# TEACHING DIGITAL SIGNAL PROCESSING IN A QUALITY ENGINEERING LABORATORY

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*Abstract: The digital signal processing laboratory play a very important role in teaching students within the metrology and quality engineering study program. The importance of DSP is outlined in this paper. The example of a student workbench is described in details and the structure and the features of a set-up are given. The advantages of the concept and some remarks how students feel about this workbench and programme of the experiment are pointed out. The specifications of the instruments used are given. One of the advantage of the set-up is a possibility to perform the experiment from different location by the use of TCP IP protocol. As an example of the possibility of the proposed workbench, both the RMS value and the average value of measuring voltage in two ways (i.e. by means of the virtual sampling voltmeter and by the analysis of harmonics) are presented.*
*Keywords: DSP, Virtual Voltmeter, Virtual Spectrum Analyser.*

## 1 INTRODUCTION

The Electrical and Electronic Measurement Laboratory use analogue methods in measuring electrical quantities. The aim of this paper is to present the new control set-up, which is making use of the digital processing of the tested signal. The set-up enables students to learn a specificity of digital measurements using sampling of the input signal.

## 2 STRUCTURE OF THE SET-UP

Structure of the set-up is presented in Figure 1.

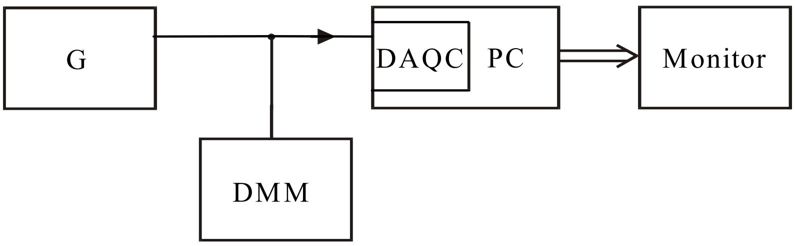


Figure 1. The block-diagram of the measurement set-up

The set-up consists of a PC computer, a control card DAQC, a multi-choice waveform generator G and a digital multi-meter, DMM. The PC computer with a specialized software controls the card and then it processes the data at the output of the control card and also displays the results of the measurement. The card ensures the choice of input signal, its amplification and its processing into digital value by the analogue-to-digital (A/D) converter. The generator, G, is used to supply a standard signal and a high precision multi-meter, DMM, to monitor the accuracy of the analog to digital signal processing.

## 3 MEASUREMENT POSSIBILITIES OF THE SET-UP

The operation of the set-up is determined by a specialised software, which is realized in Test Point environment. The software performs the following functions as shown below:

i) generation of virtual digital signals: sinusoidal, polyharmonical, triangular, rectangular and stochastic signals;

ii) monitoring of the signal transient, its modulus and its amplitude spectrum as well as its phase spectrum;

iii) measurement of the signal values: maximal, mean of modulus, root-mean-square, RMS of fundamental harmonic, RMS of higher harmonics sum as well as amplitude and frequency of facultative harmonic.

Thanks to the versatility of the software, we can examine such things as the influence of the sampling frequency and the number of samples in a measurement window onto the measurement accuracy of the following quantities: peak value, mean value, RMS value, frequency, duty factor and correlation factor. Besides, thanks to the simulation of the A/D converter, we can also examine an influence of the resolution of the converter on the measurement accuracy of the above mentioned quantities. It is also possible to observe a transient, which is first differentiated and then integrated. The result of these two operations is compared with the input transient. The software also performs many other conversions of the signal such as digital filtering and Fourier transformation.

The set-up enables converting input signals, both virtual and real. The virtual signals, which are realised by mathematical functions, are applied to assess the accuracy of the digital part of the converting line. Application of the real, standard signals enables the assessment of the conversion accuracy of the total measurement line. First, we can choose the input signal form independent on the type of the measured quantity. Next, we record the effect of every measurement for various sampling frequencies and numbers of samples in measurement window and then compute the error with reference to theoretically determined value. A small difference between the computed error and the resolution of the A/D converter confirms good selection of the two changed parameters. In case of testing metrological properties of digital Fourier transformation we can additionally observe a phenomenon of spectrum reflection, which also influences the converting accuracy.

## 4 AVERAGE VOLTAGE MEASUREMENT BY MEANS OF VIRTUAL INSTRUMENT

The measurement method results from conversion of the instantaneous values of a voltage into the sampled values according to the formula on average value of the voltage modulus

$$V_{av} = \frac{1}{pT_1} \int_0^{pT_1} |v(t)|\, dt \cong \frac{1}{T_W} \sum_{i=0}^{M-1} |v_i|\, T_S = \frac{1}{M} \sum_{i=0}^{M-1} |v_i| \qquad (1)$$

where:

- $p$ – whole number of periods $T_1$ of the first harmonic
- $T_W$ – length of measurement window ($T_W = MT_S$)
- $M$ – number of samples in $T_W$ time
- $T_S$ – sample period, otherwise inverse of sampling frequency $f_S$.

The following errors of measurement method result from approximation in formula (1):

i) the error resulting from substituting of the integral by the sum – $\delta_a$ ;

ii) the error resulting from this, that length of measurement window is not equal to the total number of the measured transient periods ($T_W \neq pT_1$) – $\delta_b$ ;

iii) the error resulting from quantisation ($v_i \neq v(t)$) – $\delta_c$.

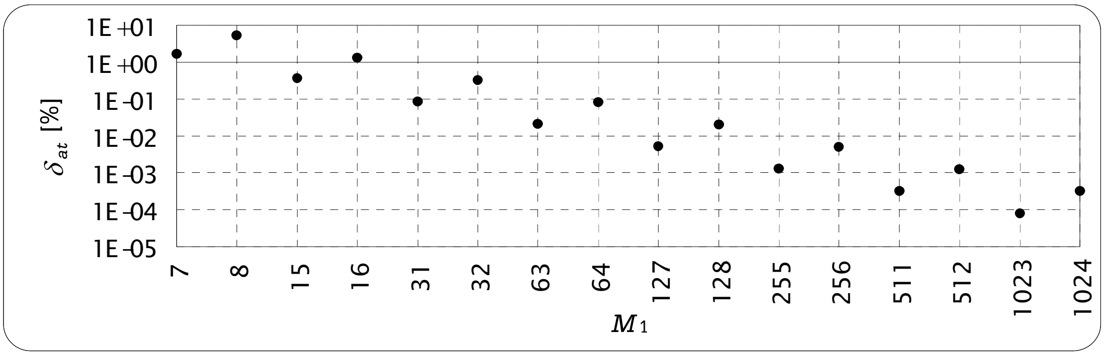


Figure 2. The dependence of the error $\delta_{at}$ on number of samples $M_1$ in period.

The measurement set-up enables determining of values of the above mentioned errors. The error $\delta_a$ we determine for sinusoidal transient with frequency $f_1$. We investigate how the error depends on sampling frequency, $f_S$, (or the number of samples in one period, $M_1$; $f_S = M_1 f_1$), phase shift of samples, $\psi$, ($\psi = 2\pi i_0 / M_1$, where $i_0 \in <0,1)$) as well as the parity or non-parity of number $M_1$. In order to verify the obtained results of measurements of the error $\delta_a$ a theoretical formula has been utilised

$$\delta_{at} = \frac{\pi}{2M_1} \sum_{i=0}^{M_1-1} \left| \sin \frac{2\pi}{M_1}(i + i_0) \right| - 1 \qquad (2)$$

The diagrams according to the formula (2) are presented in Figure 2 and Figure 3.

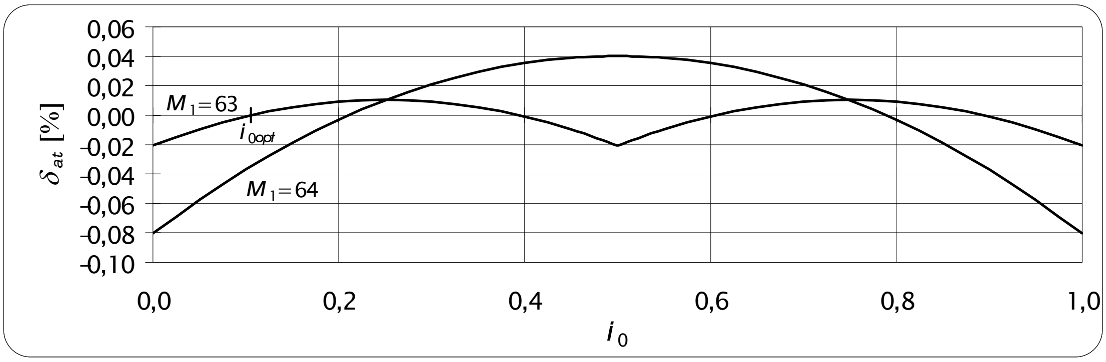


Figure 3. The dependence of the error $\delta_{at}$ on phase shift of samples .

The differences between errors determined by the set-up and errors determined theoretically do not exceed the accuracy of the computer calculations.

This also applies to the error $\delta_b$ we determine for sinusoidal signal with frequency $f_1$. We examine how the error depends on the non-integer value of number of periods, $p$, and phase shift of sinusoidal transient, $\varphi$. In order to avoid the error $\delta_a$, we apply a coefficient in phase shift of samples $i_0 = i_{0opt}$ (see Figure 3). The theoretical formula for measurement error $\delta_{bt}$ is

$$\delta_{bt} = \frac{\pi}{2pM_1} \sum_{i=0}^{pM_1-1} \left| \sin\left( \frac{2\pi(i + i_{0opt})}{M_1} + \varphi \right) \right| - 1 \tag{3}$$

The diagrams according to formula (3) are presented in Figure 4. The differences between errors of the set-up and the theoretical errors do not exceed an accuracy of the computer calculations.

The formula of theoretical error $\delta_{ct}$ is as follows

$$\delta_{ct} = \frac{k_p}{2(2^n - 1)\sqrt{3M}} \tag{4}$$

where: $k_p$ – extension factor (for confidence level $p$ = 0.99, $k_p$ = 2.58)
$n$ – number of bits of A/D converter.

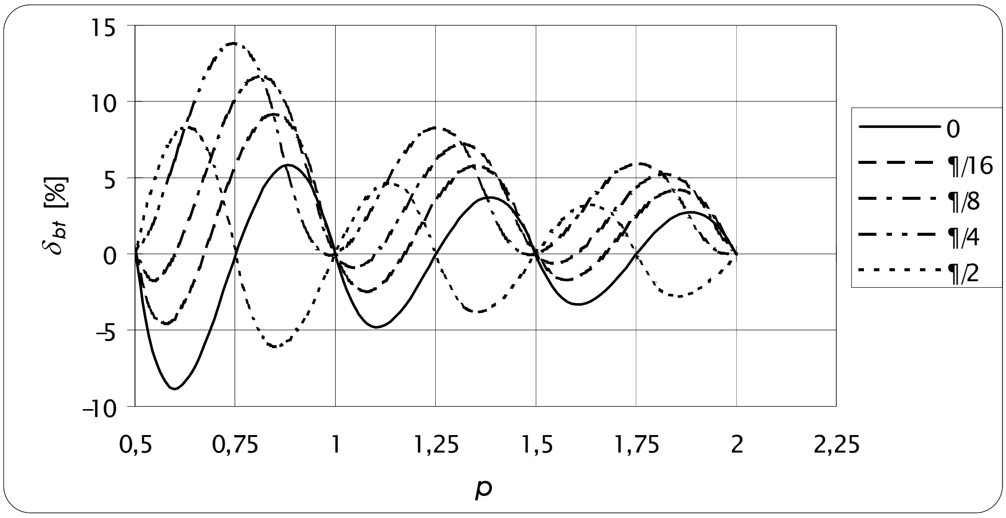


Figure 4. The dependence of the error $\delta_{bt}$ on number of periods, $p$, for different values of phase shift, $\varphi$

To emphasise the quantization error that should be evidently more than the errors $\delta_a$ and $\delta_b$, for experimental checking the above formula, a simulation of the A/D converter is made by having a small number of bits (n = 8). The number of samples, $M$, has been changed in the time of measurement by ensuring the integer number of periods, $p$, and the phase shift angle, $\psi$, have been set so that several samples $v_i$ were of different values.

The dependence of theoretical error, $\delta_{ct}$, versus number of samples, $M$, is presented in Figure 5.

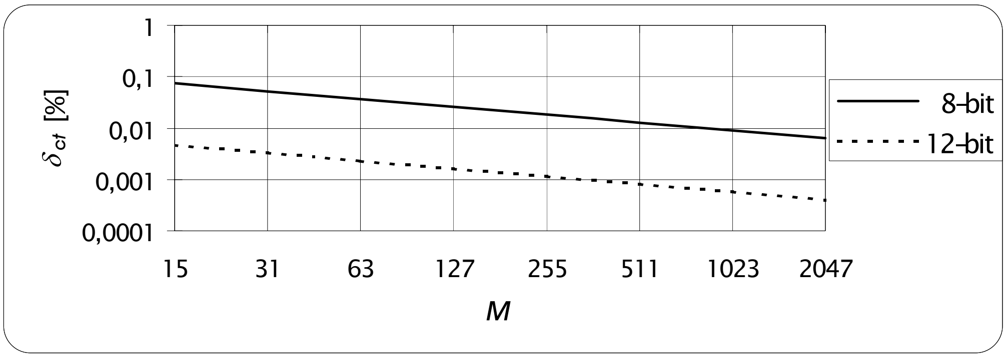


Figure 5. The dependence of the error $\delta_{ct}$ on number of samples, $M$

In case of examining the stochastic transient, the error of the average value measurement, $\delta_c$, has been conformed to the theoretical value of $\delta_{ct}$. On the other hand, this error, $\delta_c$, for sinusoidal transient has been 10 times less than the theoretical error, $\delta_{ct}$. The formula 4 can be applied only for independent set of quantisation errors of individual samples. Sinusoidal signal do not fulfil that condition.

In measurement of the average value of voltage, the component $\delta_b$ has the highest part in the total error. In order to eliminate this component we should know the accurate value of period $T_1$ to perform the rule $T_W = pT_1$. The set-up has been equipped with a digital interpolation method for measuring period $T_1$.

## 5 RMS VOLTAGE MEASUREMENT BY VIRTUAL SPECTRUM ANALYSER

We shall examine the measurement of RMS value in the most difficult case when neither the value of the first harmonic frequency $f_1$ nor number of the highest harmonic $n_{max}$ is known. Authors worked out their own method that is presented below. The method requires repeating of sampling 4-times with different values of sampling frequency and number of samples. We start with the greatest sampling frequency, $f_{Smax}$, that is permissible for the measurement card, DAQC, and with maximal number of samples, $M_{max}$, which FFT algorithm can transform in reasonable time. In that way we achieve the maximum width and the best resolution of spectrum. For example we take a measurement of $M_1 = M_{max} = 2048$ and $f_{S1} = f_{Smax} = 100$ kHz. First, we check on the virtual monitor if there is at least one period of the tested signal in the measurement window otherwise we reduce the sampling frequency. Next, we apply FFT and take notes of the frequency $f_1$ and the amplitude $A_1$ of the lowest harmonic as well as the frequency fmax and amplitude Amax of the highest harmonic as shown in the Table 1. We also note the frequencies $f_{1-}$, $f_{1+}$ and amplitudes $A_{1-}$, $A_{1+}$ of adjacent lines in spectrum.

Table 1. Results of harmonic components measurements using FFT method

| No | $f_S$ | $f_{1-}$ | $f_1$ | $f_{1+}$ | $A_{1-}$ | $A_1$ | $A_{1+}$ | $f_{max}$ | $A_{max}$ |
|---|---|---|---|---|---|---|---|---|---|
| — | Hz | Hz | Hz | Hz | V | V | V | Hz | V |
| 1. | 100000 | 0 | 48.8281 | 97.6562 | 1.2031 | 9.1698 | 2.8522 | 439.453 | 1.4815 |
| 2. | 884.9553 | 59.6308 | 60.0629 | 60.4950 | 1.1783 | 9.6802 | 1.5695 | 420.872 | 1.9973 |
| 3. | 75.0786 | 14.9204 | 14.9571 | 14.9937 | 0.4161 | 9.9728 | 0.3823 | 29.6209 | 1.7514 |
| 4. | 961.9696 | 0 | 60.1231 | 120.246 | 8E-06 | 10.000 | 2E-05 | 420.862 | 2.0000 |

Now we calculate the number of the highest harmonic, $n_{max1}$, number of periods, $p_2$, and a new sampling frequency, $f_{S2}$, applying the formulae given in Table 2 and $M_2 = M_{max}$. Then we do FFT again.

$$n_{max1} = \frac{f_{max1}}{f_{1,1}} = \frac{439.453}{48.8281} = 9.000 \quad (5)$$

$$p_2 = Ent\left(\frac{M_2 - 2}{2n_{max1}}\right) = Ent\left(\frac{2048-2}{2 \cdot 9}\right) = 113 \quad (6)$$

$$f_{S2} = \frac{M_2 f_{1,1}}{p_2} = \frac{2048 \cdot 48.8281}{113} = 884.9553\,\text{Hz} \quad (7)$$

Table 2. Formulae for the FFT parameters calculation: $f_1$ – frequency of fundamental harmonic, $f_{max} = n_{max} f_1$ – frequency of maximal harmonic, that should be measured

| Name | Formulae | Value | |
|---|---|---|---|
| | | minimum | maximum |
| Number of samples | $M = 2^{Int} \geq M_{\min} = 2(n_{\max}+1)$ | $2(n_{\max}+1)$ | $M_{\max}$ |
| Number of spectrum components | $N = \frac{1}{2}M - 1 \geq n_{\max}$ | $n_{\max}$ | $\frac{1}{2}M_{\max} - 1$ |
| Number of periods in window | $p = Ent\left(\frac{N}{n_{\max}}\right) = \frac{f_1}{f_W} = \frac{M f_1}{f_S}$ | 1 | $Ent\left(\frac{M-2}{2n_{\max}}\right)$ |
| Sampling frequency | $f_S = \frac{M f_1}{p} = M f_W \geq \frac{M}{N} f_{\max}$ | $\xrightarrow[M>>M_{\min}]{} \frac{2M}{M-2} f_{\max}$ | $M f_1$ |
| Resolution of spectrum | $f_W = \frac{f_S}{M} = \frac{f_1}{p}$ | $\xrightarrow[M>>M_{\min}]{} \frac{2f_{\max}}{M-2}$ | $f_1$ |
| Width of spectrum | $f_B = N f_W = \frac{N}{M} f_S = \frac{N}{p} f_1 \geq f_{\max}$ | $f_{\max}$ | $\frac{M-2}{2} f_1$ |
| Number of measurable harmonics | $n_{MH} = Ent\left(\frac{f_B}{f_1}\right) = Ent\left(\frac{N f_S}{M f_1}\right)$ | $n_{\max}$ | $\frac{M-2}{2}$ |
| Oversampling factor | $\lambda = \frac{f_S}{2 f_{\max}} = \frac{M}{2 p n_{\max}}$ | $\xrightarrow[M>>M_{\min}]{} \frac{M}{M-2}$ | $\frac{M}{2n_{\max}}$ |

It was found that, after doing FFT, the amplitudes of adjacent lines differ more than 1‰ from that of the amplitude of the first harmonic. It can be argued that measurement should be continued. In order to determine precise the value of frequency $f_1$ we must increase spectral concentration by applying undersampling. We adjust the sampling frequency only 25% more than frequency $f_1$

$$f_{S3} = 2\lambda f_{1,2} = 2 \cdot \tfrac{5}{8} \cdot 60.0629 = 1.25 \cdot 60.0629 = 75.0786\,\text{Hz} \tag{8}$$

Undersampling reduces all harmonics in the low frequency range. The value of sampling factor $\lambda = 5/8$ has been chosen by the authors to be the most suitable because, due to aliasing phenomenon, in the neighborhood of the first harmonic 9th, 11th, 19th, 21st et cetera harmonics, appear and the amplitudes of these harmonics are small in proportion to the amplitude of the first harmonic. We then apply FFT once more. The measured amplitudes of adjacent lines we utilize to interpolation calculation of the frequency $f_{1U}$

$$\begin{aligned} f_{1U} &= f_{1-} + \frac{A_{1+}}{A_{1-} + A_{1+}}\left(f_{1+} - f_{1-}\right) = \\ &= 14.9204 + \frac{0.3823}{0.4161 + 0.3823}\left(14.9937 - 14.9204\right) = 14.9555\,\text{Hz} \end{aligned} \tag{9}$$

The real value of the frequency $f_{1,3}$ we achieve from formula

$$f_{1,3} = f_{S3} - f_{1U} = 75{,}0786 - 14{,}9555 = 60.1231\,\text{Hz} \tag{10}$$

To the last FFT we apply values according to formulae in Table 2

$$n_{\max 3} = \frac{f_{\max 2}}{f_{1,3}} = \frac{420.872}{60.1231} = 7.000 \tag{11}$$

$$M_4 \geq 2\left(n_{\max 3} + 1\right) = 2\left(7 + 1\right) = 16 = 2^4 \tag{12}$$

$$p_4 = Ent\left(\frac{M_4 - 2}{2n_{\max 3}}\right) = Ent\left(\frac{16-2}{2 \cdot 7}\right) = 1 \tag{13}$$

$$f_{S4} = \frac{M_4 f_{1,3}}{p_4} = \frac{16 \cdot 60.1231}{1} = 961.9696\,\text{Hz} \tag{14}$$

The results of the last FFT show reaching sufficient accuracy of the measurement: amplitudes $A_{1-}$ and $A_{1+}$ are lower than $10^{-3} A_1$.

The real parameters of the examined signal had been $A_1 = 10$ V, $f_1 = 60.1230$ Hz, $n_{max} = 7$, $A_7 = 2$ V, $f_7 = 420.861$ Hz. The measured values are the same with 5-digit accuracy.

Finally, on the base of the obtained amplitudes of individual harmonics, we can get the RMS value of voltage

$$V = \sqrt{A_0^2 + \frac{1}{2}\sum_{n=1}^{N} A_n^2} \tag{15}$$

## 6 CONCLUSIONS

The teaching set-up is used to students the basis of DSP and allows to perform more advanced functions especially that the wide range of signals can be generated.

The very important feature of the elaborated set-up is the friendly used operation. The students familiarise with the operation very fast and presented examples well present the theory of DSP.

The design and features of sampling virtual voltmeter for RMS, pick value, mean value and frequency and spectrum analyser are taught at the same set-up.

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