium IMEKO TC 4, 3rd Symposium IMEKO TC 19 and 15th IWADC Workshop
Instrumentation for the ICT Era
Sept. 8-10, 2010, Kosice, Slovakia

Correction of ADC Errors by Iterative Method with Dithering using Dither with Uniform Distribution

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Abstract-For automatic correction of measurement transducer error additive iterative method (AIM) could be used. But quantization error represents limitation for correction process in application for analog-to-digital converter. Combination of AIM with nonsubtractive dithering (ND) is proposed for this kind of application and the principle of combination of both techniques is described in the paper. AIM is based on precise inverse element (IE) which in our case consists of pulse width modulation output and low-pass filter. Technique similar to deterministic dithering is employed to achieve precise processing of signal from IE. Analysis of influence of dither with uniform distribution upon the results of correction is performed with the aim to find optimal parameters of ND. The optimal dispersion of dither is found leading to meaningful error correction. Finally dependency of root mean squared error on measured value is drawn to show how AIM improves accuracy.

I. Introduction

In modern equipment self-correction functions become important. Very often analog-to-digital converter (ADC) integrated within monolithic microcontroller is used for signal level measurements. Then ADC characteristics determine the overall metrology properties of measuring channel. It is not difficult to make correction of offset and gain error of ADC. But correction of nonlinearities is problematic especially if they vary in time.

Special automatic correction of ADC is employed as combination of two methods as discussed below. This correction is focused on ADC nonlinearities [1]. Additive iterative method (AIM) uses precise inverse element (IE) for correction of nonlinear error of general analog measurement transducer (MT). Besides integral nonlinearity (INL) ADC naturally has quantisation error. Quantisation limits efficiency of AIM, therefore in designed measurement system AIM is combined with dithering. We prefer nonsubtractive dithering (ND) for this application where low price is important aspect. Implementation of ND is easier than implementation of the subtractive form. However other special kind of dithering is applied for suppression of quantization error in our system. Technique based on deterministic dithering helps to obtain precise processing of signal from IE.

II. Principle of iterative correction

One of ways for measurement accuracy improvement is the use of so-called structural-algorithmic methods [2], where the measurement errors are diminished with help of auxiliary means. Generally for the AIM three new blocks should be added to MT and then four main blocks of the system are distinguished (Fig.1). In our case designed measurement system consists mainly of integrated components of single-chip microcomputer: MT – in our system is represented by ADC; block of processing (BP) – CPU (processor) with memory; inverse element (IE) – pulse width modulation (PWM) circuits with RC-filter; switch – multiplexer (MUX).

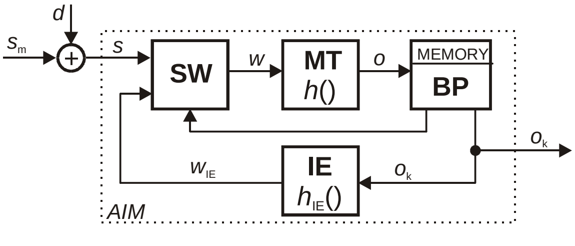


Fig. 1. Block diagram of our measuring system, the part in frame corresponds to AIM.

For our application only slow changes of measured value s_m are assumed. This value will be considered constant during one correction process and it represents the mean of actual system input $s_\mathrm{m}=E[s]$. For precise evaluation of s_m dither d is added before AIM. So in the initial step of AIM signal $s=s_\mathrm{m}+d$ is connected to the input of MT. Corresponding MT output $h(s_\mathrm{m})$ is then sent from BP to IE $o_{\mathrm{k},0}=h(s_\mathrm{m})$. At the same time BP changes the state of SW. Therefore in next steps signal from IE $w_\mathrm{IE}= h_\mathrm{IE}(o_\mathrm{k})$ is being sent to input of MT. Every next input to IE is solved in BP from actual MT output o_i and previous IE input $o_{\mathrm{k},i-1}$ according to following iterative formula

$$o_{\mathrm{k},i} = o_{\mathrm{k},i-1} + \left(o_{\mathrm{k},0} - o_i\right) \qquad (1)$$

If characteristic of IE is ideal, i.e. it is equal to inverse of ideal MT characteristic $h_\mathrm{IE}^{-1}= h_\mathrm{I}$, described algorithm changes static error of MT $\Delta h(s_\mathrm{m})=h(s_\mathrm{m})-h_\mathrm{I}(s_\mathrm{m})$ according to geometric series $\Delta h_{\mathrm{k},i}=(-C)^\mathrm{i}\Delta h$. Factor C is determined by ratio of derivatives (sensitivities) of error and ideal characteristics and if condition of convergence is satisfied

$$|C| = \left| \left.\frac{d\Delta h(x)}{dx}\right|_{x=s_\mathrm{m}} \div \left.\frac{dh_\mathrm{I}(x)}{dx}\right|_{x=s_\mathrm{m}} \right| < 1 \qquad (2)$$

then AIM suppresses MT error. Suitable ending condition for iterative correction should be used for stopping the process after sufficient number of steps, e.g. small difference between last two corrected values $|o_{\mathrm{k},i}-o_{\mathrm{k},i-1}|<\varepsilon$. Generally, convergent iterative process tends to value given by characteristics of IE $o_\mathrm{k}=h_\mathrm{IE}^{-1}(s_\mathrm{m})$. If it is not possible to have an ideal IE, error of IE will influence results of correction. Deeper investigation of IE properties can help to obtain negligible IE error.

A. Inverse element operation

Transfer characteristic of IE determines theoretically reachable accuracy of measurement output after correction. Inverse element for ADC is digital-to-analog converter (DAC) and is built by means of pulse width modulation output of microprocessor. PWM circuits are naturally precise but to get the mean of IE output $w_\mathrm{IE,m}=E[w_\mathrm{IE}]$ corresponding to precise DAC result, low-pass filter should be added. Simple RC-filter (RCF) is used. Frequency characteristic of this filter is determined by time constant τ_RC, which has to be large enough to get small amplitude oscillation of signal from IE. But large τ_RC slows down measurement process because after every step the process should wait until settling of filter output.

The block diagram of inverse element including processing of signal from IE is depicted in the Fig. 2. To speed up the process we proposed to use combination of analog and digital filter. Time constant τ_RC is selected such that output of analog RCF oscillates in the range of several LSB. As digital filter simple averaging of N samples (block AVE in the Fig. 2) in each step of the iterative correction is used. The best way would be synchronous sampling here but there could be no possibility to synchronize ADC and PWM circuits. Mathematical model of error caused by non-synchronous sampling was designed. In this model quantization is not considered but it could be used as model of IE error because this part of IE error is dominant for many values of N. Using the error model time constant of RC-filter (τ_RC=0,1 s) and number of samples (N=59) was found and proved by simulations. For appropriate number of samples N special case of quasi-synchronous sampling is achieved.

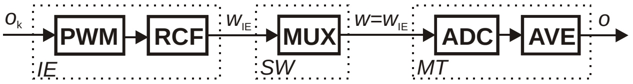


Fig. 2. Block diagram of creating and processing of signal from IE. Dotted frames indicate blocks of AIM from the Fig. 1.

III. Uniform dither and averaging

Quantization error limits measurement accuracy of an ADC corrected with AIM. To overcome this limitation in evaluating of measured value s_m, intentionally added noise d (dither) prior to quantization (prior to SW in Fig.1) can help. Then averaging of samples of signal $s = s_\mathrm{m}+d$ can lead to resolution improvement for measurement of mean $s_\mathrm{m}=E[s]$. This technique is called nonsubtractive dithering (ND), as the noise is not subtracted from the signal after quantization.

Influence of dither on quantizer could be generally investigated from theory of quantization described in [3] or [4] where technique of creating of mathematical modelled is called area sampling. However other sources like [5] avoid area sampling while evaluating a particular accuracy parameter. In [5] mean error is derived from known deterministic behaviour of quantization error and known probability density function (PDF) f_d of dither as expected value. The same result for mean error could be obtained according to theory of area sampling. Assuming dither with zero mean and symmetrical (and real) PDF it holds

$$ME(s,\sigma_d)=\sum_{k=1}^{\infty}\frac{q(-1)^k}{\pi k}\Phi_d\left(k\frac{2\pi}{q}\right)\sin\left(k\frac{2\pi}{q}s_{\mathrm{m}}\right) \tag{3}$$

where q is quantization step, Φ_d is characteristic function of dither with standard deviation σ_{d} and s_{m} is measured input of quantizer with ND. Disadvantage of (3) is dependancy on input s_{m}. To evaluate single parameter describing error of the whole range measurements it is better to solve mean squared error (MSE) [6] within one quantization step

$$MSE(s,\sigma_d)=\frac{1}{q}\int_{-\frac{q}{2}}^{\frac{q}{2}}ME^2(s_{\mathrm{m}},\sigma_d)ds_{\mathrm{m}}=\frac{q^2}{2\pi^2}\sum_{k=1}^{\infty}\frac{1}{k^2}\Phi_d^2\left(k\frac{2\pi}{q}\right) \tag{4}$$

There are several types of dither analysed in [5]. Usually uniform noise leads to best results, if peak-to-peak value D_d of noise equal to one quantization step q could be provided. But it is often assumed in theory, that the number of processed samples N is large when (3) or (4) give good error estimation. In the microcomputer application like this N is small and therefore dispersion of measurement output should be included in model of error like in [6] or [7]. The mean squared error is suitable parameter for rating of dithering and averaging performance if stochastic part of error after averaging is included into total MSE [6]

$$MSE_{\mathrm{T}}(\sigma_d,N)=\frac{\frac{q^2}{12}+\sigma_d^2}{N}+\left(1-\frac{1}{N}\right)\frac{q^2}{2\pi^2}\sum_{k=1}^{\infty}\frac{1}{k^2}\Phi_d^2\left(k\frac{2\pi}{q}\right) \tag{5}$$

Such total MSE (MSE_{T}) was theoretically evaluated in [6] for Gaussian dither. But it is difficult to generate artificial Gaussian noise. Uniform dither generally leads to better accuracy and could be generated easily e.g. by asynchronous sampling of triangular signal. Therefore we designed uniform dither for our application and derived theoretical model of total RMSE (Root MSE) for uniform dither

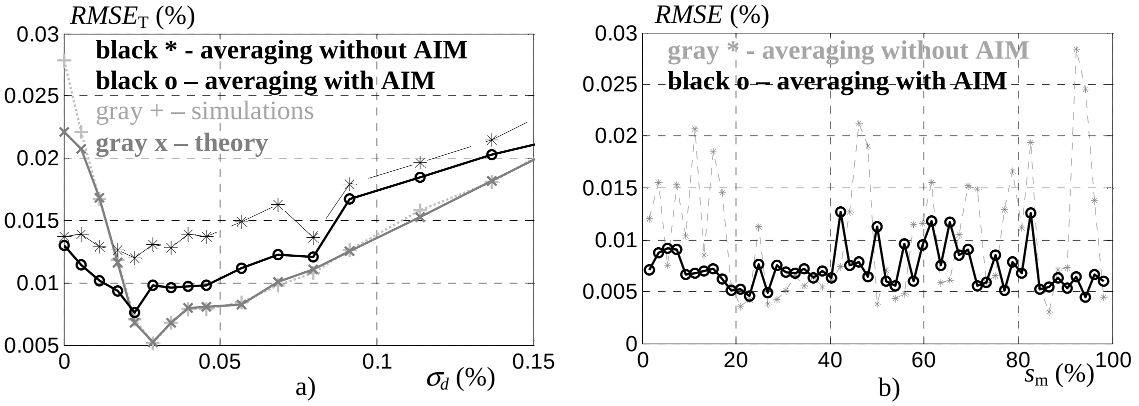
$$RMSE_{\mathrm{T}}^2(D_d,N)=MSE_{\mathrm{T}}(D_d,N)=\frac{q^2+D_d^2}{12N}+\left(1-\frac{1}{N}\right)\frac{q^2}{2\pi^2}\operatorname{sinc}^2\left(\frac{\pi D_d}{q}\right) \tag{6}$$

This formula embodies both the mean error and dispersion of measurement results. The second part of (6) reflects deterministic part of error which is zero for $D_d=l.q$ (l is positive integer) and negligible for large D_{d}. But the first part reflects dispersion of error and rises with dispersion of added noise. This part is meaningful for small number N of averaged samples, when optimal dispersion of dither is dependent on N. Optimal D_d is then less than q.

IV. Experimental results and discusion

Experiments were performed with designed measurement system, where AIM and ND with averaging was implemented for error correction of 10-bit ADC. To evaluate dispersion of results within error analysis, total RMSE ($RMSE_{\mathrm{T}}$) curves are depicted in the Fig.3a. Theoretical dependency solved from (6) is proved by simulation results seeing that light and dark gray lines are almost identical for major part of graph. Implemented method significantly corrected gain error and offset in experiments. Main contribution of designed correction is in suppression of nonlinear error component. Therefore linear error part had been subtracted from measurement results before evaluation of experimental RMSE. If only natural noise used the reduction of nonlinear error with AIM is little. According to Fig.3a dither with standard deviation close to 0.25 LSB (0.025 %) has really enhanced accuracy but without iterative correction INL of ADC has caused notable shift of experimental curve against theory. AIM corrects INL and therefore it shifts the curve closer to theoretical values. As in our case real system is tested optimal dither dispersion is lower than theoretical optimum because natural noise is present in the input signal. According to Fig.3a quasi-optimal standard deviation of dither is found $\sigma_d = 0.2333q$ (0.02279 %) as the final parameter of our design.

For the designed measurement system the RMSE (of nonlinear error part) dependency on measured value calculated from 20 measurements in each input level s_{m} is depicted in the Fig.3b. The points of minimum are influenced by dispersion of results which is higher after correction with AIM. Main contribution of AIM is suppression of peaks caused by INL. Finally error is corrected considerably under the 1/sqrt(12) LSB (0,02819 %). In our case appropriate dither enables this suppression of INL with AIM deep below 1 LSB.



a) b)

Fig. 3. RMSE of ADC with averaging before and after correction with AIM:
a) total RMSE; b) RMSE evaluated in each point of range for quasi-optimal dither dispersion.

V. Conclusion

Combination of two methods has been implemented for ADC error correction. Additive iterative method (AIM) automatically corrects integral nonlinearity and it is based on precisely designed inverse element. Non-subtractive dithering with averaging enables correction below level of 1 LSB of used 10-bit ADC. For uniform dither theoretical dependence of total root mean square error (RMSE) upon standard deviation of added noise has been proved through measurements in the whole ADC range. Quasi-optimal value of dither dispersion has been found. Finally the RMSE has been reduced significantly. Accuracy improvement evaluated in ENOB (Efective Number of Bits) is 2.26 bit with final value of 11.90 bit after correction. The designed technique requires minimal additional hardware components (only passive low-pass filter) so it may be used in each microcontroller based measuring channel for which the high speed is not the necessity. Such microcontroller applications are widely used in industrial praxis today.

Acknowledgements

Work presented in this paper was supported by the Slovak Ministry of Education under grant No. 2003SP200280802 and by the Slovak Grant Agency VEGA under grant No. 1/0551/09.

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