

Efficient Estimation of the Word Error Rate of Digitizing Waveform Recorders

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Abstract- The paper proposes a new method for Word Error Rate (WER) measurement for digitizing waveform recorders. In particular, a WER estimation procedure, based on Student's t distribution, is provided to estimate the WER and its confidence interval, minimizing the acquisition record length. The proposed test procedure has been experimentally verified by means of WER measurements on actual digitizing waveform recorders, where the test results have been compared to those achieved by implementing the method described in the IEEE Std.1057. Two measurement set-ups have been adopted in order to prove the validity of the proposed approach.

I. Introduction

There has been a rapid increase in the speed and accuracy of data conversion systems, whose characteristics have a determinant impact on the performance of digital instruments as well as on the quality of the measurement systems. Consequently the required test equipment is becoming increasingly expensive. In fact, characterization and testing activities are a major factor of cost in integrated circuit (IC) manufacturing, circuits may cover nearly 50% of the whole production budget. Apart from being a design challenge, testing of high-performance data converters has become a trouble for the engineers. For this reason, in order to provide a guide to engineers for designing test methods and systems for Analog-to-Digital Converters (ADCs) and Digitizing Waveform Recorders (DWRs), two technical standards have been developed in the years by IEEE [1,2].

The development of new faster, cheaper and/or more accurate DWR testing procedures, able to characterize high frequency converters is nowadays one of the main tasks in ADC and the related DWR research activity. The performance of the same converter under different conditions tends to vary, in some cases very strongly. Consequently, the test conditions should be similar to the expected operating ones.

The complete characterisation of data conversion systems requires a high number of parameters. Due to the significant contribution of the sampling and quantization subsystems, the main research contributions focus on dynamic testing in the time and frequency domains looking at the analog section of the converter [3]. Other contributions can be found focusing on the digital counterpart and the figures of merit that qualify it, like the Word Error Rate (WER). As a matter of fact, this parameter is of particular interest to several applications requiring high sampling rates and long acquisition records such as in telecommunications and time-domain signal analysis.

From the manufacturers' point of view, a comprehensive document which could be used as a guide to DWR testing, and meet the requirements of different applications, is required [4]. In particular, the reference [5] puts in evidence that the Word or Bit Error Rate is not usually stated on most DWR specifications while excessive digitizer Bit Error Rates (BERs) in communications applications can degrade the overall system BER. As it can be seen from [5], the WER measurement requires extensive characterization of the devices under a variety of input conditions and error limits, resulting in a complex and long procedure. Due to the typical WER values, given as parts per million, or parts per billion, a typical WER test session requires several days, thus making such assessment difficult to do within an industrial mass production scenario. For what it concerns the WER measurement in an industry scenario, the application note [6] shows a WER measurement methodology and characterization results for a particular family of converters. A statistical approach is also discussed but it is not for a general purpose use and an evaluation of the uncertainty correlated with WER measurement is missing.

By taking in account the points of view of the different stakeholders the IEEE Instrumentation and Measurement Society TC-10 "Waveform generation, measurement and analysis" standardized the WER measurement for

ADCs in the IEEE Std. 1241 [1], and the WER measurement for DWRs in the IEEE Std. 1057 [2]. In particular, the WER values can be obtained according to the Annex A of [1], and the Annex C of [2]. Both the standards specify very similar test methods and conditions.

According to the IEEE methods the WER estimation requires a relevant number of acquisitions. The WER measurements can be obtained only in very small numbers in a reasonable time, thus making the IEEE proposed model, the Gaussian one, useless for estimating an uncertainty interval.

In order to harmonize the [1] with [7] an improvement of the Annex A and a suitable method for estimating the WER uncertainty from a little number of measurements have been proposed in [8].

With the aim of harmonize the [2] with [7], this paper presents a WER estimation procedure, based on the model proposed in [8], to obtain the figure of merit and its variability for different test durations and acquisition lengths, and its experimental validation on different measurement stations. The main target of the research work is to provide a guide to choose a trade off between the test duration and the WER estimation accuracy in designing manufacturer test sessions. The digital oscilloscope has been used as main DWR platform for the experiments due to the importance that the WER has in their characterization. The next Sections recall some theoretical bases, present the improved uncertainty estimation method, and describe the experiments. Finally, the obtained results are presented and discussed in comparison with those provided by means of the IEEE method.

II. Theoretical bases

It is known that if X_1, X_2, \dots, X_n is a sample of a population with both mean, μ , and standard deviation, σ , parameters unknown, the confidence interval around μ , with a confidence level $1-\alpha$, cannot be based on the fact that the statistic $\sqrt{n}(\bar{X} - \mu)/\sigma$ is normal, but it is necessary to suppose that $\sqrt{n}(\bar{X} - \mu)/S$ is distributed as a Student's t random variable with $(n-1)$ degrees of freedom.

Considering the WER as a generic normally distributed measure M with expected value m_o and standard uncertainty u_o , according to [7], it is possible to define the uncertainty interval based on the use of the Student's t distribution of the WER measurements as follows [8]:

$$p = P\{\bar{o} - t_p s_o^- \leq M \leq \bar{o} + t_p s_o^-\} \quad (1)$$

where, t_p is given by the Student's t table, \bar{o} represents the estimate of m_o and $s_o^- = \sqrt{\sum_{i=1}^n (o_i - \bar{o})^2 / [n(n-1)]}$ is

the estimate of $\frac{u_o}{\sqrt{n}}$ obtained by n measurements (o_1, \dots, o_n) .

By applying the proposed model, it is possible to estimate the WER variability with a reduced number of measurements, this allows to implement the technique in production process, too.

It is possible to see that at a given level the width of the confidence interval, is not necessarily bigger when the variance is unknown. Its width is, as a matter of fact, equivalent to $2z_\alpha \sigma / \sqrt{n}$ when σ is known, where z_α is the quantile of normal distribution, and σ is its standard deviation. This measure is equivalent to $2t_{\alpha, n-1} S / \sqrt{n}$ when σ is unknown, where S indicates the sample standard deviation and $t_{\alpha, n-1}$ is the α -quantile of Student's t distribution with $n-1$ degrees of freedom. Comparing these two measures it is certainly possible that the sample standard deviation S turns out to be much smaller than σ . Furthermore, it is also possible to demonstrate that the average width of the confidence interval is bigger when the variance is unknown. That is, it can be rigorously demonstrated that [9]:

$$t_{\alpha, n-1} E[S] \geq z_\alpha \sigma \quad (2)$$

where, $E[S]$ represents the mean value of the sample standard deviation. In other words, considering Eq. (2), it is possible to see how the amplitude of the confidence interval is always bigger when it is evaluated by means of the Student's t distribution.

III. Improved method

The improved method relies on the previous approach and on the control charts theory [10]. The main aim of the improved method is to estimate the WER mean value \bar{X} and its confidence interval, for a given confidence level, without long historical records and without knowing μ and σ . Therefore, it is necessary to acquire a certain number k of subrecords in order to carry out this estimate. The higher is k , the higher is the accuracy that can be achieved. For example, usually $k \geq 20$ such as $n = k \cdot l \geq 100$ and l is a small integer. As it is well known, the estimator of μ is the arithmetic mean of the subsamples mean:

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_k}{k}, \quad (3)$$

while the mean of the sample standard deviation \bar{S} is

$$\bar{S} = \frac{S_1 + S_2 + \dots + S_k}{k}. \quad (4)$$

It is well known that \bar{S} is not a correct estimator of σ . However, it is possible [9] to calculate the exact value of $E[\bar{S}]$ in order to find the correction factor $c(n)$ that will enable \bar{S} to be transformed into a correct estimator of σ . Therefore, defining:

$$c(n) = \frac{\Gamma\left(\frac{n}{2}\right)\sqrt{2}}{\Gamma\left(\frac{n-1}{2}\right)\sqrt{n-1}}, \quad (5)$$

where, $\Gamma(n) = (n-1)!$, it turns out that $E[\bar{S}] = \sigma c(n)$ [9]. The Table I shows the different values of $c(n)$ for n ranging from 1 to 10.

Table I. Multiplicative factor $c(n)$ values.

n	$c(n)$
2	0.797885
3	0.886227
4	0.921318
5	0.939986
6	0.951533
7	0.959369
8	0.965031
9	0.969311
10	0.972659

It is necessary to underline that previous estimates of μ and σ from formulas (3) and (4) rely on k subrecords of l measures. Therefore, the proposed estimates are reasonable only if the process that causing the WER has remained stable during the acquisition time. In order to verify this important assumption, it is possible to calculate the values of UCL (Upper Control Limit) and LCL (Lower Control Limit) based upon the estimates, which means:

$$LCL = \bar{\bar{X}} - \frac{3\bar{S}}{c(n)\sqrt{n}} \quad \text{and} \quad UCL = \bar{\bar{X}} + \frac{3\bar{S}}{c(n)\sqrt{n}} \quad (6)$$

and subsequently to verify that the means of all the k subrecords fall within the interval [LCL, UCL]. If this isn't the case, it is necessary to reject the anomalous values (by supposing that the process has been temporary out of control) and recalculate the estimates (3) and (4) and the limits (6), by verifying once again that the inclusion condition is satisfied. The procedure should be repeated once more if necessary.

It is obvious that if the means which fall outside the check limits are too many, it would be necessary concluded that the process is out of control and the WER measurement results are not reliable at all. Consider, for example, the design of a test procedure where the sample length has to be chosen between 5 and 9. Looking at Table 1:

$$E[\bar{S}] \approx \begin{cases} 0.94 \sigma & n = 5 \\ 0.97 \sigma & n = 9 \end{cases} \quad (6)$$

Looking at the Normal and Student's t tables it can be found that $z_{0.025} \approx 1.96$, $t_{0.025, \nu=4} \approx 2.78$ and $t_{0.025, \nu=8} \approx 2.31$. From Eq. (2) the width of a confidence interval at 95% for a sample length 5 is $2 \cdot z_{\alpha} \cdot \sigma / \sqrt{n} = 2 \cdot 1.96 \cdot \sigma / \sqrt{5} \approx 1.75\sigma$ when σ is known, whereas its width is $2 \cdot t_{\alpha-1} \cdot E[\bar{S}] / \sqrt{n} = 2 \cdot 2.78 \cdot 0.94 \sigma / \sqrt{5} \approx 2.34\sigma$ when such value is unknown. As a result, the width of the confidence interval is increased of 33.7% compared to the value achievable referring to the Normal distribution. If a sample of 9 data is chosen, instead, the two values to compare are 1.31σ and 1.49σ , with an increase of 13.7%, only. From this example, it is possible to highlight that a decrease of the sample number causes an increase of the amplitude of the confidence interval, as shown previously with the Eq. (2).

IV. Test setups and results

The experimental validation of the proposed method has been carried out by using the test setup and procedures reported in [2]. Since the WER is little (usually measured in parts per million or parts per billion), a lot of samples must be collected to measure it. The test is based on a proper choice of the input and clock frequencies, so that, in the absence of word errors, the difference between two successive samples is always smaller than a given qualified level. An error is to be considered qualified if it is due to the DWR only and not to noise or external interferences. Therefore, an error detection threshold, called Qualified Error Level (QEL) has to be set depending, at least, on (i) the DWR characteristics, (ii) the test set up characteristics, and (iii) the DWR operating specifications. This should be the smallest value excluding all other sources of error from this test.

In order to develop a standard supplement, the tests should be designed without knowing the conditions (i) and (iii). As a matter of fact the choice of the correct QEL can be done by knowing exactly the inner architecture and design solutions behind the equipment under test. Such characteristics usually are known by the manufacturer only and considered confidential. The DWR operating specifications depend on the particular application it has been designed by the buyer and they can be exactly known by the buyer only.

After the QEL has been chosen, a large signal, whose rate of change is significantly less than the equivalent of 1 ideal code bin width per sample period, is applied to the ADC [1]. The largest possible record of data should be taken. Then, the differences among m successive samples are examined, and the number of times the absolute value of this difference exceeds the chosen QEL. Successive records of data are then taken keeping a running total of qualified errors until the required number of samples has been examined [2].

The WER is the number of qualified errors found through the test method and divided by the number of samples examined.

The experimental validation of the proposed method has been designed focusing on the test set up and the test procedure under the hypotheses that only the data sheet information is available from the manufacturer and that the QEL, when not specified by the manufacturer, should be found experimentally, using as reference the typical values for equipment of the same type. Due to the lack of information about the inner structure and components of the DWRs on the market, the validation has been focused on the most available platform: the digital oscilloscope. In particular two different oscilloscopes from different manufacturers have been chosen to provide the hardware platforms on whose the confidence intervals obtained by applying the proposed and the IEEE method have been compared.

The first experimental validation test setup is shown in Fig.1. In particular, the Tektronix AWG420 arbitrary waveform generator has been used to provide a sinusoidal signal to the Tektronix TDS 7704B oscilloscope. The records acquired by the oscilloscope have been then processed on the same oscilloscope or on an external PC to obtain the WER.

The test sine wave, having an amplitude of $2V_{pp}$ and a frequency of 10kHz, has been acquired with a sampling frequency of 10GS/s. As specified in [2], in fact, a significant oversampling ratio should be used for the acquisitions, in order to acquire a significant number of samples to be used for the WER computation. A very high sampling rate has been chosen for the DWR to set a worst case scenario. Before starting the record acquisitions a warm up time of 1 hour has been elapsed. Three groups of sixty successive acquisitions have been carried out, each of them made of 1M samples, spaced each other out by 1 minute interval. After each partitioned acquisition, a single record of 60M samples has been acquired and the WER results compared with those coming from the 1M acquisitions.

In order to analyse the possible effects on the test results due to the data transfer interface between the

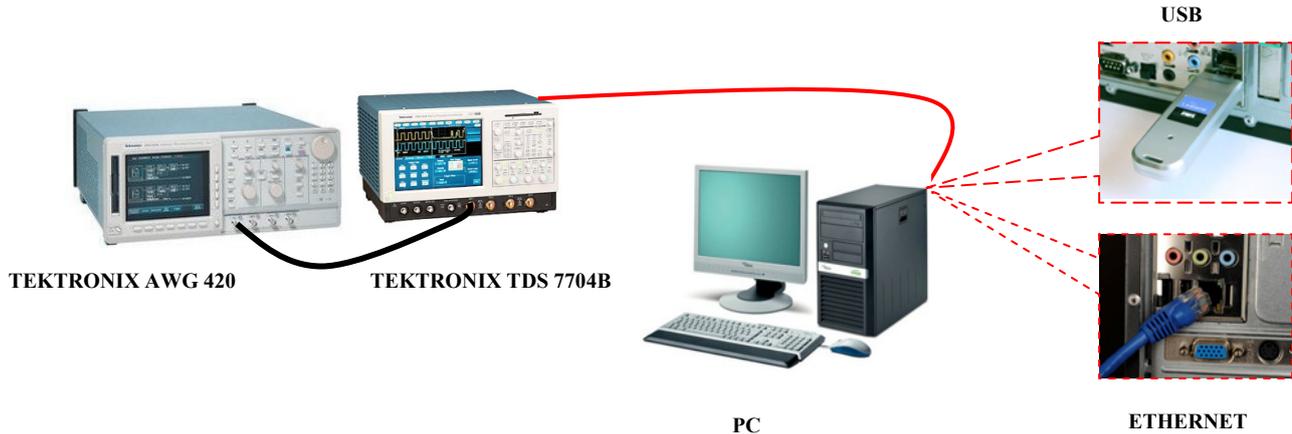


Fig. 1. First WER test setup.

oscilloscope and the PC, WER has been measured on the same input signal by a Matlab script running inside (i) the oscilloscope, (ii) the external PC connected to the oscilloscope by Ethernet, and (iii) by the external PC transferring the acquired records by a USB pen drive. The WER results obtained have been exactly the same, showing the independency of the WER measures from the data transfer interface used in the test setup.

The analysis of the WER results carried out by means of dot plots and box plots on the three data sets allowed to identify some outliers, however, they didn't lead to significant asymmetries and, therefore, it has been considered reasonable to use the Student's t distribution. The results of the statistical analysis based on this distribution are reported in Table II. The data sets, composed of 60 WER estimates coming from a 1M acquisition each, have been processed by using two QELs: 8 LSB and 9 LSB, in order to observe the influence of the QEL choice on the WER results. As it can be seen from the Table II, in fact, the QEL choice is decisive in characterizing DWR.

Table II. Student's t distribution statistical analysis.

Data set	Samples	WER mean ($\cdot 10^{-6}$)	WER StDev ($\cdot 10^{-6}$)	WER Experimental StDev ($\cdot 10^{-6}$)	Minimum WER ($\cdot 10^{-6}$)	Maximum WER ($\cdot 10^{-6}$)
Set#1 8lsb	60	18,33	28,63	3,70	0	100
Set#1 9lsb	60	0,72	1,51	0,19	0	6
Set#2 8lsb	60	9,10	15,40	1,99	0	79
Set#2 9lsb	60	0,23	0,59	0,08	0	3
Set#3 8lsb	60	13,78	19,93	2,57	0	96
Set#3 9lsb	60	0,37	0,92	0,12	0	4

By comparing Annex C [2] and the proposed Student's t distribution approach, shown in Table III, it is possible to note that by using the Student's t distribution the upper limit of the confidence interval is always greater than that obtained by following the method reported in the Annex C, in particular for 8 LSB QEL.

Table III. Comparison between the upper limits of the confidence intervals obtained by applying the proposed approach and the IEEE Std. 1057.

WER UCL (60 samples)	Set#1 ($\cdot 10^{-6}$)		Set#2 ($\cdot 10^{-6}$)		Set#3 ($\cdot 10^{-6}$)	
	<i>Annex C</i>	<i>Student's t</i>	<i>Annex C</i>	<i>Student's t</i>	<i>Annex C</i>	<i>Student's t</i>
8 LSB (95 %)	19	26	10	13	15	19
8 LSB (99 %)	20	28	10	14	15	21
9 LSB (95 %)	0	1	0	1	0	1
9 LSB (99 %)	0	1	0	1	0	1

The second group of experiments has been carried out on a different test set up to apply the proposed method in different scenarios where the instruments and the test signal characteristics change.

The test set up is shown in Fig.2. In particular, the Agilent 33220A arbitrary waveform generator has been used to provide a sinusoidal signal to the LeCroy SDA6000 oscilloscope. The records acquired by the oscilloscope have been then processed on the same oscilloscope or on an external PC to obtain the WER.



Fig.2 Second WER test setup.

The test about the digital interfaces has been first carried out as previously done for the Tektronix oscilloscope giving the same results. During this second test phase a Western Digital MyBook USB external hard disk has been used instead of the USB pen drive due to the bigger memory requirements.

Then, the tests have been carried by digitizing a test sine wave having an amplitude of $2V_{PP}$ and a frequency of 100kHz. Three data records have been taken with 30 acquisitions of 1M sample each and a 5GSa/s sampling frequency. Before starting the acquisition of the first record acquisition a 1h warm up time has been waited. The achieved results are shown in Tables IV and V for the three QEL values used during the previous experiments: 8 LSB and 9 LSB, respectively.

Table IV. Student's *t* distribution statistical analysis.

Data set	Samples	WER mean ($\cdot 10^{-6}$)	WER StDev ($\cdot 10^{-6}$)	WER Experimental StDev ($\cdot 10^{-6}$)	Minimum WER ($\cdot 10^{-6}$)	Maximum WER ($\cdot 10^{-6}$)
Set#1 8lsb	30	16,12	4,02	0,73	7	28
Set#1 9lsb	30	0,52	0,74	0,14	0	3
Set#2 8lsb	30	46,07	8,37	1,53	25	68
Set#2 9lsb	30	1,68	1,36	0,25	0	6
Set#3 8lsb	30	37,17	6,83	1,25	18	63
Set#3 9lsb	30	1,26	1,18	0,22	0	6

Table V. Comparison between the upper limits of the confidence intervals obtained by applying the proposed approach and the IEEE Std. 1057.

WER UCL (30 samples)	Set#1 ($\cdot 10^{-6}$)		Set#2 ($\cdot 10^{-6}$)		Set#3 ($\cdot 10^{-6}$)	
	Annex C	Student's <i>t</i>	Annex C	Student's <i>t</i>	Annex C	Student's <i>t</i>
8 LSB (95 %)	17	20	41	42	35	38
8 LSB (99 %)	18	22	43	46	36	40
9 LSB (95 %)	1	2	0	1	0	1
9 LSB (99 %)	1	2	1	1	1	2

The results shown in Tables IV and V highlight the same differences observed after the previous tests. Also in this case, in fact, UCL obtained by referring to the Student's *t* distribution is always greater than that obtained by following the method reported in the Annex C. This is a second proof of the validity of the proposed approach.

V. Conclusions

The aim of this paper is to propose a novel approach to assess the Word Error Rate (WER) parameter in digitizing waveform recorders. A new procedure based on Student's *t* distribution, to estimate the WER is provide, taken into account the minimization of the acquisition record length to make the approach more attractive from the industrial point of view.

The proposed test method has been experimentally verified by means of WER measurements on actual waveform recorders, where the test results have been compared to those achieved by implementing the IEEE Std. 1057 method. The validity of the proposed approach has been proven by applying it two different hardware platforms and by comparing the achieved results with those provided by means of the IEEE method. In both cases, even changing the instrument under test, the signal generator, the signal frequency and the sampling frequency the proposed method shown that the correct confidence intervals to be used in the uncertainty specifications are wider that those suggested in the Annex C of the IEEE Std. 1057. The future research activity will be focused on applying the proposed method on different hardware platforms.

Acknowledgements

The authors thank Prof. Pasquale Daponte for his significant advice and support during the research activity that led to this paper.

The research has been funded by the Italian Ministry of Education, University and Research within the project "A/D and D/A conversion metrology: standardization of the figures of merit, error estimation and correction, uncertainty evaluation", PRIN 2008, prot. 2008S9J8XE.

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