

# A RULE-BASED EXPERT SYSTEM GUIDE ON UNCERTAINTY FOR METROLOGISTS

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*Abstract: A rule based expert system (ES) has been implemented, which acts as a tutor to measurement scientists and technicians (metrologists) in the calculation of the uncertainty of measurements according to the methodology of the ISO "Guide to the Expression of Uncertainty in Measurement" (ISO Guide). The ES was implemented with the expert system shell/inference engine, "VP-Expert" (VPX) running in a DOS window. The intended target users are experienced metrologists, who lack the mathematical skills and concepts, particularly in statistical and sensitivity analysis, to implement the ISO-GUM document correctly. The ES guide uses seven main knowledge bases with a total of about four hundred rules. It allows a full, guided evaluation of the uncertainty of a measurement and uses a text-based tutoring knowledge base which is accessed either in a generic browse mode, or a context-sensitive mode based on the status of the current calculation. The ES uses background variables to maintain a consultation across knowledge bases and between interrupted sessions. The software implementation of this training is used as an analytical tool to make explicit and systematically specify the categorization and structure of the ISO Guide's fact-based and process-based knowledge content.*

## 1 INTRODUCTION

Measurement is a purposeful activity, and on it depend decisions at all levels of criticality and importance. The uncertainty of a measurement has always been considered important in this context, and taken into account either consequentially, as in design rules which allow for variability in a product or its components, or directly as in the rigorous testing of scientific theories and hypotheses by experimental data [1].

The current competitive drive for better industrial products and economic efficiency, both relying on an integrated global manufacturing base and market, have impacted on the science and practice of metrology not only in the direct delivery of measurement capability, but no less so in the more rigorous application of analytical tools, in particular the evaluation of measurement uncertainty, which now becomes one of its critical components.

A consistent, rigorous and universal methodology for the evaluation of measurement uncertainty was specified with the 1993 publication of the "Guide to the Expression of Uncertainty in Measurement" by the International Standards Organization (ISO Guide) [2]. The book was the result of 14 years' work by the leading international organizations charged with the maintenance and development of the science and technology of metrology, and has broad and international support.

The ISO Guide's procedures require a generalized conceptual understanding of the science and concepts of metrology and the mathematical techniques of the differential calculus and statistics. Technically, the Guide is a sophisticated benchmark document written in the generalized language of mathematics and it must cover all reasonable eventualities, exceptions and conditions which may arise and require treatment. This makes it, even for specialists, a rather daunting task, which requires some weeks of careful study before it can be applied at a competent level. Whilst the Guide's procedures require thoughtful and expert judgement and have a reasonable level of complexity, they are nevertheless able to be codified, defined and well specified. On the other hand, many metrologists come from the technical and engineering disciplines operating in industrial environments and often work in relatively small teams or even individually, with limited access to higher level assistance. These factors of a complex but codifiable knowledge genuinely needed by a widely dispersed potential user group make the expertise expressed by the ISO Guide suitable for implementation in the form of an expert system. In its basic pedagogical approach, the ES is based on the first-named author's expertise in running extensive training in this field.

A rule based expert system (ES) was implemented [3], to act as a tutor to measurement scientists and technicians (metrologists) in the methodology of calculating the uncertainty of measurements. The particular approach taken was to simulate a consultation between an expert and an inexperienced user of the ISO-GUM (although an otherwise experienced metrologist) in the process of developing and executing a calculation of uncertainty for a real measurement. The motivation for this approach was:

1. The assumption that effective learning is related to the learner's requirements to complete a given task and to therefore learn by the example of focus on an actual case, here the calculation of an uncertainty for a real current measurement.
2. The application of the ISO Guide to an actual measurement problem enforces a natural sequence and structure to the learning task, which is integrated with the Guide's own structure.
3. A real expert/learner interaction always allows the divergence from the current problem to consider tangential or related issues, and then a return to the current problem, so that the original thread is maintained.

The ES was implemented with the ES tool "VP-Expert" (VPX) [4], which is an expert system shell (i.e., an ES building tool) and provides an editor to create a rule base, and an inference engine by which queries are resolved and a consultation driven, under the action of "if-then" rules. VPX also importantly provides facilities for saving and reading data files, transferring control to other knowledge bases, and structure queries according to useful logical structures such as "while" loops. VPX implements fundamentally a backward-chaining strategy, by which the consequences of a rule (that is to say the successful assignment of a value to a parameter in the rule consequences) is achieved by progressively searching and assigning values to antecedents. However, an essentially forward-chaining mechanism can also be implemented by forcing a "find" action on a variable which is a consequence of an "if-then" rule.

Using a mixture of forward and backward chaining strategies and operating with seven main knowledge bases with a total of about four hundred rules, the ES is complete in that it operates from the problem definition stage to calculation of a final numerical result, and allows a fully guided evaluation of the uncertainty of a measurement. It offers a first level of context-sensitive tutorial in the form of text, references to the ISO-GUM, and internal linking of associated concepts. The textual tutoring knowledge base is accessed either through a generic entry status, in which the user can browse through the menu of concepts, or context-sensitive approach based on the current status of the evaluation model being developed and calculated. The ES uses a set of current status variables to maintain a consultation across knowledge bases and between interrupted sessions. The current implementation in VPX runs within a DOS window of the Windows operating system.

A critical outcome of this implementation has been the identification and specification of the underlying knowledge content of the ISO Guide,

## **2 OPERATION**

### **Procedural aspects of knowledge base**

The basic steps in the calculation of the uncertainty of a measurement are:

1. Specification of the measurand and the model of the measurement
2. Characterization of the dispersion parameters of the input and influence quantities of the measurement
3. Evaluation of the sensitivity coefficients for each input and influence quantity
4. Calculation of the combined (standard) uncertainty using standard equations
5. Calculation of the statistical probability interval for the values of the measurand (expanded uncertainty)

As well as this procedural knowledge, the ISO Guide specifies a terminology for uncertainty evaluation and defines a number of concepts such as "uncertainty", "standard uncertainty", "Type A" and "Type B" evaluation of uncertainty, and others. The Guide also requires the user to have knowledge about basic statistical and probability parameters, basic functional understanding of sensitivity analysis, and a basic conceptual understanding of the role of the "model" as the mathematical (or in general, logical) expression of the relationships between the input variables and the output result.

In practice, the knowledge content of these areas is presented to the user in the form of text and specific references to the ISO Guide itself, and the process content is expressed through the sequential behaviour of the ES and user interaction.

It is important to note the point clearly, that the expert system does no calculations. It guides the user to perform the calculations at each point. In effect, it acts as a procedural template, with the added feature of a substantial text help knowledge base, which can be browsed or which is accessed in a context sensitive manner.

### Structure of the expert system Knowledge base

The structure of the knowledge base is shown in Figure 1.

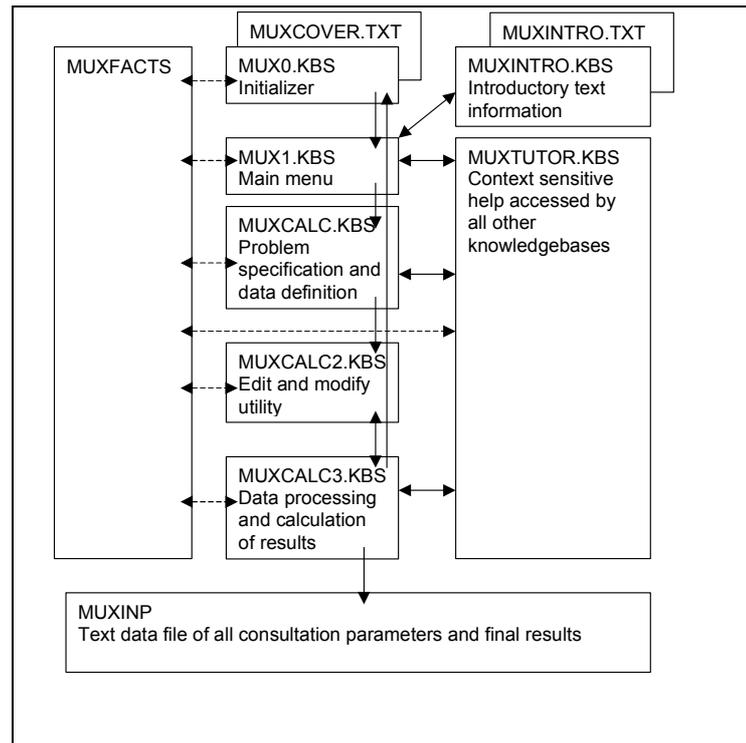


Figure 1

### Structure of field knowledge

MUX0.KBS and MUX1.KBS are the entry modules to the expert system. MUX0.KBS is a pre-processing step which initializes the data files and also checks to see whether the previous exit from the program was normal or interrupted, and if the latter, then the user is given the option of continuing from the point of interruption. The title screen (MUXCOVER.TXT) is displayed by MUX0.KBS which then passes control to MUX1.KBS for a normal start. MUX1.KBS is the first active and visible user interface, where the user is introduced to preliminary information about the system, and given a number of options to progress into a consultation. At every point in the user's interaction with the expert system, help is available. In the entry knowledge-base module, this is an explicit choice, but during the consultation, the user can reply to any question for input with the word "help" to which the system responds by transferring to the MUXTUTOR.KBS knowledge base, and in particular to the help topic relevant to the context in which help was activated.

MUXCALC.KBS, MUXCALC2.KBS and MUXCALC3.KBS are the three core procedural knowledge-bases. The first guides the user to setting up the measurement model, and specifying all the input and influence quantities parameters. The second allows the user to edit, modify or change the input and the influence quantity parameters, and the third guides the user in the processing and combination of the inputs to obtain the final value of the expanded uncertainty at the required level of confidence.

The logical sequence and structure of the ISO-GUM knowledge has a hierarchy, best represented diagrammatically in Figure 2.

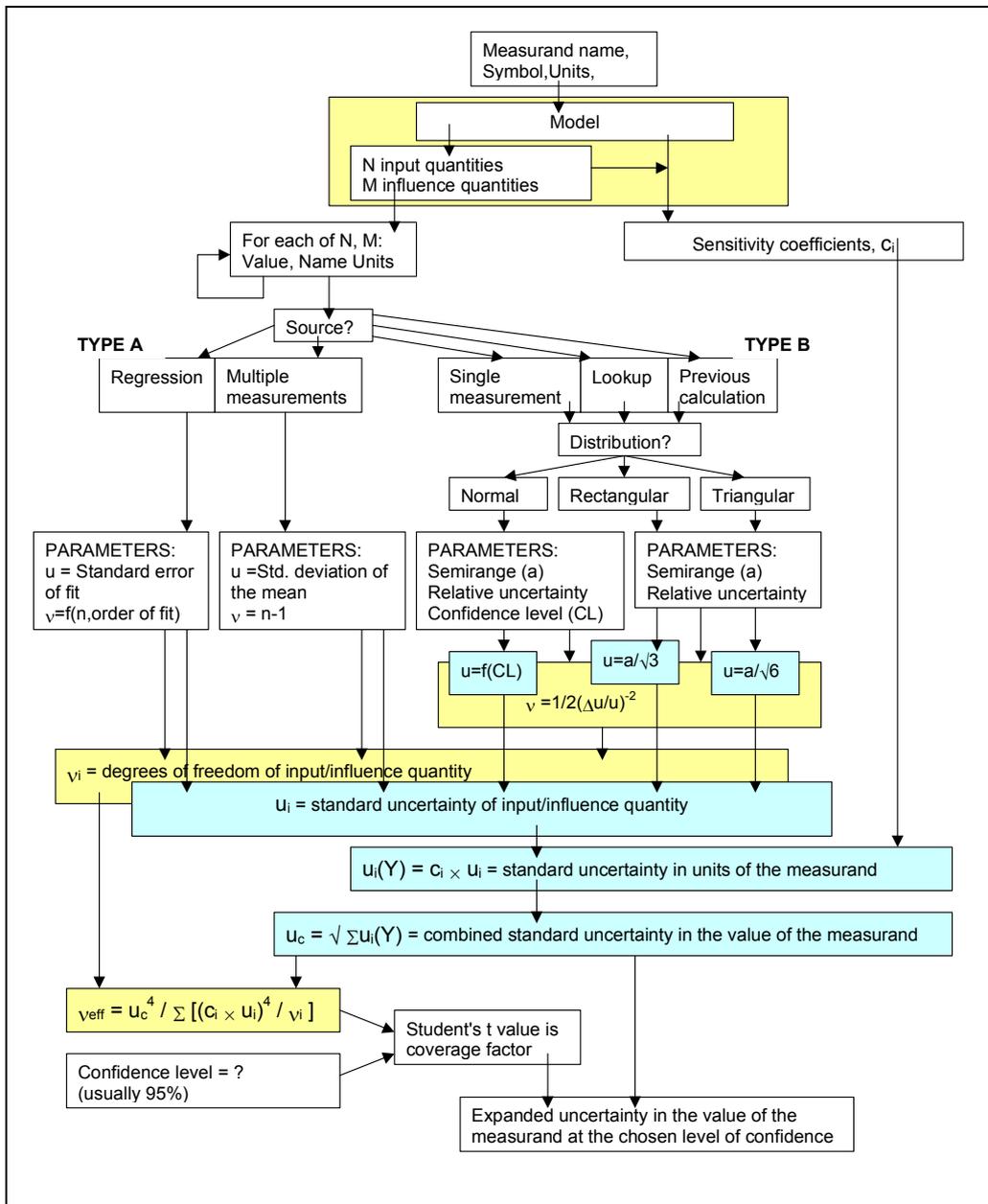


Figure 2

### Mechanism of knowledge-base

A basic template query is used at each stage of the calculation to elicit from the user the value of each parameter. For example, at a particular point in the consultation, the system may require the user to input the semirange uncertainty estimate of input quantity #5. A query variable defines the current enquiry and the process then proceeds as shown in Table 1.

Table 1 - Template for basic query function

<b>Set up the enquiry into the variable X (e.g. X = Semirange)</b>	
<i>The following is a typical statement set found in the "Action Block" of MUXCALC.KBS the first calculation knowledge-base. For multiple (array) variables, such as the input of parameters for each input or influence quantity, this basic template is contained within a "whileknown" loop structure.</i>	
query = X	<i>Specifies what the current topic of enquiry is</i>
find_X_display_action	<i>This displays to the user the appropriate prompt text relevant to the current query.</i>
find_X_action	<i>Find what action is to be taken with respect to this enquiry</i>
<b>Rules which determines what action to take with respect to the current enquiry</b>	
if query = X and X_action is unknown then	<i>Only do this if X_action has not already been done</i>
X_action = done	<i>Flag that now the X_action has been done</i>
display message	<i>Display an appropriate prompt to the user</i>
find X	<i>Now, go ahead and find the value of X</i>
<b>Rules which interact with the user, get and give responses</b>	
<b>PART A: CONDITION WHERE THE USER RESPONDS WITH A CALL FOR HELP</b>	
if query = X and response = help then	<i>Identify the current query and detect call for help</i>
X = not found	<i>Flag that X value at this stage is not found. Even though X is reset in the next line, this rule is required to make an explicit reference to X in the rule consequences.</i>
reset X_action	<i>As the user has called for help, reset this variable to indicate that it remains an unsatisfied condition.</i>
reset X	<i>As the user has called for help, reset this variable to indicate that it remains an unsatisfied condition</i>
reset response	<i>This is always reset, ready for generic re-use</i>
tutor_query = X	<i>The value of this variable is set to the current topic of enquiry to set the entry (context) condition into the tutoring knowledge-base.</i>
savefacts MUXFACTS	<i>All current state variables are saved to the datafile which allows global access, and maintains communication between knowledge-bases.</i>
reset X	<i>As the user has called for help, reset this variable to indicate that it remains an unsatisfied condition</i>
chain MUXTUTOR.KBS	<i>Control is transferred to the tutorial knowledge-base</i>
<b>PART B: CONDITION WHERE THE USER RESPONDS WITH THE VALUE OF THE REQUESTED VARIABLE. NOTE THAT A VALID RESPONSE IS RESPONSE WHICH IS NOT "HELP" AND NOT UNDEFINED.</b>	
if query = X and response <> help and response <> unknown then	<i>Identify the current query and detect that it is not a call for help and detect that it exists</i>
X = (response)	<i>Assign the user's response to the value of X</i>
reset response	<i>This is always reset, ready for generic re-use</i>
ask response; "Please enter the value of {query} or type help";	<i>This statement elicits a response from the user, with the value of the current topic for {query}</i>
	<i>Note: for array variables, such as multiple input quantities <math>i = 1..N</math>, VPX does not support indexed variables in "ask" statements, and a dummy variable "q" is therefore used in the expert system implementation to obtain this input.</i>

### Structure of key concepts

The MUXTUTOR.KBS knowledge base is a large set of interlinked text notes which explain approximately fifty identified key or core concepts. Each individual concept (such as "measurand") is linked to a small set of closely related key concepts (such as "units", "name", "model", "value", "input

quantity" and "influence quantity"). A variable named *query* is defined which at any point during a consultation has the value of the current query (such as "measurand name" or "input quantity #1 value"). If the user requests help, then control is passed to the MUXTUTOR.KBS knowledge-base with the query variable pointing to the specific topic of the current enquiry. Once control is transferred to MUXTUTOR.KBS however, the user can explore related topics or indeed any key topic for an extended learning interaction, before returning to the current calculation. Table 2 shows the key topics and the closely related key topics to which they are interlinked.

Table 2

Key Topic	Interlinked Key Topics
Measurand	Units, Name, Model, Value, Input quantity, Influence quantity
Symbol	Name, Model, Units, Input quantity, Influence quantity
Name	Measurand, Model, Units, Input quantity, Influence quantity, Value, Symbol
Units	Measurand, Input quantity, Influence quantity, Name, Value, Symbol
Type	Type A evaluation, Type B evaluation, Type A B mixed evaluation, ISO-GUM calculation
Model	Measurand, Input quantity, Influence quantity, Units, Value, Symbol, Name, Sensitivity coefficient
Source	Input quantity, Influence quantity, Type A evaluation, Type B evaluation, Type A B mixed evaluation, Repeated measurements, Single measurement, Lookup, Regression, ISO-GUM calculation
Value	Measurand, Input quantity, Influence quantity, Name, Units, Value, Model, Symbol
Input quantity	Measurand, Influence quantity, Sensitivity coefficient, Model, Name, Value, Units, Symbol
Influence quantity	Measurand, Input quantity, Sensitivity coefficient, Model, Name, Value, Units, Symbol
Uncertainty source	Input quantity, Influence quantity, Model, Type A evaluation, Type B evaluation, Type A B mixed evaluation, Repeated measurements, Single measurement, Lookup, Regression, ISO-GUM calculation
Repeated measurements	Input quantity, Influence quantity, Model, Type A evaluation, Type B evaluation, Type A B mixed evaluation, Uncertainty source, Single measurement, Lookup, Regression, ISO-GUM calculation, Random
Single measurement	Input quantity, Influence quantity, Model, Type A evaluation, Type B evaluation, Type A B mixed evaluation, Uncertainty source, Repeated measurements, Lookup, Regression, ISO-GUM calculation
Lookup	Input quantity, Influence quantity, Model, Type A evaluation, Type B evaluation, Type A B mixed evaluation, Uncertainty source, Repeated measurements, Single measurement, Regression, ISO-GUM calculation
Regression	Input quantity, Influence quantity, Model, Type A evaluation, Type B evaluation, Type A B mixed evaluation, Uncertainty source, Repeated measurements, Single measurement, Lookup, ISO-GUM calculation, Random
ISO-GUM calculation	Input quantity, Influence quantity, Model, Type A evaluation, Type B evaluation, Type A B mixed evaluation, Uncertainty source, Repeated measurements, Single measurement, Lookup, Regression
Type A evaluation	Type B evaluation, Type A B mixed evaluation, Random, Number of measurements, Standard deviation, ESDM, Dispersion parameters
Type B evaluation	Type A evaluation, Type A B mixed evaluation, Random, Number of measurements, Standard deviation, ESDM, Dispersion parameters
Type A B mixed evaluation	Type A evaluation, Type B evaluation, Random, Number of measurements, Standard deviation, ESDM, Dispersion parameters
Rectangular distribution	Distribution, Triangular distribution, Normal distribution, Semirange, Likelihood, Degrees of freedom, Welch Satterthwaite formula, Effective degrees of freedom, Reliability
Triangular distribution	Distribution, Rectangular distribution, Normal distribution, Semirange, Likelihood, Degrees of freedom, Welch Satterthwaite formula, Effective degrees of freedom, Reliability
Normal distribution	Distribution, Rectangular distribution, Triangular distribution, Semirange, Likelihood, Degrees of freedom, Welch Satterthwaite formula, Effective degrees of freedom, Reliability, Random
Distribution	Rectangular distribution, Triangular distribution, Normal distribution, Semirange, Likelihood, Degrees of freedom, Welch Satterthwaite formula, Effective degrees of freedom, Reliability
Dispersion parameters	Standard deviation, ESDM, Variance, Likelihood, Reliability, Degrees of freedom, Semirange, Effective degrees of freedom, Standard uncertainty, Random
Standard uncertainty	Dispersion parameters, Standard deviation, Expanded uncertainty, Rectangular distribution, Triangular distribution, Normal distribution, Likelihood, Reliability, Distribution
Expanded uncertainty	Dispersion parameters, Standard deviation, Standard uncertainty, Rectangular distribution, Triangular distribution, Normal distribution, Likelihood, Reliability, Distribution

Sensitivity coefficient	Model, Analytic differentiation, Numerical differentiation, Experimental differentiation, Input quantity, Influence quantity
Analytic differentiation	Sensitivity coefficient, Numerical differentiation, Experimental differentiation
Numerical differentiation	Sensitivity coefficient, Analytic differentiation, Experimental differentiation
Experimental differentiation	Sensitivity coefficient, Analytic differentiation, Numerical differentiation
Effective degrees of freedom	Degrees of freedom, Student's t distribution, Welch Satterthwaite formula
Degrees of freedom	Effective degrees of freedom, Student's t distribution
Welch Satterthwaite formula	Degrees of freedom, Effective degrees of freedom
Standard deviation	Variance, Standard uncertainty, ESDM,
ESDM (experimental standard deviation of the mean)	Standard deviation, Variance, Standard uncertainty
Variance	Standard deviation, ESDM, Standard uncertainty
Number of measurements	Degrees of freedom, Effective degrees of freedom, ESDM, Student's t distribution
Student's t distribution	Degrees of freedom, Effective degrees of freedom
Semi range	Rectangular distribution, Triangular distribution, Normal distribution, Dispersion parameters, Likelihood, Reliability
Reliability	Degrees of freedom, Effective degrees of freedom, Dispersion parameters, Likelihood
Standard error	Regression, Standard deviation, Standard uncertainty
Likelihood	Normal distribution, Semirange
Confidence interval	Student's t distribution, Degrees of freedom, Effective degrees of freedom
Random	Type A evaluation
Sample standard deviation	Standard deviation, ESDM, Number of measurements

The set of linked key topic for any key topic were determined by considering what the user might need or wish to know about at a particular point in the consultation, given that he had requested help on the primary key topic.

## Testing

The expert system was tested with a group of five users. The five individuals were professional metrologists, working in fields of engineering, dimensional, physical and electrical metrology, and performing roles from calibration to scientific officer. A software disk with a basic set of installation and operating instructions were provided (refer to Appendix below) and nothing else. The test group's evaluations were obtained on a standard pro-forma questionnaire, and addressed issues of installation performance; adequacy of start-up information on first use; usability, difficulty and completeness of tutoring module; operability of calculation modules; ease of use of editing facility; adequacy of guidance on calculation of results module; and an opinion on the level of user at which the expert system would be most appropriate.

Users reported satisfactory installation, except in one case of the absence of the Microsoft "Windows" operating system, where more complete instructions were suggested. While the level and content of the explanations were considered adequate to good by the test group users, there was an expectation that the system would perform more of the calculations, rather than merely guide the user through them. The use of the keyboard, rather than a mouse and the separate "help" screens, rather than the Microsoft style of "layered" windows were also considered less desirable.

## 3 DISCUSSION

The critical educational value of this work is seen to be the clear specification of the procedural and factual knowledge content of the ISO Guide protocols, and their structure. The implementation in a rule-based user-responsive context is of interest, but only to the extent that the tool satisfies the requirement of effective training outcome.

The controlling mechanism which achieves the consultation's aim is realized in the Action Block of the expert system. In this part of each knowledge base, a set of queries are required to be satisfied in a fixed sequence. The set of if-then rules which the inference engine operates on, are then the user interface, by which questions are put, information sought, and responses acquired. The structure of the expert system software implementation then makes explicit the two types of knowledge which make up the ISO-GUM training module. The *if-then rules* embody the knowledge of the meaning of the ISO-GUM terminology through their use of words and labels attached to questions and input

statements, whilst the *Action blocks* embody the knowledge of the process of the ISO-GUM through their sequential structure.

Although the diagram of Figure 2 shows a strictly sequential structure of knowledge content, this is more properly regarded as a "first order approximation" to the topic, with the true operational structure of the associated knowledge being more diffuse, overlapping and bi-directional in parts. For example, the estimation of relative uncertainties for Type B estimates of uncertainties, which are necessary to estimate the associated degrees of freedom, are only fully meaningful after the user understands that the coverage factor, which relates a confidence interval to a standard deviation is a function of the degrees of freedom associated with the statistic being considered. The structure of the knowledge bases allows for a flexible, non-sequential and iterative learning mode, by allowing the user to freely access all topics at any time, even though initial entry is conditioned by the entry topic context. When a query is completed however, the user is returned to continue the consultation at the precise point where the consultation was interrupted.

## 4 CONCLUSION

The knowledge content in the ISO Guide for the Expression of Measurement Uncertainty has been characterized into two main components; procedural and factual, and successfully implemented in a rule-based expert system.

The results of preliminary testing suggest that whilst the structure and technical usability of the expert system are operationally satisfactory, user's expectations are that programs will conform more closely to the familiar "windows" style of graphical interface, and that the system should perform more of the numerical calculations.

The system was framed in the context of an actual consultation in which the user is guided through the development of a measurement model for the system under consideration, and where the user can ask for help at any time, which is presented in a context-sensitive mode, and from which the entire knowledge base of ISO-GUM terminology can be accessed, before returning to the consultation at the point where the help was requested, to continue with the current consultation.

The definition of terminology and specification of the basic conceptual elements of the Guide are expressed in a knowledge base consisting of text-based explanations and references to the Guide itself, whilst the procedural knowledge content of the Guide is expressed in the program structure of the actions performed in the Actions Block component of the expert system.

## REFERENCES

- [1] Klein H.A., *The Science of Measurement - A Historical Survey*, Dover Publications Inc. New York, 1974
- [2] *Guide to the Expression of Uncertainty in Measurement*, International Standards Organization, Geneva, Switzerland, 1995
- [3] Giardini W., *Measurement Uncertainty Expert*, MEngSc. minor thesis, Monash University, 1996
- [4] *Paperback Software International*, California, VP-Expert, USA, 1987

## Appendix - Software user guide and sample consultation

### Installation and operating instructions

The expert system is presented as a set of files encapsulated into the file MUX.ZIP which should be downloaded and stored to a directory of the hard drive (suggest C:\MUX\).

The download site is:

[http://www.tip.csiro.au/cscripts/relationships/render.asp?page\\_id=1957&left=1623](http://www.tip.csiro.au/cscripts/relationships/render.asp?page_id=1957&left=1623)

The file MUX.ZIP should be uncompressed and the set of files listed in Table 3 should be present in the chosen directory.

Table 3 - run files - (see Fig. 1 also)

mux.bat	user run file starts system
mux0.kbs	Initialize system
mux1.kbs	Main menu for start up
muxcover.txt	Cover page shows at startup
muxintro.kbs	Introduction knowledge-base
muxintro.txt	Introductory text content
muxfacts	File stores and transfers system variables
results	Result record of consultation parameters and results
muxcalc.kbs	Calculator consultation set up and data entry
muxcalc2.kbs	Consultation editing tool
muxcalc3.kbs	Calculation of results from input data
Muxtutor.kbs	ISO-GUM Facts knowledge base
Sample Pressure Calc.txt	Sample file re-named from "results" file
vpz.exe	VPX Expert system run files
vpz.cnf	
vpz.msg	
vpze.exe	
vpzh.exe	
vpzi.exe	
vpzm.exe	
vpzt.exe	

To start a consultation, open a DOS window in the chosen directory and type MUX. The file mux.bat will start the consultation by running VPX.EXE and the startup knowledge-base MUX0.KBS. The knowledge-bases are ASCII files and all files may be read with any text editor.

### Sample Consultation

Consider the measurement of pressure using a piston gauge (pressure balance) in simplified form, refer to Figure A1. The pressure is obtained by equation (1) and it is influenced by temperature, which changes the area of the piston-cylinder assembly, and hence the generated pressure. The operation and inputs to the expert system are shown in Table A1.

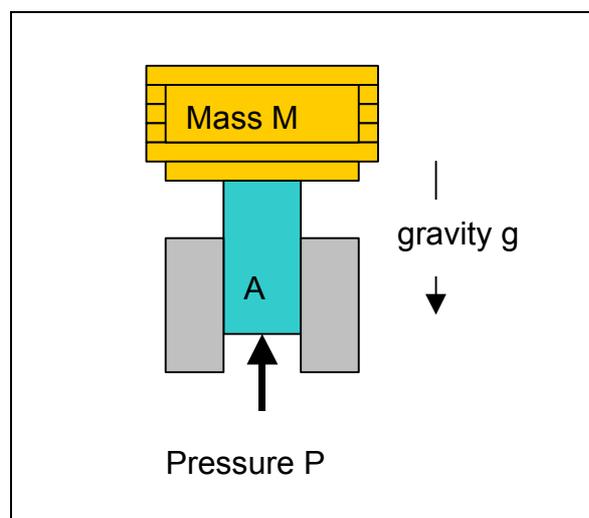


Figure A1

A piston gauge, in which the pressure of the fluid P, acting on the piston base of area A is balanced by the weight load Mg

$$P = \frac{Mg}{A} \quad (1)$$

Table A1 Sample Data Run

<b>MUXCALC.KBS</b>		
<b>Query</b>	<b>Input response</b>	<b>Input entry</b>
Measurand	Pressure	Pressure
Symbol	P	P
Units	Pa	Pa
Value	$1.4 \times 10^7$	1.4E7
Model specification	P=Mg/A	-
Number of input quantities	3	3
Number of influence quantities	1	1
Input quantity 1		
Name	mass	mass
Symbol	m	m
Value	4.5	4.5
Units	kg	kg
Source	repeated measurements	(menu choice)
number of measurements	5	5
degrees of freedom	4	4
sample standard deviation	0.000 03	0.000 03
experimental std. dev. of mean	$1.34 \times 10^{-5}$	1.34E-5
sensitivity coefficient	$3.27 \times 10^6$	3.27E6
sensitivity coefficient	$1.5 \times 10^6$	1.5E6
Input quantity 2		
Name	Area	Area
Symbol	A	A
Value	$3 \times 10^{-6}$	3E-6
Units	m <sup>2</sup>	square metres
Source	regression	(menu choice)
standard error	$2 \times 10^{-10}$	2E-10
degrees of freedom	7	7
sensitivity coefficient	$4.9 \times 10^{12}$	4.9E12
Input quantity 3		
Name	gravity	gravity
Symbol	g	g
Value	9.8	9.8
Units	m/s <sup>2</sup>	metre per second squared
Source	lookup	(menu choice)
Type of distribution	Rectangular	(menu choice)
semirange	0.000 1	0.000 1
standard uncertainty	$5.8 \times 10^{-5}$	5.8E-5
reliability	10%	10%
degrees of freedom	50	50
Influence quantity 1		
Name	Temperature	Temperature
symbol	t	t
semirange infl. quantity units	0.1 (degrees)	0.1
standard uncertainty in measurand units	$23 \times 10^{-6} \times 0.1 \times P = 33.81$ (Pa)	33.81
<b>MUXCALC2.KBS allows review and editing of any data</b>		

<b>MUXCALC3.KBS guides user in calculation of results from input data</b>	
The system displays the standard uncertainty and the sensitivity coefficient for each input parameter and then prompts the user to multiply them together to obtain the weighted standard uncertainty. The results of the calculations are entered by the user.	
Weighted standard uncertainty Input quantity 1 (mass)	43.81
Weighted standard uncertainty Input quantity 2 (area)	980
Weighted standard uncertainty Input quantity 3 (gravity)	90.48
The influence quantity does not have to be calculated since it is already in the units of the measurand, and in this implementation is always assumed to have infinite degrees of freedom.	
Root square sum of the weighted standard uncertainties	985.7
Welch-Satterthwaite denominator	1.32E11
Effective Degrees of Freedom	7.15
Selected level of confidence	95%
Value of Student's t, for edf = 7.77 and conf level of 95%	2.36
Value of the expanded uncertainty	2326
The system then presents the final result, and offers the choice of a full recalculation, an edit followed by a full recalculation, or exit. The data is all saved in a text file named "results". This file should be renamed immediately following the consultation, so that it is not lost when overwritten after the next consultation. The file is presented below, renamed to "Sample Pressure Calc.txt"	

**Sample Pressure Calc.txt**

```

change_what_disp = influence_quantity_measurand_std_unc CNF 100
combined_standard_uncertainty = 985.7 CNF 100
confidence_level = 95% CNF 100
degrees_of_freedom[1] = 4 CNF 100
degrees_of_freedom[2] = 7 CNF 100
degrees_of_freedom[3] = 50 CNF 100
dispersion_parameters[1] = found CNF 100
dispersion_parameters[2] = found CNF 100
dispersion_parameters[3] = found CNF 100
distribution[3] = Rectangular CNF 100
DOFeff = 7.15 CNF 100
exit_action = end CNF 100
exit_response = End CNF 100
exp_standard_deviation_of_mean[1] = 1.34E-5 CNF 100
Expanded_uncertainty = 2326.2 CNF 100
infl_qty_mrnd_std_unc[1] = 33.81 CNF 100
infl_qty_name[1] = temperature CNF 100
infl_qty_semrge[1] = 0.1 CNF 100
infl_qty_sym[1] = t CNF 100
intro_display2_action = done CNF 100
intro_display_action = done CNF 100
measurand_name = Pressure CNF 100
measurand_params_intro_display_action = done CNF 100
measurand_symbol = P CNF 100
measurand_units = Pa CNF 100
measurand_value = 1.4E7 CNF 100
model = specified_for_user CNF 100
model_action = done CNF 100
name[1] = mass CNF 100
name[2] = Area CNF 100
name[3] = gravity CNF 100
number_of_influence_quantities = 1 CNF 100
number_of_influence_quantities_action = done CNF 100
number_of_input_quantities = 3 CNF 100
number_of_input_quantities_action = done CNF 100
number_of_measurements[1] = 5 CNF 100
parameters_spec_intro_msg_action = done CNF 100

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prompt2\_show[1] = done CNF 100  
prompt2\_show[2] = done CNF 100  
prompt2\_show[3] = done CNF 100  
prompt2A\_show[1] = done CNF 100  
prompt3\_show = done CNF 100  
prompt5\_show = done CNF 100  
prompt6\_show = done CNF 100  
prompt7\_show = done CNF 100  
query = Expanded\_uncertainty CNF 100  
query2 = degrees\_of\_freedom CNF 100  
reliability[3] = 10 CNF 100  
sample\_standard\_deviation[1] = 0.000\*03 CNF 100  
semi\_range[3] = 0.000\*1 CNF 100  
sensitivity\_coefficient[1] = 3.27E6 CNF 100  
sensitivity\_coefficient[2] = 4.9E12 CNF 100  
sensitivity\_coefficient[3] = 1.5E6 CNF 100  
service = calculation3 CNF 100  
source[1] = Repeated\_measurements CNF 100  
source[2] = Regression CNF 100  
source[3] = Lookup CNF 100  
standard\_error[2] = 2E-10 CNF 100  
standard\_uncertainty[1] = 1.34E-5 CNF 100  
standard\_uncertainty[2] = 2E-10 CNF 100  
standard\_uncertainty[3] = 5.8E-5 CNF 100  
status = running CNF 100  
Students\_t = 2.36 CNF 100  
symbol[1] = m CNF 100  
symbol[2] = A CNF 100  
symbol[3] = g CNF 100  
type[1] = A CNF 100  
type[2] = A CNF 100  
type[3] = B CNF 100  
units[1] = kg CNF 100  
units[2] = square\*metres CNF 100  
units[3] = metre\*per\*sec\*squared CNF 100  
value[1] = 4.5 CNF 100  
value[2] = 3E-6 CNF 100  
value[3] = 9.8 CNF 100  
weighted\_standard\_uncertainty[1] = 43.81 CNF 100  
weighted\_standard\_uncertainty[2] = 980 CNF 100  
weighted\_standard\_uncertainty[3] = 90.48 CNF 100  
WSDenom = 1.32E11 CNF 100