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Analysis of the variability of systematic errors in ADC-based instruments

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Abstract – The paper deals with the problem of assessing the uncertainty due to systematic errors, especially in ADC or ADC-based instruments. The problem of defining and assessing systematic errors is briefly discussed, and the conceptual scheme of gage repeatability & reproducibility is adopted. Experiments are conducted in various conditions, and it is shown that modeling the variability of systematic errors is more problematic than suggested in the ISO 5725 norm. The solution to the problem of assessing the overall uncertainty is outlined through an in-depth analysis of all the separate causes of variations in measurements.

Keywords: uncertainty, systematic effects, digital instruments, A/D converters

I. INTRODUCTION

The concept of "systematic effect" or "systematic error" (SE), as well as that of "random error" (RE) lies at the basis of metrology; nevertheless, a clear and universally agreed upon definition for them is not available in the literature, nor is available a clear and universally agreed upon way of taking into account SEs in the overall measurement uncertainty.

According to the latest edition of the International Vocabulary of Metrology (VIM) [1], a SE is a "component of measurement error that, in replicate measurements, remains constant or varies in a predictable manner". This definition implies the accepting of the conceptual scheme called, in the VIM itself, "Error Approach" or "Traditional Approach" or "True Value Approach", thus abandoning the alternative scheme called "Uncertainty Approach", which is, in turn, is at the basis of the Guide to the Expression of Uncertainty of Measurement (GUM) [2]. The Error Approach, with its clear definition of reference (true) value and of measurement error, greatly simplifies matters (with respect to the Uncertainty Approach), but the definitional problem remains – surprisingly – far from being solved.

What does it mean, indeed, that a component of the measurement error is constant in replicate measurements? A truly "constant" error could be determined once and for all with a single effort, and corrected in all the measurements. This operation is contemplated in the VIM and in the GUM, but it is not the typical scenario. SEs of instruments, even those of high-class instruments, are not negligible with respect to random ones. Why manufacturers do not correct for them? The reason is that SEs, actually, vary.

As a matter of fact, the ISO norm 5725 [3] clarifies that the "constant" systematic error is not at all constant. The norm deals with errors of a standard measurement method implemented in different laboratories. The basic hypothesis is that, while repeated measurements in the same laboratory are affected by a constant systematic error, an analogous set of measurements performed in a different laboratory is affected by a constant but *different* systematic error. This makes the difference between "repeatability" and "reproducibility", i.e., between the intra-laboratory and the inter-laboratory dispersion of the measurements [4]. Clearly, the total variance of the measurements is the sum of intra-laboratory and inter-laboratory variances, and this indisputable conclusion agrees with the GUM way of treating systematic errors (but not with alternative approaches like those suggested in [5] or in [6]). In short, the ISO 5725 model seems to be the basis for a correct understanding of SEs, not only in terms of definition, but also in terms of contribution to the overall measurement uncertainty. The norm explains, actually, how to carry out a full Type A evalution of the uncertainty of a standard measurement method, taking into account REs and SEs as well.

In a previous work [7] the authors have discussed the possibility of extending the concepts of ISO 5725, which are relevant to *laboratories* and *measurement methods*, to the much more common case of SEs in *instruments*, and particularly in digital instruments. Obviously, the repeatability and reproducibility concepts have been already applied to instruments, at least since the appearance of the fundamental work [8]. However, there are many differences between the proposal in [7] and the customary *gage repeatability & reproducibility* (GRR) study, as described, for example, in the tutorial work [9], or in the Statistics Toolbox of the popular Matlab software [10], and in countless other papers and manuals.

GRR is, indeed, a statistical tool to assess the variability of measurements *in production processes*. As a consequence, the reproducibility is almost always referred also to *operators*, not only to equipments; but, more importantly, it is by definition the variability among the set of instruments actually used in production, in the special conditions encountered in production. If only one instrument of a certain type is used in the production process, the repeatability is referred merely to appraisers. The "intra-factory" repeatability does not express a general property of the instrument, differently from the uncertainty. Indeed, the "intra-factory" repeatability must be evaluated periodically, due to drifts, etc., and it is used merely to verify that, at a given moment and for a given production process, the available measurement system complies to a specific metrological performance..

The proposal in [7] is to extend the repeatability concept *to the whole population of existing instruments* of a certain model. The properties of the population must be inferred through specific tests on several instruments of the same kind, possibly situated in different and far apart laboratories. This broader "inter-instrument" repeatability should consider all the reasonably allowed working conditions of the instrument, should be evaluated once and for all, and should be used to assess, in a general sense, the overall instrument uncertainty.

Special considerations must be done for *digital instruments for measurements of electrical quantities*, based on an analog-to-digital converter (ADC). These instruments allow the accumulation of a large number of measurements without human intervention; this makes possible a much deeper error analysis than the customary GRR. Currently, GRR studies are based on complex and rigorous statistical approaches (mainly ANOVA [11]) but, usually, they must rely on a limited set of measurement data. Therefore, a standard GRR yields the best possible quantification of different variation components in the measurements; nevertheless, because of the limited set of data it cannot extract information about particular effects, such as stability, trends, etc.

On such bases, the present paper is focused on an experimental analysis of variability of one of the main SEs affecting digital instruments and ADCs, i.e offset errors. The analysis is not a customary GRR, because the correctness of the ISO 5725 error model is not taken for granted; SEs are examined on the basis of a very high number of measurements, performed automatically in various conditions on different instruments. It is shown that SEs, actually, are even more variable and unpredictable than the model predicted by ISO 5725.

II. THEORETICAL FUNDAMENTALS

The equation at the basis of the ISO 5725 norm can be written as:

$$y_{ik} = x + b + B_i + er_{ik} \Leftrightarrow e_{ik} = b + B_i + e_{ik} = es_i + er_{ik} \tag{1}$$

where x is the accepted reference value or "true value", y_{ik} is the k-th measurement of the i-th laboratory, b is the bias or SE introduced by the method, B_i is the bias or SE introduced by the i-th laboratory, and er_{ik} is the RE of the measurement. The total error $e_{ik} = y_{ik} - x$ is therefore the sum of three errors with different degree of variability: b is completely constant; B_i is constant within the laboratory, but variable among laboratories; and er_{ik} is completely variable. The sum $b + B_i$ is the overall SE es_i, which depends only on the laboratory.

When dealing with A/D converters and digital instruments, one must also take into account the standard way of modelling SEs of this kind of devices. No manufacturer or user consider the SE of an A/D converter as a whole; on the contrary, it is decomposed in different contributions that are usually referred to as gain, offset, and nonlinearity error:

$$es = e_G \cdot x + e_O + nl(x) \tag{2}$$

(index i is dropped here, and $nl(x)$ takes into account both integral nonlinearity and quantization error, which is a nonlinearity error in nature). Nonlinearity errors have great importance in many measurements (e.g. in spectral measurements), but in simpler and more common measurements, like those of absolute voltage, peak-to-peak voltage, ratio between voltages, etc., gain and offset errors have usually the greater effect. This study is especially focused on offset errors e_O.

According to the error model of ISO 5725, e_O should be constant in the same instrument, and should vary among instruments. Besides, in multi-range instruments, SEs errors are expected to depend also upon the selected range. Therefore, an analysis of the offset error for a given instrument requires, first of all, to obtain many repeated estimates of the errors for many instruments of the same model, with different selections of the input range. If the index i identifies the instrument/range pair, $\hat{e}_{O_{ik}}$ can denote the k-th estimates. After deriving $\hat{e}_{O_{ik}}$, it is possible to answer some questions.

- Are $\hat{e}_{O_{ik}}$ substantially independent of k ?

- If not, what are the causes of the (additional) variation of the systematic error?
- How can the overall variability of the systematic error be characterized?

III. EXPERIMENTAL SETUP

A. Basic scheme of operations

The basic setup adopted to carry out the experiments is depicted in Figs. 1-2.

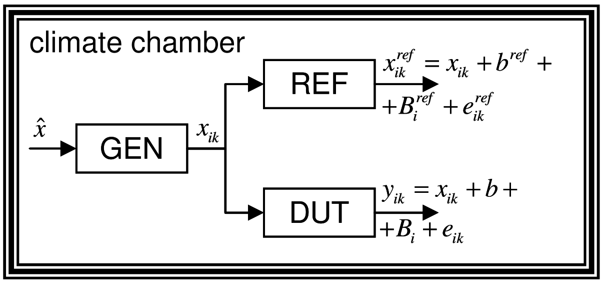

Fig. 1. Basic setup for a single instrument experiment in controlled environment (climate chamber).

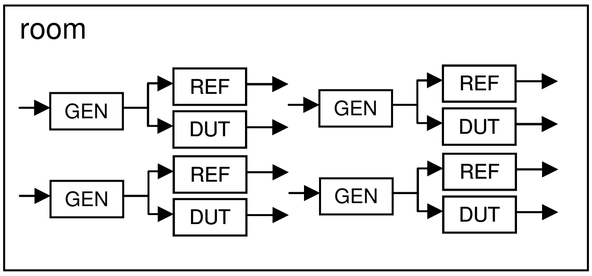

Fig. 2. Basic setup for a multiple instruments experiment in an ordinary room.

Fig. 1 refers to experiments performed on a single instrument in controlled environmental conditions, while the second depicts a similar experiment carried out on multiple instruments in a generic (non-controlled) environment. GEN is a non-ideal generator: the input is the nominal output $\hat{x}$, while the output are, in repeated measurements, the unknown voltages x_{ik}. The device under test DUT yields the measurements y_{ik}, which should theoretically obey the model in (1). The reference instrument REF yields the measurements x_{ik}^{ref}, and, thanks to its highly accurate metrological performance, the corresponding error $|x_{ik}^{ref} - x_{ik}|$ is negligible with respect to the errors $y_{ik} - x_{ik}$ that are to be characterized. The fulfillment of this condition can be inferred from the manufacturer's specifications of DUT and REF.

B. Technical data of the appliances

The conceptual scheme represented in Fig. 1-2 has been practically realized using the following instruments:

Climate chamber: Angelantoni Industrie, mod. Challenge 250
DUT: Agilent digital oscilloscope, model 54600B;
REF: Agilent digital multimeter, model 34401A;
GEN: Agilent generator, model 33220A;

The Challenge 250 climate chamber can keep a constant temperature chosen in the range –40/+180 °C, with a maximum variation of ±0.3 °C over the experiment time; the relative humidity is also kept constant within ±3% (typical variations are within ±1%) [12]. The results reported herein have been obtained at the temperatures of 20 °C, 25 °C and 30 °C, respectively, and for a constant relative humidity of 40%. The three temperatures have been chosen with the aim of representing typical working conditions for an instrument used in a room.

The device under test is an 8-bit digital oscilloscope, while the reference instrument is a 6 ½ digit multimeter. As aforementioned, the uncertainty of the latter can be considered negligible with respect to that of the DUT. The DUT has been tested with two different selections of the vertical gain: 1 V/div and 0.1 V/div, which yielded a full-scale range (FSR) of [–0.4, +0.4] V and [–4, +4] V respectively. Therefore, six separate tests (two vertical gains, at three temperatures) have been performed in the climate chamber. Each test in the climate chamber lasted 6 hours. Additionally, to address a comparative analysis, a single test on multiple instruments has also been carried out in an ordinary laboratory environment, i.e. without control on temperature and humidity. This test lasted 42 hours.

C. Technical data relevant to offset error measurements

The offset errors of the DUT have been measured on the basis of a straight line passing by two test points, put symmetrically in the vertical range, at ±90% of the maximum voltage (i.e., at ±0.36 V at 0.1 V/div, and at ±3.6 V at 1 V/div). The voltage of each test point has been generated by GEN, and has been measured 10 000 times by DUT, and 5 times by REF. The average of the measurements has been used to obtain the systematic difference between the measured and the reference voltages.

IV. EXPERIMENTAL RESULTS AND CONSIDERATIONS

A. Results

Experimental results are reported in Figs. 3-6.

Fig. 3 shows the offset error measured on one of eight identical instruments in the same laboratory environment, with the vertical gain at 1 V/div, during a 42 hours long test. The test was carried out in a laboratory at different environmental temperatures, obtained with the ordinary heating and air conditioning appliances. Of course this kind of experiment does not allow to control accurately the ambient conditions, and therefore to make a reliable study of their effects on the error. However, it demonstrates that the error actually varies in normal working conditions, while it should be constant, and therefore truly "systematic", with constant temperature and humidity conditions. For example, in the interval between hour 10 and hour 15 temperature and humidity were approximately constant, and so is the offset error.

Fig. 4 shows the offset errors of the eight different tested DUTs in the interval between hour 10 and hour 15. (among which the one in Fig. 3). It can be noted that the instruments, although identical externally and placed in the same room (approximately equal ambient conditions), show clearly different offset errors. This can be considered the outcome of an "inter-instrument" experiment, which show the variability of the error among different devices in the same conditions.

To observe the variability of the error in the same device an "intra-instrument" experiment is carried out. This consists in six separate 6 hour-long tests carried out in the accurately controlled ambient conditions of the climate chamber; in fact, the climate chamber allows only to test one instrument at a time.

Fig. 5 and 6 show the test results at 1 V/div and at 0.1 V/div respectively, at the three different temperatures of 20 °C, 25 °C and 30 °C, and at the relative humidity of 40%. Results show that the offset error depends strongly both on temperature and on the selected scale. The dependency on temperature appears to be quite deterministic (as it could be expected). Offset errors vary quickly in the first 20÷30 minutes, which is the normal warm-up of the instrument. Besides, it can be note that even in controlled environment conditions, offset error appears to be affected by small "drifts" and instabilities, which should be accounted for in some way.

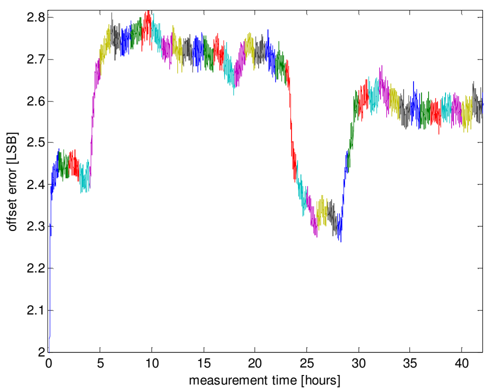

Figure 3. Offset error in one of eight identical DUTs in the same environment

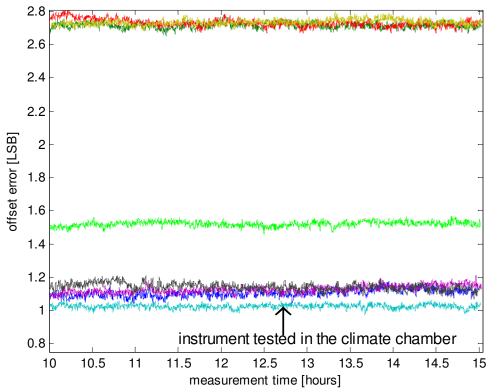

Figure 4. Offset errors of eight different DUTs (among which the one in Fig. 3), in the interval between hour 10 and hour 15

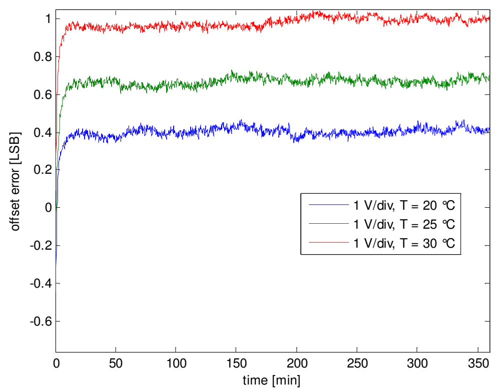

Figure 5. Offset errors of a DUT at different controlled temperatures and FSR = [−4, +4] V

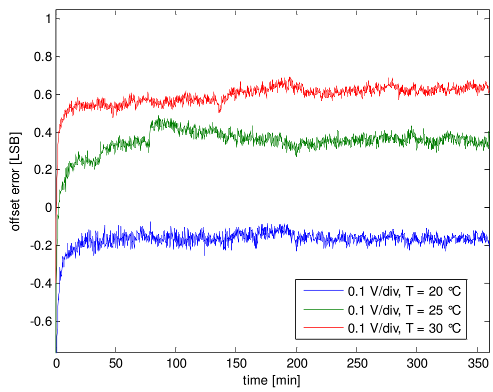

Figure 6. Offset errors of the same DUT at the same temperatures and FSR = [−0.4, +0.4] V

B. Considerations

The obtained results suggest some considerations.

1) Systematic errors in "random" environmental conditions are almost "random" (Fig. 3). In general, there are many experimental conditions causing meaningful "intra-instrument" variations in the systematic errors (warm-up, drifts, etc.). Therefore, the simple error model (1) at the basis of ISO 5725 must be considered valid only in very special cases, i.e. considering constant environment conditions, and a comparatively short time, in which the instrument is "stable" (Fig. 4).

2) Conditions necessary to fulfill the error model (1) must not be confused with actual conditions in which instruments are used. In other words, making measurements at controlled environmental conditions could be recommended for maximum precision; but it is not correct to suppose that these conditions are met when assessing the uncertainty of an instrument.

3) A meaningful uncertainty assessment seems to require *a detailed analysis of all the causes of variations in the measurements*; in this paper four principal causes (the equipment; the selected range; the temperature; the time elapsed from the power-on instant) are pointed out. These factors should all be evaluated, besides the magnitude of random errors.

V. CONCLUSIONS

The paper reports some experimental results on the variability of systematic errors in an ADC-based instrument, and specifically of the offset error. A great number of measurements, on many instruments, with different selections of the input range, and in different ambient conditions, have been performed and reported. Experiments show that "constant" systematic errors present a meaningful and unpredictable variability, due to many factors, such as the specific instrument, the selected input range, the temperature variations, and drifts. Actual experimental data, therefore, indicate that all the causes of variations in measurements should be separately quantified in the characteristic sheets of the instrument. This could allow the final user to evaluate the uncertainty of measurements on the basis of the particular measurement conditions, by adding up the relevant variance (and, possibly, covariance) terms.

ACKNOWLEDGEMENTS

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