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A New Statistical Approach to Word Error Rate Measurement in Analog-to-Digital Converters

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Abstract **-** The paper is focused on the Word Error Rate (WER) estimation in Analog to Digital Converters (ADCs) as specified in Annex A of the IEEE Std. 1241. The aim is to give a contribution for the next version of such standard. A new statistical approach for WER estimation, that can better integrate what it is stated in the standard, is proposed. In particular, Student's t and chi-square distributions have been introduced for a more accurate WER measurement in the case of *n* successive observations. The proposed method has been experimentally verified by means of WER measurement, where the testing results were compared by standard method.

I. Introduction

Testing of ADCs is crucial in the most part of electronic equipment interfacing digital processing units with the analog world. In order to provide a guide to engineers designing ADC test methods and systems, technical standards have been provided in the years from IEEE [1,2]. Along with the test accuracy, the test speed is the main concern in industrial environment, due to the economic relevance of the time to market of a new product. To that aim several research papers can be found proposing test methods to improve the standard ones in order to obtain higher speed with the same accuracy or better accuracy with the same speed [3,4]. The main research contributions focus on dynamic testing in the time and frequency domains looking at fast assessment of Signal to Noise Ratio, Signal to Noise and Distortion, Effective Number of Bits, Total Harmonic Distortion, Spurious Free Dynamic Range, Integral and Differential Nonlinearities [3-6]. All of the previous figures of merit concern the analog circuitry of the ADCs. Less contributions can be found on the digital counterparts and the figures of merit that qualify them, like the Word Error Rate (WER). The WER is defined by the IEEE Std. 1241 as the probability of receiving an erroneous code for an input, after correction is made for gain, offset, and nonlinearity errors, and a specified allowance is made for noise [1]. Typical causes of word errors are metastability and timing jitter of comparators within the ADC. In particular, the Annex A of [1] discusses the presence of statistical errors associated with WER measurement. In this context the source of word errors is assumed to be purely random and both the number of observed word errors and the total number *n* of trial samples are supposed to be statistically significant. Even when users are not interested in knowing an exact WER, but they would be satisfied with an upper limit the standard [1] suggests to acquire at least ten times the number of samples expected to be affected by a word error. With such assumptions the WER tests can be very time expensive due to the significant number of acquisitions needed to use the standard model. It's important to note that only when the samples belong to normal population with known standard deviation a normal distribution may be assumed and the WER values can be obtained using the method suggested in the Annex A.

Taking into account that the WER measurement can be obtained only in very small numbers in a reasonable time, in order to determine the WER and its uncertainty interval it's not possible to use the Gaussian model (as required by IEEE Std. 1241): to this aim other statistical techniques [7], which can better integrate what is stated in [1] and in [8], based on Student's t and chi-square distribution, fixing the number of degrees of freedom and using the t table to determine the confidence level, are proposed in the paper. In the next Section, after a brief recall of the theoretical bases, the proposed approach for the WER estimation is presented. In order to validate the proposal, several experiments have been carried out on a actual devices. The test setup and achieved results are reported and discussed in Section III, before providing the preliminary conclusions.

II. Estimation of the WER for n successive observations

It is well known that a generic normally distributed measure $M=N(m,u)$ with expected value m and standard uncertainty u, can be expressed in a reduced form using the $\frac{M-m}{u}=N(0,1)$ normally distributed, with expected value zero and unitary standard uncertainty. It is also known that in the presence of a number ν of normal random variables given in the reduced form $N_1(0,1),\cdots,N_\nu(0,1)$, mutually independent and independent from M, the sum of squares of such variables, that is $\chi_\nu^2=\sum_{i=1}^{n}N_i^2(0,1)$ is distributed like a chi-square distribution with ν degrees of freedom. The ratio between the original measure expressed in the reduced form $\frac{M-m}{u}=N(0,1)$ and the positive square root of χ_ν^2 divided by degrees of freedom ν, can be expressed by the following variable:

$$T_\nu=\frac{N(0,1)}{\sqrt{\sum_{i=1}^{\nu}N_i^2(0,1)/\nu}} \tag{1}$$

T_ν follows a Student distribution (or t distribution) with ν degrees of freedom.
The confidence level p and the corresponding uncertainty interval $[-t_p,+t_p]$, centred around zero for the Student variable with ν degrees of freedom, is determined as it follows:

$$p=P\{-t_p\le T_\nu\le t_p\}=\int_{-t_p}^{t_p}f_\nu(t)\,dt \tag{2}$$

The Student table of quartiles (see for instance GUM [9] – Table 2.2, p.66) offers different values of t_p for different numbers of degrees of freedom corresponding to the various confidence levels p.
Let consider n independent successive observations $(o_1,\cdots,o_n)$ of the same measurand (trial samples), obtained by the same measurement process implemented in identical conditions of repeatability (***inherent variability of the measurement process***). In the hypothesis - that each observation of the word error is a normally random variable with expected value m_o and standard uncertainty u_o, then, $O_i=N(m_o,u_o)\ \forall i=1,\dots,n$.
The arithmetic mean $\overline{O}=\sum_{i=1}^{n}\frac{o_i}{n}\approx N\left(m_o,\frac{u_0^2}{n}\right)$ is an estimator, that is a random variable with the same expected value m_o and variance reduced by a factor *(1/n)*.
The normal arithmetic mean expressed in its reduced form is: $\frac{\overline{O}-m_o}{\frac{u_o}{\sqrt{n}}}=N(0,1)$. An estimator $S(\overline{O})$ of $\frac{u_o}{\sqrt{n}}$ exists, and it is given by the so-called experimental standard deviation of the mean, according to the following formula:

$$S(\overline{O})=\sqrt{\sum_{i=1}^{n}\frac{(o_i-\overline{O})^2}{[n(n-1)]}}=\frac{u_o}{\sqrt{n}}\sqrt{\frac{\chi^2}{n-1}} \tag{3}$$

being:

$$\chi_{n-1}^2 = \sum_{i=1}^{n} \frac{\left(o_i - \overline{O}\right)^2}{u_o^2} = \sum_{i=1}^{n} N_i^2(0,1) \tag{4}$$

Eq. (4) denotes the well known chi-square with *(n-1)* degrees of freedom, with the $N_i(0,1)$ mutually independent and independent from $\overline{O}$ and, therefore, from $N_o(0,1)$.
Consequently, it can be verified that:

$$T_{n-1} = \frac{\overline{O} - m_o}{S\left(\overline{O}\right)} = \frac{\left(\overline{O} - m_o\right) / \left(u_o / \sqrt{n}\right)}{\sqrt{\sum_{i=1}^{n} \frac{\left(o_i - \overline{O}\right)^2}{u_o^2}}} = \frac{N_o(0,1)}{\sqrt{\chi_{n-1}^2 / (n-1)}} \tag{5}$$

is a Student variable with *(n-1)* degrees of freedom.
Considering the uncertainty interval introduced in (3), assuming $\nu = n-1$ and adopting (5), it is possible to write:

$$p = P\left\{-t_p \le T_{n-1} \le t_p\right\} = P\left\{m_o - t_p S\left(\overline{O}\right) \le \overline{O} \le m_o + t_p S\left(\overline{O}\right)\right\} \tag{6}$$

which represents an estimation of the confidence level shown in Eq. (2).
The arithmetic mean $\overline{O}$ can be interpreted as a final measure, however, the necessary additional information about the uncertainty interval as stated in Eq. (6) is missing, being m_o generally unknown and the estimator $S\left(\overline{O}\right)$ of Eq. (4) a random variable.
An approximate solution can be obtained by introducing (see GUM [9] G.3.1) the Student variable:

$$T_{n-1} = \frac{M - \overline{o}}{s_{\overline{o}}}$$

where the observed word errors $\left[o_1, \cdots, o_n\right]$ are introduced. In this case $\overline{o}$ represents the estimate of m_o while $s_{\overline{o}} = \sqrt{\sum_{i=1}^{n} \left(o_i - \overline{o}\right)^2 / \left[n(n-1)\right]}$ is the estimate of $\frac{u_0}{\sqrt{n}}$, a value of the estimator $S\left(\overline{O}\right)$ obtained by a session of measurement.
The uncertainty interval of the final measure *m* of the word error, although approximated because of the adopted estimate procedures, becomes:

$$p = P\left\{\overline{o} - t_p s_{\overline{o}} \le M \le \overline{o} + t_p s_{\overline{o}}\right\} \tag{7}$$

where t_p is still given by the above recalled Student table.

III. Test setup and results

The experimental validation of the proposed method has been carried out by using the test setup shown in Fig.1. The Tektronix Arbitrary Waveform Generator AWG420 has been used to provide a sinusoidal signal to the Tektronix oscilloscope TDS 7704B. The records acquired by the oscilloscope have been then processed by a PC to compute the WER. Since the WER is small (usually measured in parts per million or parts per billion), a lot of samples must be collected to test for it [1]. Before starting the test, a qualified error level (QEL) has to be chosen. This should be, according to [1], the smallest value that excludes all other sources of error from this test. Particular attention should be paid to excluding the tails of the noise distribution as a source of word errors. Therefore, the WER test has been carried out several times changing the considered QEL to find the value complying with the IEEE Std. 1241 requirements. In particular, several groups of six successive acquisitions have been carried out, each of them made of 10M samples, spaced each other out by 1 minute interval. After each partitioned acquisition, a single record of 60M samples has been acquired. The test sine wave, having an amplitude of $2V_{PP}$ and a frequency of 100kHz, has been acquired with a sampling frequency of 10GS/s. Before starting the record acquisitions a warm up time of 1 hour has been elapsed.

The proposed method has been applied to the WER values obtained by means of the test procedure described above, the results have then been compared with the ones obtained by applying the method described in [1].

Fig.1. WER test setup.

In Table I the experimental WER measurement are shown, considering two QEL, equal to 8 and 9 LSB, respectively.

Table I. Experimental WER measurement.

WER 8 LSB	samples 8 LSB	wrong words 8 LSB	WER 9 LSB	samples 9 LSB	Wrong words 9 LSB
$7{,}7 \cdot 10^{-6}$	10M	77	$2 \cdot 10^{-7}$	10M	2
$7{,}6 \cdot 10^{-6}$	10M	76	$4 \cdot 10^{-7}$	10M	4
$6{,}4 \cdot 10^{-6}$	10M	64	$3 \cdot 10^{-7}$	10M	3
$8{,}6 \cdot 10^{-6}$	10M	86	$2 \cdot 10^{-7}$	10M	2
$7{,}1 \cdot 10^{-6}$	10M	71	$8 \cdot 10^{-7}$	10M	8
$6{,}1 \cdot 10^{-6}$	10M	61	$4 \cdot 10^{-7}$	10M	4

In order to use the Student's t-distribution for a more accurate WER measurement, the population distribution should be hypothesized approximately normal. When sample size is big enough, it isn't necessary to analyze the nature of the population, because the central limit theorem guarantees that the expected value of the sample will be approximately distributed as a Gaussian. But when, as in this case, the sample size is small, less than approximately 30÷35 samples, an assessment of the sample is required. A possible way is to build a box plot of the sample. If this representation does not reveal any significant asymmetries or outliers, it is reasonable to use a Student's t-distribution [10].

Figs. 2 and 3 show the dot plot and the box plot trend of the measured values. The dot plots report in a bidimensional graph the number of times a given WER has been observed (y axis) for each WER value (x axis), as it can clearly be seen by comparing the values in Table I and Fig.2.

The box plot or boxplot (also known as a box-and-whisker diagram or plot) is a convenient way of graphically depicting groups of numerical data through their five-number summaries: the smallest observation (sample minimum), lower quartile (Q1), median (Q2), upper quartile (Q3), and largest observation (sample maximum). A boxplot may also indicate which observations, if any, might be considered outliers.

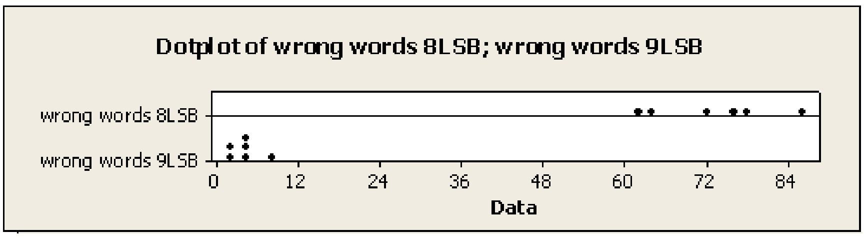

Fig.2. Measurement dot plot trend.

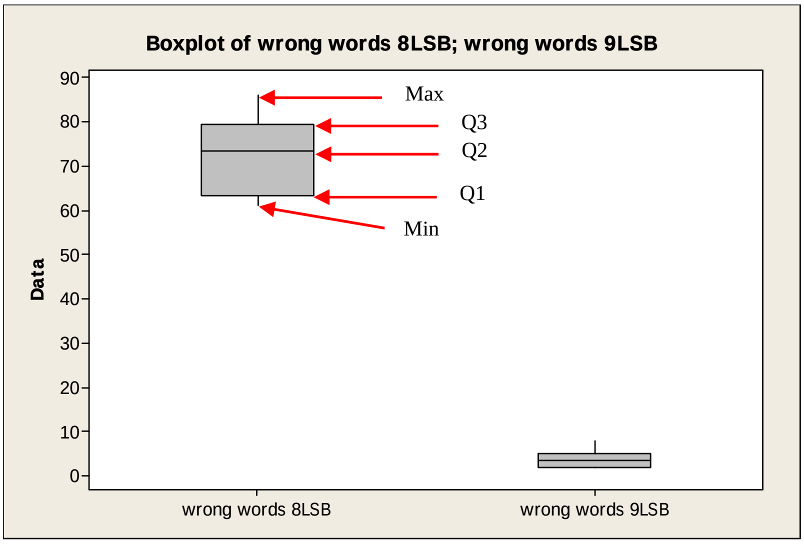

Fig.3. Measurement box plot trend.

Boxplots display differences between populations without making any assumptions of the underlying statistical distribution: they are non-parametric. Box and whisker plots are uniform in their use of the box: the bottom and top of the box are always the 25th and 75th percentile (the lower and upper quartiles, respectively), and the band near the middle of the box is always the 50th percentile (the median). The spacing between the different parts of the box help indicate the degree of dispersion (spread) and skewness in the data, and identify outliers.
The representations in Figs. 2 and 3 of data given in Table I lead to the conclusion that the trends don't show a significant drift from normality, in particular, the plots are not strongly asymmetric and contain no outliers. Therefore, the use of Student's t has been considered appropriate giving the results reported in Table II.

Table II. Student statistical analysis.

Variable	Total Count	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
wrong words 8LSB	6	72,5	9,2	61,0	63,2	73,5	79,2	86,0
wrong words 9LSB	6	3,8	2,2	2,0	2,0	3,5	5,0	8,0

For a confidence level of 95% and 99% with 5 degrees of freedom, the corresponding uncertainty intervals, determined from Eq. (7), have been reported in Table III.

Table III. Uncertainty intervals vs confidence levels.

One-Sample T: wrong words 8LSB (95%)

Variable	N	Mean	StDev	SE Mean	95% CI
wrong words 8LSB	6	72,5000	9,1815	3,7483	(62,8646; 82,1354)

One-Sample T: wrong words 8LSB (99%)

Variable	N	Mean	StDev	SE Mean	99% CI
wrong words 8LSB	6	72,5000	9,1815	3,7483	(57,3862; 87,6138)

One-Sample T: wrong words 9LSB (95%)

Variable	N	Mean	StDev	SE Mean	95% CI
wrong words 9LSB	6	3,83333	2,22860	0,90982	(1,49456; 6,17211)

One-Sample T: wrong words 9LSB (99%)

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Variable          N     Mean    StDev  SE Mean       99% CI
wrong words 9LSB  6  3,83333  2,22860  0,90982  (0,16480; 7,50187)
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The results obtained by applying the proposed method, appropriately rounded, can be compared with the calculation of worst-case error rate as indicated by the Annex A of the IEEE Std. 1241.
Only when the sample of WER observations is extracted from a normal population with known standard deviation, it's possible to use the quatiles of normal distribution and, therefore, the uncertainty intervals can be obtained using the method specified in the Annex A.

Table IV. Comparison between the upper limits of the confidence intervals obtained by applying the proposed approach and the IEEE Std, 2141 one. Only the significant digits have been used.

	Student's t	Annex A
8 LSB (95 %)	82	78
8 LSB (99 %)	88	81
9 LSB (95 %)	6	5
9 LSB (99 %)	8	6

From the results shown in Table IV, instead, it's possible to note that by using the Student's t-distribution the upper limit of the confidence interval is always greater that that obtained by following the method reported in the Annex A, in proportion to the QEL, and this for 8 LSB and 9 LSB cases. In both cases, in fact, using a t distribution with 5 degrees of freedom it is enough to obtain a confidence interval wider than that obtained using normal distribution, as suggested by the standard.

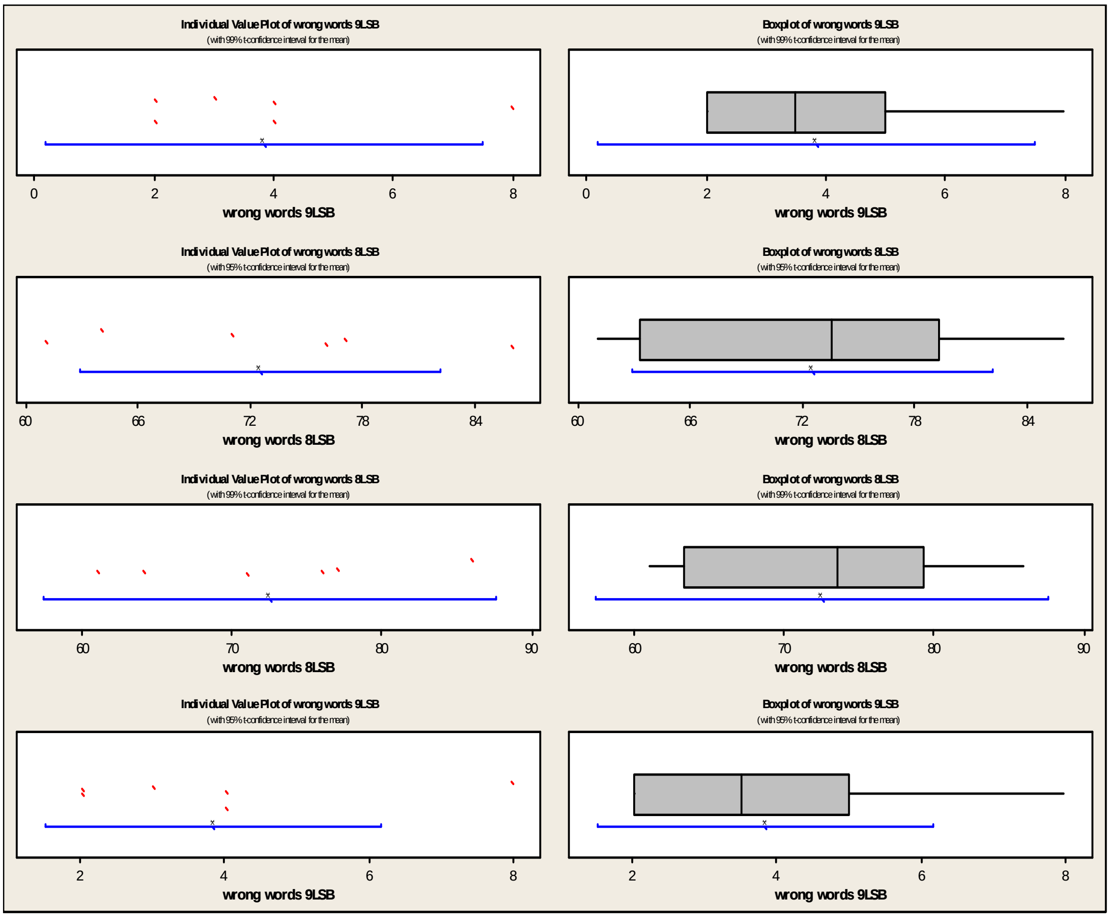

Fig.4. Confidence intervals, dot plot and box plot graphs.

Fig.4 shows the uncertainty intervals referring to the relative dot plot and box plot graphs, considering two QEL, equal to 8 and 9 LSB, respectively. The blue line represents the uncertainty interval, for a confidence level of 95% and 99%, which was calculated previously in Table III.
This figure allows to understand how are distributed the measurement compared to the uncertainty interval. Obviously, the amplitude of the 95% level is lower than the 99% level and this is due to the probability that the number of wrong words belongs to the estimated interval.

IV. Conclusions

The aim of this paper is to introduce a new method to measure the Word Error Rate for improving the Annex A of the IEEE Std. 1241 [1]. This is made according with GUM [9] and its supplements [11] carried out by BIPM. It has been quantified the uncertainty level with relative confidence level in the case of *n* successive observations using Student's t and chi-square distributions, with the final aim to give a contribution for a new draft of such standard.
The proposed method has been applied to the WER values obtained by the measurement setup described above, the results have then been compared with the ones obtained by applying the method described in [1] to validate the new statistical approach.

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