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SENSOR DIAGNOSTICS BASED ON DYNAMIC CHARACTERISTIC CURVE ESTIMATION

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Abstract – This paper presents a method for optimal detection of need for re-estimation of the parameters of characteristic curves based on infrequent and irregular reference measurements. We assume that the validity of parameters deteriorates through a stochastic process. In particular, we give analytic distributions of parameters when deterioration is through random-walk diffusion. The detection of need for re-estimation of parameters is similar to two-variable Statistical Process Control scheme.

Method is demonstrated and its detection capabilities analyzed in cases where the actual deterioration is either a step change, a linear drift or a random walk.

Keywords: Calibration, probability, diagnostics, Bayes, Statistical Process Control.

1. INTRODUCTION

The industrial practice of updating the characteristic curves of on-line measurements is rather casual. For example, in papermaking industry the parameters of fiber concentration measurements that determine the main material flows are changed to match the laboratory measurements simply by changing either the bias or the slope, and are done differently at different mills [1].

This article tackles re-estimating and dynamic updating of parameters in a characteristic curve of the sensor. This task can serve both continuous automatic tuning of characteristic curve parameters and diagnostics of sensors with fixed parameters.

2. PROBABILITY DENSITY FUNCTION FOR PARAMETERS OF THE CHARACTERISTIC CURVE

In this paper we analyze the dynamic validation of the parameters in a characteristic curve of a measurement. The characteristic curve relates the signal value from the on-line measurement and the estimate of the value of measurand through a model parameterized with vector $\overline{a}$. The validation of these parameters is based on that:

1) Originally the parameters are estimated from a large data set consisting of pairs signal values s and reference measurement values x, and

2) Occasionally we receive one or several – usually quite few – reference *validation* measurement values, associated to a signal value at a time instant, that is, a data set at t_1; $(s(t_1),\{x^{(i)}(t_1)\}_{i=1..n})$.

We address the question: when does the information accumulated from the validation measurements lead us to conclude that the original values of parameters are no longer valid [2]?

We adopt the Bayesian statistical point of view [3, 4, 5] on measurements. The results of the measurement and *our knowledge* on the measurand and on the properties (parameters) of the characteristic curve are all treated probabilistically, although the measurand and the characteristic curve can be considered as non-stochastic properties of reality. However, the Bayesian approach is fruitful due to its symmetry of updating the measurement properties when additional information about the measurand or characteristic curves is obtained, and of updating the estimates of measurand when new on-line measurement values become available.

The on-line measurement is characterized by the conditional distribution of signal values, given the true value of measurand and the parameters of the characteristic curve:

$$f^{(s)}(s\,|\,x_{true},\overline{a}) \tag{1}$$

In this paper we are interested in the probability distribution of the parameter vector $\overline{a}$, given the initial estimation of parameters and all pairs of simultaneous observations of the signal and the reference measurement, i.e. $\{s(t_j),\{x^{(i)}(t_j)\}_{i=1..n}\}_{j=1..m}$. the distribution of parameter vector develops in time through two mechanisms:

1) it changes discontinuously at time instants t_j when signal, reference value pairs, $s(t_j),\{x^{(i)}(t_j)\}_{i=1..n}$, become available;
2) between these time instants the distribution develops continuously through a mechanism describing the increase of uncertainty.

The updating of the distribution of the coefficients at time instants t_j, i.e. the mechanism 1) above, is given by Bayesian statistics as [4]

$$\begin{aligned} &f^{(a)}\left(\overline{a}(t_1)\,|\,s(t_1),\left\{x^{(i)}(t_1)\right\}_{i=1..n}\right)= \\ &Nf_{ap}^{(a)}(\overline{a}(t_1))\prod_{i=1}^{n} f^{(s)}(s(t_1)\,|\,x^{(i)}(t_1),\overline{a}(t_1)) \end{aligned} \tag{2}$$

Here N is a normalization constant, $f_{ap}^{(a)}(a(t_1))$ is the distribution of coefficients should there be no reference measurements at t_1.

Let time t_0 denote the preceding instant of updating of distribution of coefficient $\overline{a}$, according to reference data that became available. The mechanism 2), defines a continuous development

$$f^{(a)}\left(\bar{a}(t_0)\mid s(t_0),\left\{x^{(i)}(t_0)\right\}_{1=1..n}\right)\mapsto f_{ap}^{(a)}(\bar{a}(t_1)) \qquad (3).$$

Stochastic differential equations [6] are the natural framework for modelling information degradation, such as mechanism for Eq. (3). We shall consider equations of the form

$$\frac{d\bar{A}}{dt}=\bar{F}(\bar{A})+\bar{\Gamma}(t) \qquad (4)$$

where $\bar{A}$ is the stochastic vector of parameters of characteristic curve, $\bar{\Gamma}(t)$ is the (vector) white noise Gaussian process, uniquely determined through functions $<\bar{\Gamma}(t)>_{\Gamma}=0$; $<\Gamma_i(t)\Gamma_j(t')>_{\Gamma}=D_{ij}\delta(t-t')$

It can be shown [7] that the distribution of the stochastic dynamic variable $\bar{a}$, evolving according to Eq. (4), is determined through partial differential equation

$$\frac{\partial f(\bar{a},t)}{\partial t}=O_{\bar{a}}\left[f(\bar{a},t)\right] \qquad (5)$$

where O_a is a differential operator with respect to $\bar{a}$ and determined by the function $\bar{F}(\bar{a})$. If we have measurement information about $\bar{A}$ at time t_0 in the form of $f(\bar{a},t_0)$, we can solve Eq. (4) with this initial condition and obtain the measurement information about $\bar{A}$ at any time $t>t_0$. This is the mapping, Eq. (3), which together with updating with new reference data – Eq. (2) – determines the probability distribution of the parameter vector $\bar{a}$ at any time instant. In particular, the distribution provides us the maximum likelihood estimate of the parameter vector and the corresponding covariance.

The scheme of Eqs. (2-5) is rather difficult to implement for general characteristic curve, general distribution Eq. (1) and/or general degradation process $\bar{F}(\bar{a})$. From now on in this paper we discuss the case where characteristic curve is linear, the corresponding coefficient vector is two-dimensional, $\bar{a}=\begin{bmatrix}a_0 & a_1\end{bmatrix}^T$, the degradation is a pure diffusion process ($\bar{F}(\bar{Z})\equiv 0$) and the uncertainty described is normal. Then

$$f^{(s)}(s\mid x,\bar{a})=(2\pi(\sigma_s^2+\sigma_{ref}^2)a_1^{-2})^{-1/2}\exp\left[-\frac{(a_1 s+a_0-x)^2}{2\left(\sigma_s^2+\sigma_{ref}^2\right)}\right] \qquad (6),$$

where σ_s and σ_{ref} are the standard deviations due to uncertainty in the on-line measurement (with exact coefficients), and the reference measurement, respectively.

The distribution of coefficients after the initial calibration is well approximated by a two-dimensional normal distribution. With pure diffusion, the degradation is given as

$$\frac{\partial f(\bar{z},t)}{\partial t}=\sum_{i,j=0}^{1}D_{ij}\frac{\partial^2 f(\bar{z},t)}{\partial z_i\partial z_j} \qquad (7),$$

with $\bar{\bar{D}}$ as the diffusion matrix.

The information degradation, Eq. (7), with two-dimensional normal distribution is solved with a two-dimensional distribution with constant expectation values and covariance matrix increasing linearly in time. Furthermore, the distribution function after the reference measurement data at time t_1 has become available, Eq. (2), is a product of two-dimensional normal distributions, and of the same form itself. Therefore the distribution of interest, that of the parameters of the linear characteristic curve, can be calculated iteratively after each time instant that new reference measurements become available [2]:

$$\begin{aligned}&\left[\mu_0(t_0),\mu_1(t_0),\sigma_0(t_0),\sigma_0(t_0),\rho(t_0)\right]\\&\Rightarrow\left[\mu_0(t_1),\mu_1(t_1),\sigma_0(t_1),\sigma_0(t_1),\rho(t_1)\right]\end{aligned} \qquad (8).$$

The exact form of this recursion is given in Appendix. For detailed derivation, see [2].

3. DETECTION OF NEED FOR RE-ESTIMATION OF PARAMETERS

Let us assume that we use for the parameters of characteristic line the values obtained in initial parameter estimation, $\bar{a}'$. However, the analysis of Section 2 shows that our knowledge about the parameters, $f^{(a)}(a(t_1))$, is described by a two-dimensional normal distribution the parameters of which are updated according to Eq. (8) and Appendix. Mimicking Statistical Process Control, we ask if $\bar{a}'$ is abnormal with respect to $f^{(a)}(a(t_1))$. When the parameters are found abnormal, the need for their re-estimation has been established.

Using the properties of two-dimensional normal distributions, the abnormality set reads as

$$S(C,t_1)=\left\{\bar{a}':(\bar{a}'^T-\bar{\mu}^T(t_1))\bar{\bar{B}}(t_1)^{-1}(\bar{a}'-\bar{\mu}(t_1))>C\right\} \qquad (9a)$$

with

$$\bar{\bar{B}}(t_1)=\begin{bmatrix}\sigma_0(t_1)^2 & \rho(t_1)\sigma_0(t_1)\sigma_1(t_1)\\ \rho(t_1)\sigma_0(t_1)\sigma_1(t_1) & \sigma_1(t_1)^2\end{bmatrix} \qquad (9b)$$

The coefficient C in (9a) is determined through the probability of detection error, p_0 - the probability that $\bar{a}'$ is not abnormal although condition (9a) finds it abnormal. The function $C=C(p_0)$ is defined through

$$\int_{\bar{a}\in S(C,t_1)}f^{(a)}(\bar{a}(t_1))da_0da_1=p_0 \qquad (10a).$$

This solves for time-independent C as

$$C = 2 * \log(1/p_0) \qquad (10b)$$

Hence our method of determining the validity of parameters of characteristic curves consists of updating the distribution parameters of *a* through (8) and then applying the abnormality detection (9a), with limits (10b).

4. SIMULATION

In this chapter, we shall analyze the performance of parameter updating of our algorithm with simulated cases. Also the detection capabilities of this algorithm are analysed. These simulations include the response of the parameter updating algorithm to fault situations, demonstrating potential and actual situations happening in real factory.

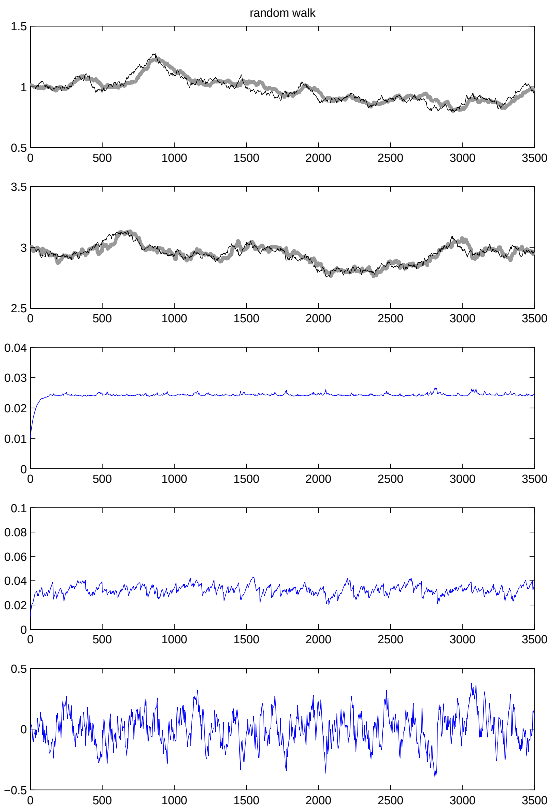

Figure 1. Updating of distribution parameters, assuming diffusion parameter D_{00}=0.00001, D_{11}=0.00003, D_{12}=0. Original parameters a_0 =1, a_1 = 3. Top: μ_0 (grey) and true (simulated) a_0 (black). Second from top: μ_1 (grey) and true (simulated) a_1 (black). Middle: σ_0^2. Second from bottom σ_1^2. Bottom: ρ.

Figure 1 shows a simulated example of updating parameters when the true parameters undergo a diffusion. Original estimation of parameters at $t = 0$ is based on 100 signal-reference observation pairs. The reference values become available individually at irregular intervals evenly distributed between 2 and 4 time units. The diffusion coefficients for parameter estimation are selected as 0.00001 and 0.00003 for diagonal elements and 0 for off-diagonal.

The simulation results show that the estimated parameters follow the true parameters rather closely although the diffusion in true parameters is about 10 times faster than assumed in the parameter updating.

The uncertainty of the estimates increase in the beginning of the simulation as original estimation is based on 100 observation pairs, whereas only some 30 reference measurements become available in 100 time units. Smaller variances would be obtained with smaller diffusion coefficients in the updating. However, then the method would not be able to track the diffusion of true parameters. This can be seen at figure 2, where diffusion coefficients are one decade smaller.

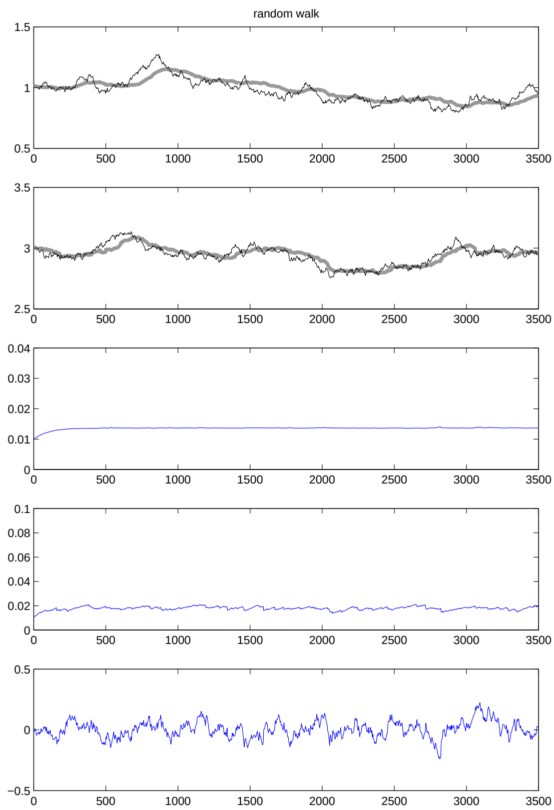

Figure 2. Updating of distribution parameters, assuming diffusion parameter D_{00}=0.000001, D_{11}=0.000003, D_{12}=0 Original parameters a_0 =1, a_1 = 3. Top: μ_0 (grey) and true (simulated) a_0 (black). Second from top: μ_1 (grey) and true (simulated) a_1 (black). Middle: σ_0^2. Second from bottom σ_1^2. Bottom: ρ.

We simulated the detection capability of our algorithm in four cases. The time series of the test variable of the cases are shown in figures 1 and 2. The cases were:

- In the first case the true coefficients (a_0 =1, a_1 = 3) were constant throughout the analysis period.

- At the second case both true coefficients undergo a random walk, such that the standard deviation of per unit time of the random step size is 0.003. An example of the random walk is shown in figure. 1. (Same random walk is in second from top in figure 3.)

- At the third case the true coefficient are constant till time t=1060 and then start to drift at the rates of + 0.0002/time unit in a_0 and -0.0002/time unit in a_1.

- And the fourth case, where step change occurs at time t=1745. True coefficients change (a_0) from 1 to 1.5 and (a_1) from 3 to 2.5.

As in previous simulation example the original estimation of parameters at $t = 0$ is based on 100 signal-reference observation pairs. In each of these cases we generated pairs of reference values obeying the instantaneous values of the coefficients and added a random noise of variance 0.01 to the relationship. Such reference pairs became available at random intervals uniformly distributed between 2 and 4 time units.

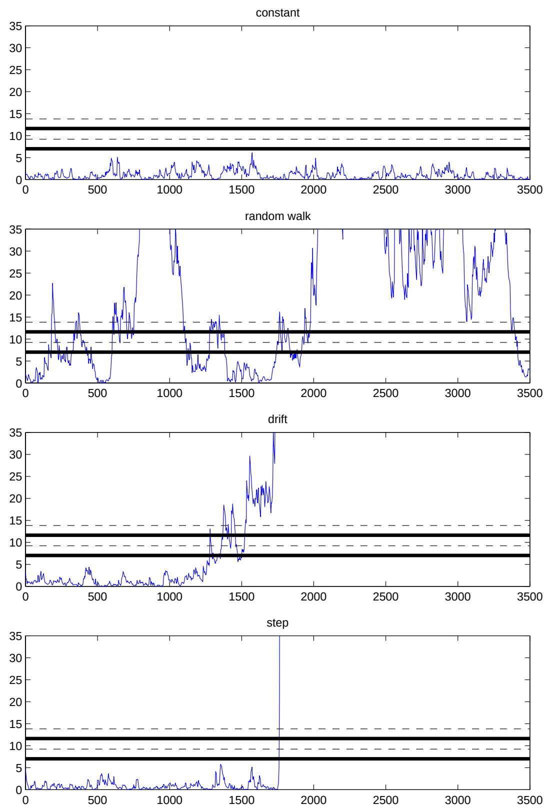

Figure 3. Detection of abnormalities at characteristic curve parameter estimation, assuming diffusion parameter D_{00}=0.00001, D_{11}=0.00003, D_{12}=0. From top to bottom: no change in calibration parameters, random walk, linear trend and step change.

The detection limits were calculated with Eq. 10b, p_0 getting values 0,03 % (bottom black line), 0,01 % (bottom black dashed line), 0,003 % (top black line) and 0,0001 % (top black dashed line).

In figures 3 and 4 are the same data set for simulation; the only difference is at the diffusion coefficients. On figure 3 the diffusion coefficients for parameter estimation are selected as 0.00001 and 0.00003 for diagonal elements and 0 for off-diagonal and on figure 4 as 0.000001 and 0.000003 for diagonal elements and 0 for off-diagonal, respectively.

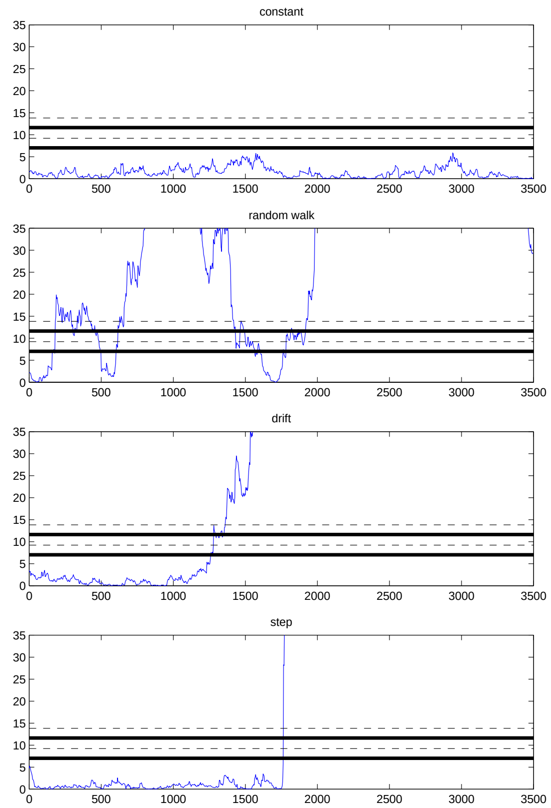

Figure 4. Detection of abnormalities at characteristic curve parameter estimation, assuming diffusion parameter D_{00}=0.000001, D_{11}=0.000003, D_{12}=0. From top to bottom: no change in calibration parameters, random walk, linear trend and step change.

The abnormality test signal was calculated with (9a) and it was compared to the detection limits.

Detection algorithm finds step change fast and reliable (see also figure 5). Abnormality test signal is increasing about 10 time units later than step change in true coefficients occurred and all limits are crossed about 15 time units later compared to change. With smaller diffusion coefficients the detection is couple of time units slower.

At the case of linear drift, abnormality test signal begins increasing about 50 time units later than drift starts. All limits are crossed about 300 time units later compared to change in true coefficient. With smaller diffusion coefficients the detection is slightly slower but there is less noise in the test signal

The difference in diffusion coefficients seen in figures 1 and 2 can easily be seen at abnormality test signal in random walk case. In the case of larger diffusion coefficient the estimated parameters response is much faster compared to the small diffusion coefficients.

Case where true coefficients were constant, if the limits are too tight the rate of false alarms increases depending on the random part and measurement uncertainty. With smaller diffusion coefficients the abnormality test signal is more tranquil.

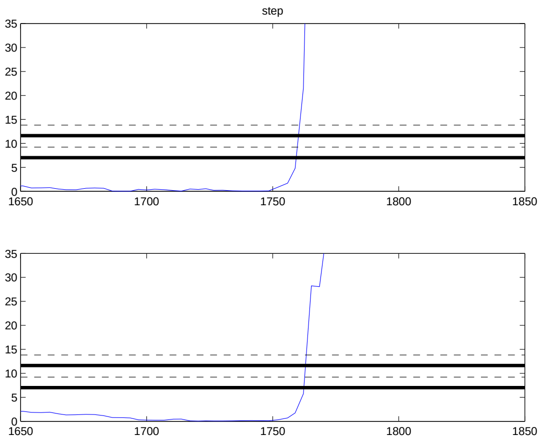

Figure 5. Enlargement of the step change detection from figure 3 (upper) and figure 4 (lower).

5. CONCLUSIONS

This paper discussed the dynamic estimation of the parameters of characteristic curves and presented a method for this parameter updating.

The scheme is based on Bayesian statistical approach to measurements and is using infrequent and irregular reference measurements to update the calibration parameters. Detection of abnormalities was simulated as well as updating the parameters of characteristic curve.

By comparing the current characteristic curve parameters at the sensor and the estimated parameters $\overline{a}$

- the need of re-estimation of the parameters can be detected,
- early warnings system for operators or engineers can be generated,
- parameters can be tuned/updated continuously,
- on-line sensor faults can be detected and isolated.

With aid of this tool the sensor will work more accurately, thus the whole process is more efficient.

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APPENDIX

After information in reference measurements at τ_1 has been taken into account, the joint distribution of the parameters of linear characteristic curve are given by:

$$f(a_0,a_1;\tau_1)=\left[2\pi\sigma_0(\tau_1)\sigma_1(\tau_1)\left((1-\rho(\tau_1)^2)\right)^{1/2}\right]^{-1}$$
$$*\exp\left[-\frac{1}{2(1-\rho(\tau_1)^2)}\left\{\frac{(a_0-\hat{a}_0(\tau_1))^2}{\sigma_0(\tau_1)^2}-2\rho(\tau_1)\frac{(a_0-\hat{a}_0(\tau_1))(a_1-\hat{a}_1(\tau_1))}{\sigma_0(\tau_1)\sigma_1(\tau_1)}+\frac{(a_1-\hat{a}_1(\tau_1))^2}{\sigma_1(\tau_1)^2}\right\}\right] \quad \text{(A 1a)}$$

With

$$\sigma_i(\tau_1-)^2=\sigma_i(\tau_0)^2+D_{ii}*((\tau_1-)-\tau_0) \quad \text{(A 1b)}$$
$$\rho(\tau_1-)\sigma_1(\tau_1-)\sigma_2(\tau_1-)=\rho(\tau_0)\sigma_1(\tau_0)\sigma_2(\tau_0)+D_{12}((\tau_1-)-\tau_0)$$

with the parameters related through

$$\frac{1}{(1-\rho(\tau_1)^2)\sigma_0(\tau_1)^2}=\frac{1}{(1-\rho(\tau_1-)^2)\sigma_0(\tau_1-)^2}+\frac{n}{\sigma_m^2}\equiv a$$
$$\frac{1}{(1-\rho(\tau_1)^2)\sigma_1(\tau_1)^2}=\frac{1}{(1-\rho(\tau_1-)^2)\sigma_1(\tau_1-)^2}+\frac{nz_4}{\sigma_m^2}\equiv b$$
$$\frac{\rho(\tau_1)}{(1-\rho(\tau_1)^2)\sigma_0(\tau_1)\sigma_1(\tau_1)}=\frac{\rho(\tau_1-)}{(1-\rho(\tau_1-)^2)\sigma_0(\tau_1-)\sigma_0(\tau_1-)}-\frac{nz_1}{\sigma_m^2}\equiv c \quad \text{(A 1c)}$$
$$\frac{\hat{a}_0(\tau_1)}{(1-\rho(\tau_1)^2)\sigma_0(\tau_1)^2}-\frac{\rho(\tau_1)\hat{a}_1(\tau_1)}{(1-\rho(\tau_1)^2)\sigma_0(\tau_1)\sigma_1(\tau_1)}=$$
$$\frac{\hat{a}_0(\tau_0)}{(1-\rho(\tau_1-)^2)\sigma_0(\tau_1-)^2}-\frac{\rho(\tau_1-)\hat{a}_1(\tau_0)}{(1-\rho(\tau_1-)^2)\sigma_0(\tau_1-)\sigma_1(\tau_1-)}+\frac{nz_2}{\sigma_m^2}\equiv d$$
$$\frac{\hat{a}_1(\tau_1)}{(1-\rho(\tau_1)^2)\sigma_1(\tau_1)^2}-\frac{\rho(\tau_1)\hat{a}_0(\tau_1)}{(1-\rho(\tau_1)^2)\sigma_0(\tau_1)\sigma_1(\tau_1)}=$$
$$\frac{\hat{a}_1(\tau_0)}{(1-\rho(\tau_1-)^2)\sigma_1(\tau_1-)^2}-\frac{\rho(\tau_1-)\hat{a}_0(\tau_0)}{(1-\rho(\tau_1-)^2)\sigma_0(\tau_1-)\sigma_1(\tau_1-)}+\frac{nz_3}{\sigma_m^2}\equiv e$$

And z_i as the well-known square sums of the new observation pairs

$$z_1=\frac{1}{n}\sum_{j=1}^{n}s^{(j)}(\tau_1)$$
$$z_2=\frac{1}{n}\sum_{j=1}^{n}x^{(j)}(\tau_1) \quad \text{(A 1d)}$$
$$z_3=\frac{1}{n}\sum_{j=1}^{n}x_{lab}^{(j)}(\tau_1)s^{(j)}(\tau_1)$$
$$z_4=\frac{1}{n}\sum_{i=1}^{n}\left(s^{(j)}(\tau_1)\right)^2$$

In Eqs. (A 1c) all the right hand sides are known. The updated distribution parameters can be solved exactly as in the standard maximum likelihood parameter estimation of joint normal distribution- with the short-hand notation introduced in Eq. (A 1c) – as

$$\rho(\tau_1)=\frac{c}{\sqrt{ab}}$$
$$\sigma_0(\tau_1)^2=\frac{b}{ab-c^2}$$
$$\sigma_1(\tau_1)^2=\frac{a}{ab-c^2} \quad \text{(A 2)}$$
$$\hat{a}_1(\tau_1)=\frac{ea+cd}{ab-c^2}$$
$$\hat{a}_0(\tau_1)=\frac{d+c\hat{a}_1}{a}$$