Improved Estimate of Parametric Models for Analogue to Digital Converters by Using Weighted Integral Nonlinearity Data

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Abstract- Error modelling has played a major role in generating post-corrections of analogue to digital converters (ADC). Benefits by using parametric models for post-correction are that they requires less memory and that they are easier to identify for arbitrary signals. However, the parameters are estimated in two steps; firstly, the integral nonlinearity (INL) is estimated and secondly, the model parameters. In this paper we propose a method to improve the performance in the second step, by utilizing information about the statistical properties of the first step.

I. Introduction

For an ideal ADC the output is a discrete time and discrete amplitude representation of the analogue input signal with fixed time and amplitude steps. The output from a practical ADC has deviations from the ideal steps. Due to these imperfections in practical ADCs the performance of an ideal ADC can never be achieved. An ADC-model contains information on the size of the amplitude deviation as a function of the output code, which can be used to correct the digital output, and, thus, give a better representation of the analogue input.
The most commonly used measure of amplitude deviation is the INL. The imperfections can be compensated by using a post-correction method, for instance, by the use of a look-up table (LUT). However, a LUT requires a large memory size and is time consuming to train. Instead, parametric models of the INL can be used. In [1], the INL was split up into one low code frequency (LCF) part, representing a smooth curve over the code range, and one high code frequency (HCF) component represented by a saw-tooth wave. This was further developed in [2-4] to include dynamic behaviour. In [5], a post-correction method based on a parametric INL model was suggested. The performance of the post-correction algorithm depends on the accuracy of the parametric model, which in turns depends on the accuracy of the INL estimates. In this paper we will improve the accuracy of the parameter estimates by making use of known uncertainties in the INL estimates.

II. Theory

A. Estimate INL

The ideal transition level, T, can be written in terms of the output code k of the ADC, as

$$T_k = Q\left(k-1\right) + T_1, \tag{1}$$

where Q is the ideal width of a code bin; in other words, the full-scale (FS) range of the ADC is divided by the total number of codes $(V_{max} - V_{min})/2^N$, where N denotes the number of bits. Further, T_1 is the ideal voltage corresponding to the first transition level. The code k spans $k = 1, \ldots, 2^N - 1$. For a practical ADC subject to a gain G and offset V_{OS}, the INL $i[k]$ is defined as [6]

$$i\left[k\right] = \frac{T_k - GT\left[k\right] - V_{OS}}{Q}, \tag{2}$$

where $T[k]$ is the transition level of the practical ADC.

The test of ADCs with statistical analysis is based on the construction of a histogram, which gives the number of occurrences of each code at the output of the converter. This histogram is then compared with the probability density function (pdf) of the stimulus signal. The obtained deviation is a measure of the ADC nonlinearity where the INL and differential nonlinearity (DNL) are the most commonly used measures. Test methods for ADCs are standardised by IEEE [6]. The choice of stimuli depends on the statistical properties of the signal, how easy it is to generate, its noise sensitivity, and for what application the resulting histogram will be used. The most commonly used test signal is probably the sine wave [7], due to the easiness with which sine waves can be generated and with their spectral purity. Consider the sine wave signal that has the pdf f and the distribution function F as follows:

$$f\left(T_k\right)=\begin{cases}\dfrac{1}{\pi\sqrt{FS^2-T_k^{\,2}}} & \left|T_k\right|\leq FS\\ 0 & \left|T_k\right|>FS\end{cases}, \tag{3}$$

$$F\left(T_k\right)=\begin{cases}0 & T_k<-FS\\ \dfrac{1}{2}+\dfrac{1}{\pi}\arcsin\left(\dfrac{T_k}{FS}\right) & \left|T_k\right|\leq FS\\ 1 & T_k>FS\end{cases}. \tag{4}$$

The histogram test is an unbiased estimator, where the transition levels are estimated by [8]

$$\hat{T}\left[k\right]=F^{-1}\left(\hat{p}_k\right), \tag{5}$$

where p_k is the cumulative histogram for output code k.
The statistical efficiency of an unbiased estimator can be evaluated by comparing its variance with the corresponding Cramér-Rao lower bound (CRLB). The CRLB for a histogram with arbitrary stimuli is [8]

$$CRLB\left(T_k\right)=\frac{1}{NI\left(T_k\right)}=\frac{1}{N}\frac{F\left(T_k\right)\left[1-F\left(T_k\right)\right]}{f\left(T_k\right)^2}, \tag{6}$$

where N is the record length and $I(T_k)$ is the, so called, Fisher information [9].

B. Estimate a parametric model of the INL

The estimated INL can be used to generate a look-up table for post correction. However, due to drawbacks with large LUTs and the difficulties to train them, a parametric model of the INL may be preferable, especially if the dynamic behaviour of the ADC performance is taken into account. In [4], the measured INL was described by a parametric model $i_{m,k}$ where m corresponds to the test frequency (that is, f_m in Hz) and k to the ADC output code:

$$i_{m,k}=h_k+l_{m,k}. \tag{7}$$

The term h_k is the HCF component, and $l_{m,k}$ the dynamic LCF component. The HCF component h_k is modelled by a set of P disjoint segments centred around zero, while the LCF, by a smooth polynomial over the code range for every frequency under test. The HCF model in a specific interval K_p can be written as [4]

$$h_k\triangleq h_{k,p}=\left(k-\frac{k_{p-1}+k_p-1}{2}\right)\eta_p, \tag{8}$$

$$K_p:\left\{k\,\middle|\,k_{p-1}\leq k\leq k_p\right\} \tag{9}$$

for an output code k belonging to the segment p, where $p=1,\ldots,P$. In (8), η_p is the slope of the segment p.
The set of coefficients, η_p, is gathered in a vector $\boldsymbol{\eta}=\left(\eta_1,\ldots,\eta_P\right)^T$ of size P. Then the HCF can be written as $\mathbf{h}\boldsymbol{\eta}$, where $\mathbf{h}$ is a $(2^N-1)\times P$ block matrix [4].

The LCF component is modelled as an L:th-order polynomial

$$l_{m,k} = \theta_{0,m} + \bar{k}\theta_{1,m} + \ldots + \bar{k}^L\theta_{L,m}, \tag{10}$$

for a given input frequency f_m, where m =1,… ,M. Furthermore, $\bar{k} = \left(k - 2^{N-1}\right) \big/ \left(2^{N-1} - 1\right)$ is a normalized ADC output code introduced to ensure numerical robustness. In short, using (10)

$$l_{m,k} = \mathbf{l}_k^T \boldsymbol{\theta}_m, \tag{11}$$

where $\boldsymbol{\theta}_m = \left(\theta_{0,m},\ldots,\theta_{L,m}\right)^T$ and $\mathbf{l}_k = \left(1,\bar{k},\ldots,\bar{k}^L\right)^T$.

The parametric INL data model in (7) can be gathered in a vector $\mathbf{i}_m = \left(i_{m,1},\ldots,i_{m,2^N-1}\right)$. The measured INL can be described as

$$i\left[m,k\right] = i_{m,k} + e_{m.k}, \tag{12}$$

or for in matrix form for all transitions levels for a certain frequency

$$\mathbf{i}\left[m\right] = \mathbf{h}\boldsymbol{\eta} + \mathbf{l}\boldsymbol{\theta}_m + \mathbf{e}_m, \tag{13}$$

where $\mathbf{l} = \left(\mathbf{l}_1,\ldots,\mathbf{l}_{2^N-1}\right)^T$ and $\mathbf{e}_m = \left(e_{m,1},\ldots,e_{m,2^N-1}\right)^T$.

The model (13) is linear in the parameters. Accordingly, the process of estimating the unknown parameters from measured INL data may be obtained by a least-squares fit [5]:

$$\underbrace{\begin{pmatrix} \mathbf{i}[1] \\ \vdots \\ \mathbf{i}[M] \end{pmatrix}}_{\mathbf{i}} = \underbrace{\begin{pmatrix} \mathbf{h} & \mathbf{l} & & \\ \vdots & & \ddots & \\ \mathbf{h} & & & \mathbf{l} \end{pmatrix}}_{[\mathbf{HL}]} \underbrace{\begin{pmatrix} \boldsymbol{\eta} \\ \boldsymbol{\theta}_1 \\ \vdots \\ \boldsymbol{\theta}_M \end{pmatrix}}_{\begin{bmatrix} \boldsymbol{\eta} \\ \boldsymbol{\theta} \end{bmatrix}} + \underbrace{\begin{pmatrix} \mathbf{e}_1 \\ \vdots \\ \mathbf{e}_M \end{pmatrix}}_{\mathbf{e}}, \tag{14}$$

The least square solution is obtain by

$$\begin{bmatrix} \hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\theta}} \end{bmatrix} = \left(\begin{bmatrix} \mathbf{H}^T \\ \mathbf{L}^T \end{bmatrix} \begin{bmatrix} \mathbf{H} & \mathbf{L} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{H}^T \\ \mathbf{L}^T \end{bmatrix} \mathbf{i}. \tag{15}$$

C. Improved accuracy of the parameter estimator

Equation (15) can be written in a more general form that is also applicable for other parametric models obtained by a least square fit

$$\begin{bmatrix} \hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\theta}} \end{bmatrix} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1} \mathbf{G}^T \mathbf{i}, \tag{16}$$

where $\mathbf{G}$ = [**H L**].

In order to improve the performance of the estimator, the model error $e_{k,m}$ will be further investigated. The divergence can be considered due to two parts; one part, $\varepsilon_{k,m}$, that is due to the ‘true’ deviation between the model in (7) and the actual INL, and one part, $v_{k,m}$, that is due to the estimation error in the INL estimate in (5). Accordingly, we write

$$e_{k,m} = \varepsilon_{k,m} + v_{k,m}. \tag{17}$$

For the estimation error $v_{k,m}$ some properties are known; the histogram test is unbiased and the estimation errors are uncorrelated, $v_{k,m} \sim N(0, c_{k,m})$. Moreover, the lower bound of $c_{k,m}$ is given in the CRLB in (6), which also is frequency independent, thus, $c_k \triangleq c_{k,m}$. Now, let

$$\mathbf{c}_m = \mathbf{c} = \begin{pmatrix} c_1 & & \\ & \ddots & \\ & & c_{2^{N-1}} \end{pmatrix} \tag{18}$$

and **C** be the block diagonal matrix constructed from *M* subsequent **c** matrices. Then the Gauss-Markov Theorem (see e.g. [9]) can be applied in order to improve the performance of the estimator. That is, the improved INL model parameters are obtained by

$$\begin{bmatrix} \hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\theta}} \end{bmatrix} = \left(\mathbf{G}^T\mathbf{C}^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{C}^{-1}\mathbf{i}\,. \tag{19}$$

By using (19) instead of (16) the variance of the estimated parameters will be reduced.

III. Conclusions

This paper presents a theoretical method to improve the performance of an estimator for an INL-based parametric model of an ADC utilizing the Gauss-Markov theorem. The method makes use of known uncertainties in the INL estimates. The method is applicable to an arbitrary parametric model based on INL data; even though the paper presents a specific parametric model.

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