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# Improvement of spectral properties of quantization noise by multiresolution quantization

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***Abstract*** **-** In the area of electromagnetic interference (EMI) time-domain EMI measurement systems significantly reduce measurement time. As EMI measurements require high dynamic range or signal-to-noise ratio and wide frequency range, a multiresolution structure of available fast sampling analog-to digital converters (ADC) is used inside such systems. The combination of several ADCs allows to increase overall dynamic range of measuring device. In the paper spectral properties of the quantization noise of system with two ADCs are analyzed by simulations as well as experimentally for harmonic input signal. Multiresolution measurements were realized by two channels of data acquisition card and PC postprocessing. The noise suppression achieved by experiments is presented.

## I. Introduction

Traditional EMI measuring devices offer high dynamic range for precise electromagnetic disturbance spectrum measurements. But inside them the measured interference signal is at first filtered by intermediate-frequency filter (resolution bandwidth) and then fed to analog-to-digital converter (ADC) input. So the bandwidth processed at the time is limited. Therefore simultaneous view of full bandwidth signal in time and frequency domain is not possible.

In measurement laboratories data acquisition cards (DAQ) or oscilloscopes could be used for full bandwidth measurement up to frequencies guaranteed by a device manufacturer. And the measurement could be accomplished both in time domain and in frequency domain after Discrete Fourier transform (DFT) application. The large real-time frequency bandwidth is achieved by using fast ADC. But its low resolution provides only low dynamic range.

For higher dynamic range within full bandwidth real-time spectral measurements a special multiresolution time-domain EMI measurement system has been designed. It was presented e.g. in [1]. Like depicted in the Figure 1, the input signal is split by a power splitter (PWS) into three channels and sampled and converted in all channels simultaneously by fast ADC. Then the resulting sample value is created by extracting the output from the ADC, in which the signal shows the maximal nonclipped value. As spectrum is of interest DFT is finally calculated. Similar idea could be used for measurements with DAQ or oscilloscope if more than one channel is available and some postprocessing is added. Improvements of spectral properties of quantization noise by multiresolution quantization (MQ) are presented in the paper for simple sinusoidal input signal. Two channels multiresolution system are considered, the second channel with range equal to half of the first one is supposed to suppress the noise level of the first channel. This configuration is possible for many acquisition devices and our experiments were realized with DAQ.

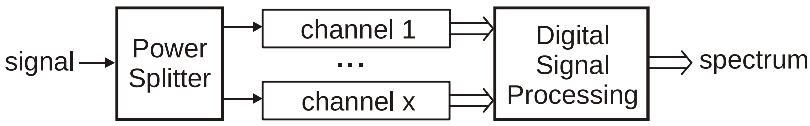


Figure 1. Block structure of multiresolution system

## II. Theory of simple quantization

Only very basic analyses of quantization error could be found in the literature for MQ. Therefore for this step of our investigation our goal was to select simple circumstances for which some existing mathematical basis is available in published literature. Consider simple harmonic signal with amplitude $U$ and frequency $f_u$ on the input of a measurement system

$$u(t) = U \sin(2\pi f_u t) \tag{1}$$

We will discus about mathematical model of quantization error for simple quantization (SQ), which could be used later as a reference for interpretation of simulation and experimental results obtained for MQ. For evaluation of overall quantization error root mean square error (RMSE) is suitable. If amplitude $U$ is large compared to quantization step $q$ mean squared error (MSE) approaches good known value of $q^2/12$. But if the ratio $U/q$ is low, better theoretical estimation should be used like in [2]. For assumed zero offset deterministic quantizer input (Eq. 1) and mid-tread quantizer the MSE is

$$MSE = \frac{q^2}{12} + \frac{q^2}{\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} J_0\left(\frac{k 2\pi U}{q}\right) \tag{2}$$

where $J_0$ (.) is the ordinary Bessel function of order zero.

If spectrum of quantization error is of interest, several approaches are suggested in [3]. For a single ADC quite simple approximation was achieved by [4] based on modulation principle. From that power spectral density of quantization noise could be theoretically approximated by

$$S_e(f) = \frac{q^3}{2\pi^3} \sum_{k=1}^{\infty} \frac{1}{k^2} \frac{1}{kU 2\pi f_u \sqrt{1 - \left(\frac{qf}{kU 2\pi f_u}\right)^2}} \quad \text{for } |f| < 2\pi f_u \frac{U}{q} k \tag{3}$$

Amplitude spectrum of quantization error $|\mathbf{A}_e|$ could be estimated from $S_e$. As could be seen from the equation it depends on ratio of signal amplitude to quantization step $U/q$. Every spectral component of quantization noise decreases with decreasing $q$, similarly with total RMSE. But according to Eq. 3 the spectral distribution of quantization error is not flat. Peaks occur at frequencies (for integer $k$)

$$f_{p,k} = k.2\pi \frac{U}{q} f_u \tag{4}$$

Exactly this effect could be observed in the Figure 2a where amplitude spectrum of quantization error $|\mathbf{A}_e|$ obtained from simulation of SQ is depicted together with results of model (3). One might suspect that after addition of a second ADC by MQ system the first peak near to $f_{p,1}$ can still significantly deform measured spectrum for signal amplitude just above range of this second ADC.

## III. Simulation of MQ

In our analyses of MQ we consider a two-channel system equipped with 8-bit bipolar ADCs. The range of the first channel $\pm U_{ref1}$ is above amplitude of input signal $U$ and it determines entire range of our measurement system. The range of the second channel $\pm U_{ref2}$ is half of the first one $U_{ref2} = U_{ref1}/2$.

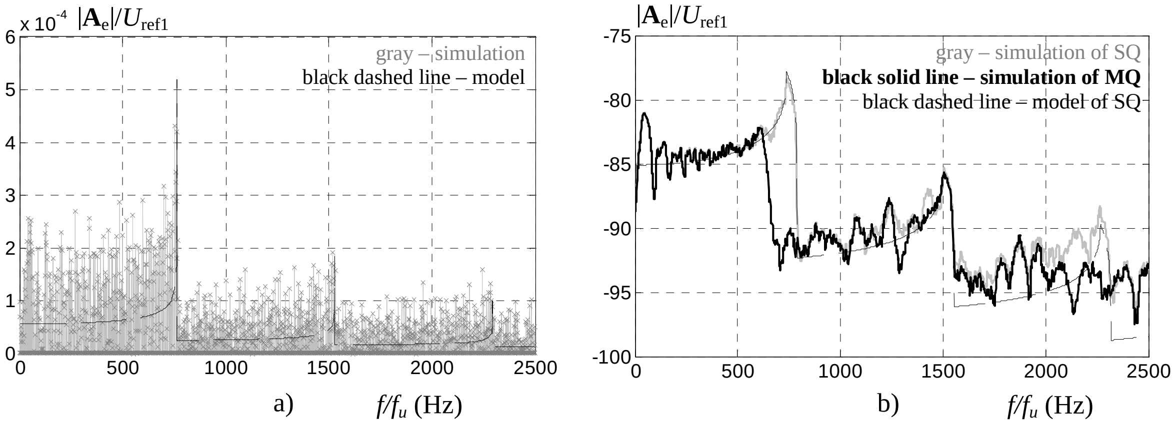


Figure 2. Simulated and modelled spectrum of quant. error: a) for SQ; b) smoothed spectrum for SQ and MQ

For signal of frequency $f_u$ oversampling ratio 2500 is used (against Nyquist frequency) resulting in sampling frequency of $f_s$ =5000$f_u$. For one spectrum estimation $N$=10000 samples has been acquired corresponding to two periods of the input signal. Spectrum of simulated quantization error is depicted in the Figure 2a and compared with mathematical model for signal amplitude $U$=0.95$U_{ref1}$. The simulated spectrum has quite complicated character and roughly follows the shape of mathematical approximation. Better correspondence could be observed from curves obtained after smoothing of the spectrum by calculating floating root mean squared value from interval of 50$f_u$. Smoothed spectrum is depicted in dB scale in the Figure 2b. Here results of MQ simulation are added and visibly the most critical peak at frequency $f_{p,1}$ = 764$f_u$ has been cut off by MQ. For lower frequencies there is no contribution of MQ as the quantization step near the extremes of sinusoid determines the low-frequency content [5]. But one can make an assumption that the peaks correspond to quantization near the middle of sinusoid where the waveform changes fast. This could explain why the second peak $f_{p,2}$ = 1528$f_u$ is not suppressed. This would be the first peak of SQ if fine step of second ADC were used for the whole range. However the next peak $f_{p,3}$ = 2292$f_u$ is truncated again.

## IV. Experiments

Experiments similar to simulations were realized using data acquisition card. Two channels of DAQ were employed for multiresolution measurement. Signal processing according to multiresolution principle and DFT evaluation was done by software in PC. Resulting whole measured spectrum |**A**| is depicted in the Figure 3a and compared with experimental data of SQ. The desired spectral component merges with vertical axis of the graph. However spurious peaks close to $f_{p,1}$, $f_{p,2}$ could be ease distinguished in results from SQ. And it could be seen that the spectrum of quantization noise near to $f_{p,1}$ = 764$f_u$ was significantly suppressed by MQ. The both spectra were smoothed and they are shown again in the Figure 3b. Under such conditions little error improvement could be observed also near to frequency $f_{p,3}$ = 2292$f_u$, which corresponds with the third theoretical peak. On the other hand the second peak $f_{p,2}$ = 1528$f_u$ retains also for MQ as it is natural component of quantization error spectrum for fine quantization step of the second channel.

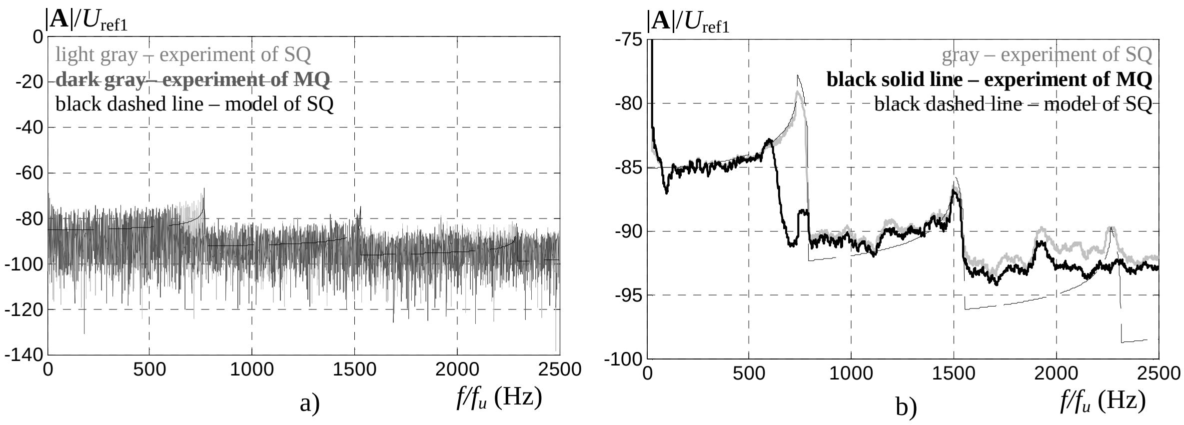


Figure 3. Comparison of spectrum obtained by SQ and MQ for harmonic input with amplitude $U$=0.95$U_{ref1}$: a) spectrum measured by experimental system; b) smoothed experimental spectrum

For better demonstration of positive contribution of multiresolution quantization we did other experiment with lower signal amplitude. If input signal of system with SQ just exceeds some range of any measurement system, the user has to switch to first available higher range e.g. with two times higher quantization step. In MQ system second channel does fine quantization and may be that only for several samples above the range of this channel the rude quantization of the first channel is used. If we set $U$=0.6$U_{ref1}$, it is close to reference of second ADC $U_{ref2}$=0.5$U_{ref1}$. Therefore from experiments under such circumstances better quantization error improvement could be expected. From the Figure 4 it could be seen that for this lowered signal amplitude the area of error spectrum improvement is wider compared to previous results close to the first and third spectral peak of SQ (for this case $f_{p,1}$ = 483$f_u$ and $f_{p,3}$ = 1448$f_u$). Again even peaks near to $f_{p,2}$ = 965$f_u$ and $f_{p,4}$ = 1930$f_u$ have retained also after addition of second channel used for MQ.

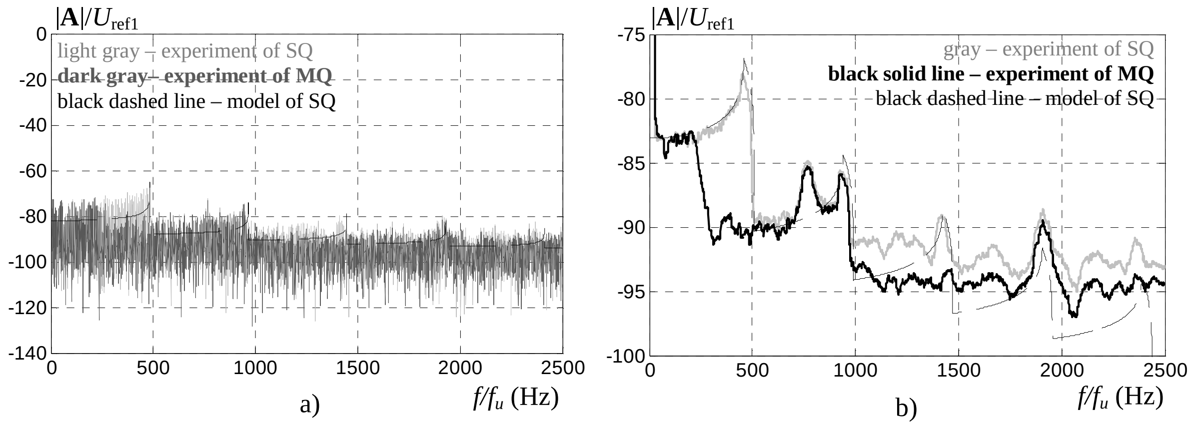


Figure 4. Comparison of spectrum obtained by SQ and MQ for harmonic input with amplitude $U$=0.6$U_{\mathrm{ref1}}$: a) spectrum measured by experimental system; b) smoothed experimental spectrum

## V. Conclusion

Simulation of multiresolution quantization (MQ) using two 8-bit analog-to-digital converters (ADC) and applied on harmonic signal have shown positive contribution of addition of the second ADC for wide bandwidth spectral measurements. The error improvement was demonstrated experimentally for two channels of data acquisition card and PC postprocessing.

For 8-bit bipolar ADC with referent voltage $U_{\mathrm{ref1}}$ theoretical root mean squared error (RMSE) caused by SQ is 0.00230$U_{\mathrm{ref1}}$ for signal amplitude $U$ = 0.95$U_{\mathrm{ref1}}$. If second ADC with reference $U_{\mathrm{ref2}}$ = $U_{\mathrm{ref1}}$/2 has been added RMSE has dropped slightly to 0.00193$U_{\mathrm{ref1}}$ for such MQ system. For lower signal amplitude $U$ = 0.6$U_{\mathrm{ref1}}$ better accuracy could be achieved with RMSE improvement from 0.00227$U_{\mathrm{ref1}}$ to 0.00167$U_{\mathrm{ref1}}$. But real contribution of MQ could be observed after evaluation of measured spectrum. Here the higher spurious component has been significantly reduced. Good approach for examination of quantization noise properties is local view on smoothed spectrum which represents dependency of rescaled RMSE on frequency. In mentioned most critical point around 20 dB improvement of error component could be observed. In addition it has been shown that also other spurious components corresponding to theoretical spectral peaks of quantization noise could be corrected. For our case only odd peaks from SQ have been suppressed by MQ because of chosen ratio $U_{\mathrm{ref1}}/U_{\mathrm{ref2}}$=2. This results from the rules of quantization error behaviour as every uniform quantizer even with fine quantization step (step of our second ADC) shows its spectral peaks.

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## References

[1] S. Braun, P. Russer, "A Low-Noise Multiresolution High-Dynamic Ultra-Broad-Band Time-Domain EMI Measurement System," *IEEE Transactions on Microwave Theory and Techniques*, vol. 53, no. 11, pp. 3354-3363, 2005.

[2] K. Hejn, A. Pacut, "Generalized Model of the Quantization Error - A Unified Approach," *IEEE Transactions on Instrumentation and Measurement*,, vol. 45, no. 1, pp. 41-44, 1996.

[3] B. Widrow, I. Kollár, *Quantization Noise: Roundoff Error in Digital Computation, Signal Processing, Control, and Communications,* Cambridge University Press, Cambridge, 2008.

[4] T. A. C. M. Claasen, A. Jongepier, "Model for the Power Spectral Density of Quantization Noise," *IEEE Transactions on Acoustics, Speech and Signal Processing,* vol. ASSP-29, no. 4, pp. 914-917, 1981.

[5] N. M. Blachman: "The Intermodulation and Distortion due to Quantization of Sinusoids," *IEEE Transactions on Acoustics, Speech and Signal Processing,* vol. ASSP-33, no. 6, pp. 1417-1426, 2005.