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About uncertainty estimation for ADC spectral figures of merit

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Abstract - The paper deals with the uncertainty evaluation for the most used figures of merit of analog to digital converters in technical standards. Total Harmonic Distortion (THD), Spurious-Free Dynamic Range (SFDR), Signal to-Noise And Distortion ratio (SINAD) and Signal to Noise Ratio (SNR), as defined in IEEE Std.1057, have been considered in order to derive practical uncertainty propagation formulas. The proposed formulas have been validated on simulated and actual signals generated by three different waveform generators and acquired by three waveform digitizers.

I. Introduction

In general, no measurement is perfect and the imperfections give rise to uncertainty of the results. Consequently, the result of a measurement is only an approximation to the value of the measurand and is only complete when accompanied by a statement of such approximation.

This is true also in the case of ADC characterization. There are several reasons involving the need of evaluating the uncertainty of ADC figures of merit. First of all, information about uncertainty allows the comparison of results coming from different laboratories, or within the same laboratory in different times, or with reference values given in specifications or standards. This can often prevent unnecessary repetition of tests.

Moreover, the uncertainty of the ADC test results should be taken into account by a customer in order to correctly interpret the information provided with the data sheets.

Finally, an evaluation (or at least a full consideration) of the components, including random effects from human operators, that contribute to the overall measurement uncertainty provides a means of establishing that the ADC test procedure will produce valid results [1].

Evidence of the importance of uncertainty estimation in ADC testing is the great number of scientific papers dealing with this topic produced in these years [2-9]. The main research contributions in that field focus on specific categories of waveform digitizers, looking at the measurement uncertainty due to their use, or on specific test methods, looking at the uncertainty on the ADC parameter characterization. In particular, the evaluation of measurement uncertainty of ADC-based devices like an oscilloscope, a plug-in data acquisition (DAQ) board [2], with and without using dither [3], as well as A/D modules used in virtual instruments [4] have been studied.

ADC test methods, histogram, sine fitting and frequency domain testing uncertainty issues have been analyzed. For example, (i) the testing parameter selection satisfying desired bounds on the estimation uncertainty [5] and the uncertainty interval determination for the independently based gain and offset error [6] have been studied when applying the ADC histogram test; (ii) the uncertainty of the digital data estimates in presence of additive noise when performing the ADC sine fitting test has been focused in [7,8]; and (iii) a frequency-domain based ADC testing procedure characterized by robustness to model uncertainties, high accuracy, low computational burden, and the ease of use has been proposed, too [9].

Different approaches to analyse the uncertainty propagation such as the Monte Carlo approach, the propagation law of the JCGM 100 [10] and the random-fuzzy variables have been also proposed in the years [11].

From the standardization point of view, with the exception of IEC Std. 62008, that includes a section devoting to the measurement uncertainty estimation, also providing some examples of calculation of modular DAQ system uncertainty no other standard, both at international and category levels, includes measurement uncertainty calculation procedures. This is mainly due to the complexity of the measurement uncertainty estimation as in [10] for the several figures of merit characterizing ADCs. The uncertainty estimation process, in fact, needs to be carried out by experts and it is difficult to generalize. As a consequence, the uncertainty evaluation has been kept out of standards thought to be used by the most part of technicians, requiring clear and repeatable procedures easy to be executed by means of test software, and left to scientists.

Some scientific contributes put in evidence the lack of uncertainty estimation procedures as per [10] in the most used ADC international standards. The authors of [11], for example, among the large number of parameters proposed by the various standards, focus on a minimum set of figures of merit allowing a correct uncertainty evaluation for a generic measurement performed by using an ADC. The paper [12], instead, analyses the uncertainty of the estimated standard deviation of the random noise in ADCs, using the test suggested in the IEEE 1057-94 Standard [12].

However, few scientific papers deal with the estimation of the uncertainty associated to the ADC parameters, especially in the case of the frequency domain ones, such as Total Harmonic Distortion (THD), Spurious-Free Dynamic Range (SFDR), Signal to-Noise And Distortion ratio (SINAD). The different proposed approach to analyse the uncertainty propagation, in fact, mainly rely on the manufacturer specifications, (performing a type B evaluation of the standard uncertainties associated with each error source), as the starting point to assess the uncertainties in the least expensive, the least time consuming and, often, the most accurate way [11].

In the present work, on the contrary, the uncertainty associated to ADC spectral parameters is indirectly estimated starting from their formulas by applying the law of propagation as reported in [10] and showing the validity of the resulting formulas taking as reference the type A uncertainty evaluation method, according to [10]. Purpose of the present paper is, in fact, to propose an easy procedure, suitable to be included in a standard, to compute the uncertainty estimation of the main used ADC spectral figures of merit, in the case of non-coherent and coherent frequency domain testing. In the following, the ADC considered parameters, that are SFDR, THD, SNR and SINAD, are presented together with their formulas as described in the IEEE Std. 1057 [13], since the other ADC standards deal only with the case of coherent sampling. A procedure for ADC parameter uncertainty computation based on the application of the law of propagation of uncertainty, as reported in [10] is then discussed for the case of non-coherent sampling. In particular, the uncertainty approximations of estimators provided by the energy based method, along with bias correction factors according to the procedure proposed in [14] have been used to make easier the computations. The derived formulas have been implemented in a software in order to assess their applicability in a test laboratory with minimal involvement of researchers and validated in a simulation environment first. Then, an experimental validation has been carried on actual ADCs. In the abstract, the first simulation results are analysed. In the full paper, the full simulation and experimental analysis concerning the non-coherent sampling will be presented.

II. ADC spectral figures of merit uncertainty estimation in the non-coherent sampling condition

The figure of merits representing the ADC performance in the frequency domain considered in this work are: *SFDR*, *THD*, *SNR* and *SINAD*. Considering the non-logarithmic scale and adopting a different notation the formulas in the case of non-coherent sampling reported in IEEE Std.1057 can be simplified as:

$$SFDR = \frac{S_1}{S_{\max}}, \tag{1}$$

$$THD = \frac{\sqrt{\sum_{h=2}^{h_{\max}} S_h^2}}{S_1}, \tag{2}$$

$$SNR = \frac{S_1}{\sigma_v}, \tag{3}$$

$$SINAD = \frac{S_1}{\sqrt{\sigma_v^2 + \sum_{h=2}^{h_{\max}} S_h^2}}, \tag{4}$$

where S_h is the root mean square (rms) value of the h-th harmonic component, $S_{\max}$ is the rms value of the maximum spectral component (except dc and fundamental components), and σ_v is the noise standard deviation.

The uncertainty associated to these parameters can be computed by applying the law of propagation of uncertainty described in [10].

In particular, when the result y of a measurement depends on N other measurement results x_i, $1 \le i \le N$, the following relationship can be considered:

$$y = f\left(x_1, x_2, \ldots, x_i, \ldots, x_N\right). \tag{5}$$

If the hypotheses of the central limit theorem are met, that is, if i) the relationship $f(\bullet)$ is linear (at least in a suitable interval about the measurement result, which generally happens when the uncertainty values are reasonably small), ii) the number N of the measurement results x_i tends to infinity (or at least is high enough to approximate this condition), and iii) none of them is prevailing over the others, then the final result y can be supposed to distribute according to a normal probability distribution, whose combined standard uncertainty is given, supposing the measurement results totally uncorrelated, by

$$u_c(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)}, \tag{6}$$

where $u(x_i)$ is the standard uncertainty associated with the measurement result x_i [10].
The uncertainties of the ADC parameters can be then computed using partial derivation of all components:

$$u_{SFDR}^2 = \frac{u_{S_{\max}}^2}{S_{\max}^2} + \frac{S_1^2 u_{S_1}^2}{S_{\max}^4}, \tag{7}$$

$$u_{THD}^2 = \frac{THD^2}{S_1^2} + \frac{\sum_{h=2}^{h_{\max}} S_h^2 u_{S_h}^2}{S_1^4 THD^2} \approx \frac{\sum_{h=2}^{h_{\max}} S_h^2 u_{S_h}^2}{S_1^4 THD^2}, \tag{8}$$

$$u_{SNR}^2 = \frac{u_{S_1}^2}{\sigma_v^2} + \frac{S_1^2 u_{\sigma_v^2}^2}{4\sigma_v^6} \approx \frac{S_1^2 u_{\sigma_v^2}^2}{4\sigma_v^6}, \tag{9}$$

$$u_{SINAD}^2 = \frac{S_1^2}{4\left(\sigma_v^2 + \sum_{h=2}^{h_{\max}} S_h^2\right)^3} \left(\frac{4u_{S_1}^2}{S_1^2} \left(\sigma_v^2 + \sum_{h=2}^{h_{\max}} S_h^2 \right)^2 + u_{\sigma_v^2}^2 + 4\sum_{h=2}^{h_{\max}} S_h^2 u_{S_h}^2 \right) \approx \frac{S_1^2}{4\left(\sigma_v^2 + \sum_{h=2}^{h_{\max}} S_h^2\right)^3} \left(u_{\sigma_v^2}^2 + 4\sum_{h=2}^{h_{\max}} S_h^2 u_{S_h}^2 \right), \tag{10}$$

where the uncertainty of the fundamental harmonic component, u_{S1}, was neglected in approximation formulas when possible. This approximation comes from the practice where the fundamental is dominant and its uncertainty, caused by additive noise, is very small.
The most common algorithm of estimation of spectral components and noise from the frequency spectrum is the energy based method, mentioned in the IEEE Std.1057 [13]. This method evaluates the tone power by computing the energy falling inside a frequency band approximately covering the window spectrum mainlobe. This requires only overall window spectrum specifications and it is characterized by a moderate computational burden [14]. According to this method the rms values are computed from the power spectrum and window parameters:

$$S_j = \sqrt{\frac{2}{NNPG} \sum_{i \in K_j}^{N/2} M^2(i)}, \tag{11}$$

$$\sigma_v = \sqrt{\frac{2}{NNPG} \sum_{i=L+1, i \notin K_j}^{N/2} M^2(i)}, \tag{12}$$

where K_j are the frequency bins within the main lobe of the component S_j and j are the significant harmonic and spurious components. The dc components are generally not taken into account. Since dc component is always coherent, only the zeroth and following L frequency lines are removed, where L is the window order.
These two mentioned estimates are biased. The reason of biased signal estimate is that the power spectrum $M^2(i)$ is given not only by signal power but also by noise power. An unbiased estimate can be get by the subtraction of the contribution of noise power:

$$S_{j,corr} = \sqrt{S_j^2 - 2\frac{N_j}{N}\sigma_v^2}, \tag{13}$$

where N_j is the number of frequency bins in K_j (usually $N_j = 2L+3$) and white noise is assumed. The bias of noise standard deviation is caused by excluding strong spectral components from noise computation. If the noise is white, noise power in signal frequency bins can be determined by scaling the computed noise power:

$$\sigma_{v,corr} = \sqrt{\frac{1}{NNPG} \frac{N}{N_v} \left(\sum_{i=L+1, i \notin K_j}^{N/2-1} M^2(i) + \frac{1}{2} M^2\left(\frac{N}{2}\right) \right)}, \tag{14}$$

where $N_v = N/2 - j_{max}N_j - L$ is the number of samples from which the noise estimate is computed, j_{max} is the number of spectral components excluded from noise computation and $NNPG$ is the normalized noise power gain of the window. The effective number of bins to compute the noise standard deviation, N_v, is obtained by subtracting the bins used for harmonic and spurious components ($j_{max}N_j$) and those corresponding to the dc offset (L). The frequency bin $N/2$ incorporates the sum of powers of both halves of frequency spectrum at this frequency.
The uncertainty of the unbiased signal estimate is given using partial derivations of (11) as:

$$u^2_{S_{j,corr}} = \left(\frac{S_j}{S_{j,corr}} u_{S_j} \right)^2 + \left(\frac{2N_j}{NS_{j,corr}} u_{\sigma_v^2} \right)^2, \tag{15}$$

where u_{S_j} can be computed from

$$u_S^2 \approx u_v^2 \frac{ENBW_0}{N}, \tag{16}$$

where u_v is noise standard deviation and $ENBW_0$ is the equivalent-noise bandwidth of the squared window. Neglecting very small contribution of the second term of (15) this uncertainty can be approximated by:

$$u^2_{S_{j,corr}} \approx \sigma_v^2 \frac{ENBW_0}{N} \frac{S_j^2}{S^2_{j,corr}}. \tag{17}$$

To be exact, the corrected noise power estimate, $\sigma^2_{v,corr}$, and its uncertainty, $u_{\sigma^2_{v,corr}}$, should be used in (13), (15) and (17). The approximation of the uncertainty of unbiased noise estimate, $u_{\sigma^2_{v,corr}}$, was derived in [14] as:

$$u^2_{\sigma^2_{v,corr}} \approx \sigma_v^4 \frac{ENBW_0}{N_v}. \tag{18}$$

As it can be seen, the uncertainty of unbiased noise clearly depends on the chosen window.

IV. Experimental results

The proposed approach to bias correction and uncertainty estimation was verified by simulations as well as real data records. The simulations were performed on a simple model consisting of a pure sine wave of a given sampling frequency of 500 kHz, signal frequency of 61.17 kHz, random initial phase and three harmonic components with amplitudes of 40,000 (fundamental), 10 (2nd harmonic component) and 100 (3rd harmonic component). 2048 samples and 1000 repetition cycles were chosen for both simulation and real data computations. Since incoherent sampling was applied, time windows had to be applied. The Blackman-Harris 4-term window was used for the simulations and the Blackman-Harris 7-term window was used for real data evaluation. The reason of usage of different windows for simulations and real data was to show the effect of limited side lobe level in the first case and the correct parameter estimation in the other case. Monte-Carlo simulations were performed by varying the following parameters; SNR, third harmonic distortion, number of samples, and incoherency. In this abstract only the results versus SNR are shown because of limited space. However, the other parameters are in agreement with the expected values and they will be reported in the final paper.
The achieved results are displayed in Figs. 1–2 by plotting: the preset simulation parameters (*actual* plots), the mean value or standard deviation of parameters (*experimental* plots) and the mean parameters estimated from each single data record (*estimated* plot) by applying Eqs. (7)–(10). Fig. 1 shows the preset and measured parameters while Fig. 2 reports the relative uncertainties. Each figure is divided in four subfigures, one for each parameter. The uncertainties have been reported relatively to the parameter value in dB units; expressed in percents they were in the range of 10% and 0.1%. As can be seen by comparing red and blue plots in Figs. 1 and 2, the simulation results are in a very good agreement with the theoretical values. For example, in Fig. 1 it can be seen that the maximum experimental *SFDR* is different from the actual one by less than 1% while the uncertainty estimation of the *SDFR* in Fig. 2 differs from the experimental one by less than 5% (corresponding to –26 dB). The same differences were found between the estimated and experimental uncertainties of all other figures of merit. Moreover, from Fig. 2 it is possible to verify that the worst relative uncertainty on all figures of merit is about 5%.
The *THD* and *SFDR* showed noticeable bias at low *SNR* values (see Fig. 1) due to inaccurate correction of

weaker harmonic components for noise power. At higher *SNR*s the effect of the chosen window can be clearly seen. In fact, the peak side lobe at –92dB caused a bias of the *SNR* and wrong estimate of the uncertainty of all the parameters (see Figs. 1–2).

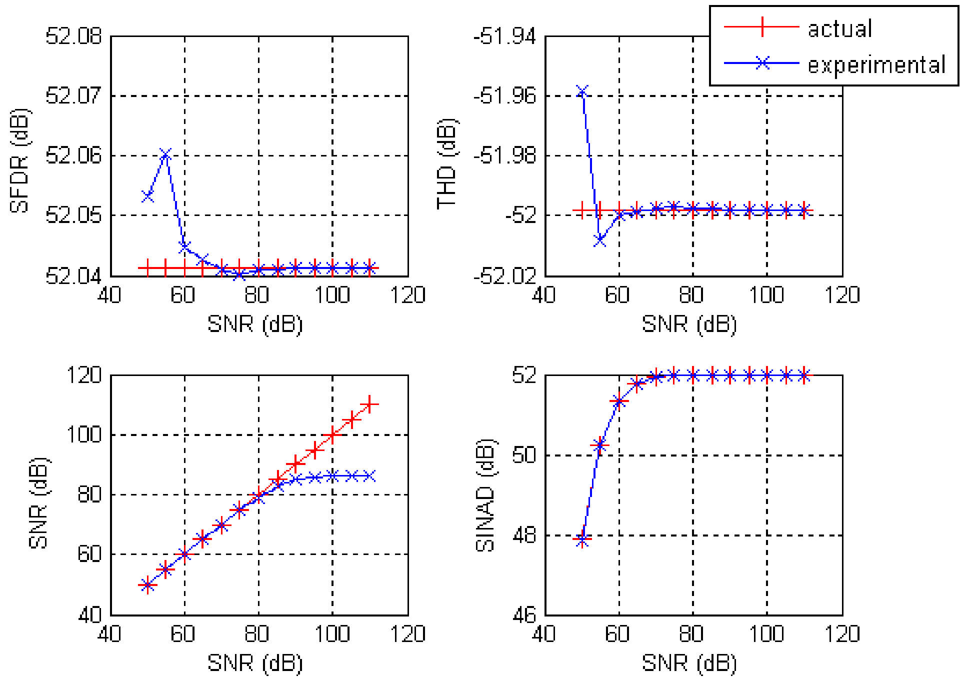

Fig. 1. Preset and mean values of computed parameters expected (actual) and observed (experimental) on the simulated signals.

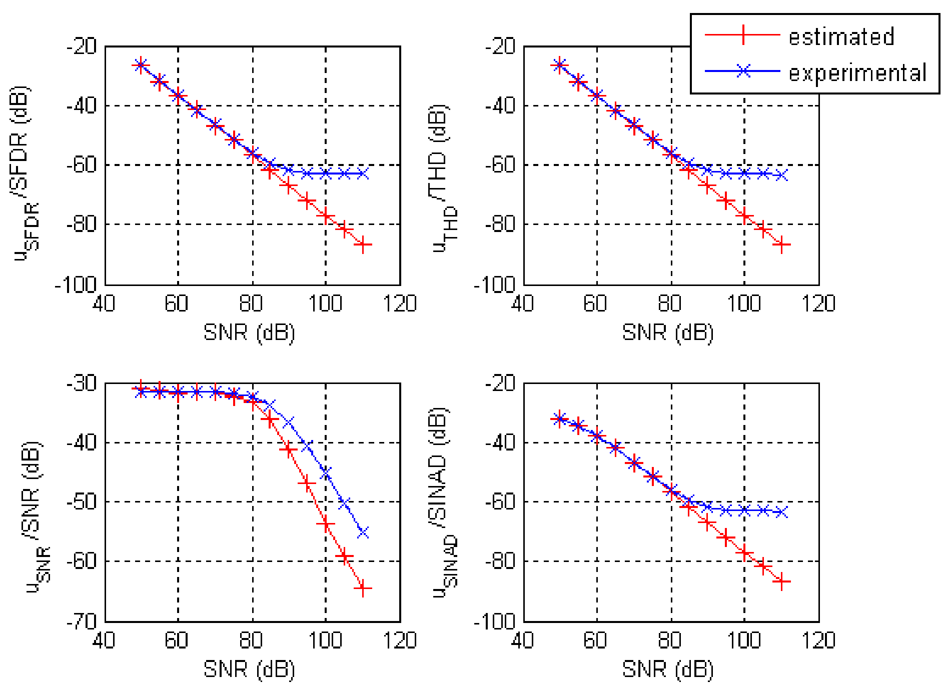

Fig.2. Relative uncertainties ($u_{SFDR}/SFDR$, u_{THD}/THD, u_{SNR}/SNR, $u_{SINAD}/SINAD$) estimated and experimentally

observed on the simulated signals.

The experimental validation on real data was performed on 9 long data records divided into 1000 data segments 50% overlapped, each consisting of 2048 samples. All data records were obtained by sampling a sine wave at the output of a band-pass filter. Two Agilent arbitrary waveform generators, model 33120A and 33220A, and a Stanford Research DS360 function generator were used for providing the test signals, whose frequencies ranged from 20.19 kHz to 19.507 MHz. Three data acquisition systems were used to sample the filtered sine waves: (i) a National Instruments PXI-5122 digitizer, (ii) a NI PXI-5922 digitizer, and an HP VXI-E1430A. The sampling frequencies ranged from 625 kSa/s for the lowest frequency sine waves to 100 MSa/s for the highest frequency ones. Data records 1 and 2 were acquired by NI PXI-5122 digitizer, 3, 4 and 5 were acquired by NI PXI-5922 digitizer and 6, 7, 8 and 9 were acquired by HP VXI-E1430A digitizer.
As can be seen from Fig. 3, the experimental uncertainties are very close to the ones estimated using Eqs. (7)–(10). Even if the values corresponding to data records 2 and 6 are higher than 10% in this experiment the estimated and experimental uncertainties are in a good agreement as well. The deviation of uncertainties of the *SNR* and *SINAD* in case of data sampled by NI PXI-5922 digitizer is caused by not fulfilling the assumption of white noise. This digitizer has so low inner noise that phase noise of the input signal becomes significant.
More details about the experimental evaluation and a complete discussion will be provided in the final version of the paper.

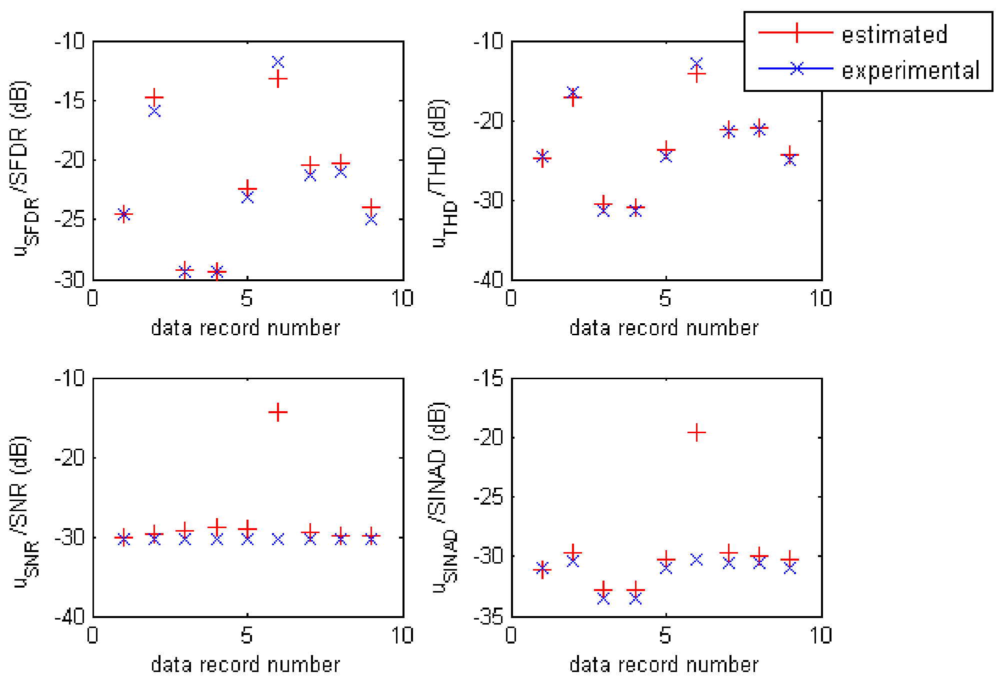

Fig.3. Relative uncertainties estimated and observed on the signals coming from actual data acquisition devices.

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