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Error propagation and signal post-processing in rocket-based experiments

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Abstract- This paper describes the error treatment of REXUS rocket-borne measurements of ion density in the middle atmosphere, using so called Gerdien condenser. The main challenge is that measurements in the range of 25-85 km can be carried out only with a sounding rockets or balloons. Therefore not only the measurement time is limited but a lot of noise distorts the data. Another problem is that the measurement model cannot be validated accurately due to the hard accessibility, so apart from a few exceptions we had to rely on probabilistic estimates. Uncertainties and the influencing effects distorting the measurement or endangering the device were estimated, with special attention to environmental effects like shock waves, radiation, photoelectric effect, etc. Finally a full error propagation model has been established and validated through simulations.

Keywords- Gerdien condenser, Ion mobility, Middle atmosphere, Uncertainties

I. Introduction

Rocket-borne experiments are very sensitive to uncertainties due to the short measuring time and fairly unpredictable environment. Moreover every particular measurement setup requires a different perspective, being highly dependent on the actual situation like the placing of the measuring device on the rocket body, the altitude where the measurements are carried out, etc. To perform a meaningful error analysis first all the potential sources of uncertainties must be identified and evaluated, then quantitatively most serious errors should be fused into the compound measurement error acc. to the data processing required by the measurand. The specific experiment described in this paper was the Gerdien condenser based ion density-, and mobility measurement [1]. It was organized by the European Space Agency(ESA), Swedish National Space Board(SNSB) and Deutsche Zentrum für Luft- und Raumfahrt (DLR) and carried out in 2013 at the Esrange Space Center in Sweden. The experiment aimed at measuring ion- and electron densities in the middle atmosphere in the 25-85 km altitude range. A few such measurements and uncertainty analysis had been made in the past [2,3,4]. The main novelty of this paper is that a full error propagation model has been developed which can be applied to various kinds of measurement setups and can be easily extended to other measurements of this kind. The model takes into account the possible distortion effects, and according to the specific environmental parameters e.g. daytime, weather, etc. it can easily be adjusted to provide a valid error estimate.

A. Experiment setup

The measuring device can be seen in Fig. 1. It is a cylindrical condenser with an inner and an outer electrode. When a bias voltage is applied to the electrodes, the positively and negatively charged particles entering the condenser space are deflected and when they reach the inner electrode a current can be measured.

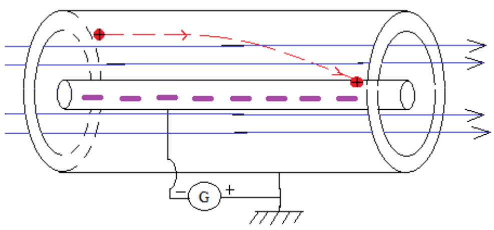

Figure 1. Gerdien condenser: positively charged ions (red), and the neutral gas flow (blue)

From the measured current values one can determine the ion and electron mobilities and densities. The physical background is discussed in more details in the next section. Two condensers had been constructed to measure ions and electrons separately. Acc. to the polarity of the applied voltage they capture positively or negatively charged particles (e.g. if the inner electrode is positive with respect to the outer electrode, the negatively charged particles are affected and the negative current related to the electron density and mobility is measured). For this purpose two precision amplifiers are needed, to measure positive and negative currents with high precision. The two condensers were placed on the opposite sides of the rocket. Due to the special flight conditions the condensers are made from a highly heat resistant material. The precision amplifiers had to measure currents in the range of pA to nA, so a very careful placement and very low noise level was needed. The whole system and the measuring method are described in details in [5].

B. Preliminaries and the physical background

Consider a charged particle of mass m_i, and charge q_i which moves in an electric field of strength E. We can define the particles mobility μ_i which measures the ability of the charged particle to move through the medium in response to an electric field. From the applied voltage and the speed of the flow we can define a critical mobility, which means that if an ion has a mobility smaller than the critical value, it cannot be collected by the cathode and thus it does not contribute to the measured current. The expression for the critical mobility can be seen in Eq. 1.

$$\mu_c = \frac{(b^2 - a^2)\ln(b/a)}{2VL} u \qquad (1)$$

where V is the applied voltage, L is the length of the inner electrode, b is the radius of the outer cylinder, a is the radius of the inner cylinder, and u is the speed of the flow in the condenser. The voltage used is derived from the the critical mobility in a way that we first determine which are the main ions in the specific atmospheric regions [6], then from their estimated mobility, the flow speed and the geometry we calculate the required voltage needed to collect the estimated maximal number of ions of each kind. Though these are only estimates, they are required because the current estimate was necessary to design a good measuring device. The measured current can be expressed as follows:

$$I = \frac{2\pi eVL}{\ln(b/a)} \sum_i \mu_i n_i \qquad (2)$$

where e is the elementary charge, n_i is the particles charge density, and μ_i is the mobility. We summed up the generated currents for every ion group present. From the estimated critical mobilities straightforward calculations (see paragraph above) yielded voltage range of 0-60 V, and the current in pA-nA range. 128 measurements/sec are carried out. In the first second a specific pre-calculated voltage sub-sweep is applied e.g. 12-20V, then in the remaining four seconds a full 0-60 V sweep is carried out. 5 seconds measurement means one characteristics related to a specific altitude range, spanning approximately 5 km-s. From the measured currents we obtain a full mobility spectra in the 25-85 km altitude range.

C. Overview of the distortion effects during the flight

The actual measured characteristics can differ greatly from the theoretical one due to a number of effects distorting the measured current. These will be reviewed in this section and the relevant ones are discussed in the subsequent sections in more detail. Uncertainties can be separated into the environmental and the instrumentation related uncertainties. The environmental uncertainties can come from radiation (sun, telemetry), rocket movement (high speed, rotational stability), temperature changes, recombination, scattering, space charge effects, turbulence, coronal effect, additional ionisation etc. The instrument noises come from the cables, precision amplifier, power supply, AD converter etc. We also had to examine which parameters are affected in the expression of the current (Eq.1) so that the GUM (Guide to the expression of uncertainty in measurement) [7] method can be used to evaluate the magnitude of the errors. To construct a confidence interval we also determined the pdf (probability density function) of the measured quantities which is also necessary if we want to construct a Maximum Likelihood estimator.

II. Uncertainties

A. Photoelectric effect

In our case the photo-emission will affect the measurement if an electron is emitted in time when the current is generated. For the radiation spectrum we consider an ideal black body spectrum with the temperature of 6000 K and for the atmospheric absorption we used the Beer-Bouguer-Lambert law [8] to determine the actual spectrum at different altitudes. The necessary parameters e.g. absorption cross section, and neutral gas concentration are available from previous measurements [9,10]. To know the value of the photocurrent we have to calculate Eq. 3. which describes the proportionality between the total radiated power and the photo current.

$$I = e\int_{\lambda 1}^{\lambda 2} C(\lambda)\frac{8\pi Ac}{\lambda^4}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}\exp\left(-\sum_j \sigma_j^a N_{j0}\int_h^{\infty} e^{-\frac{h'}{H_j}}\sec(\kappa)dh'\right)d\lambda \qquad (3)$$

where λ is the wavelength, e is the elementary charge, k_B is the Boltzmann constant, c is the speed of light, T is the temperature, σ_j^a is the absorption cross section of the j-th particle, N_{j0} is the particle density, h is the altitude, H_j is the scale height of the *j-th* particle ($Hj=kT/m_jg$), A is the surface of the condenser, κ is the zenith distance, and $C(\lambda)$ is the wavelength dependent electron emission coefficient, which can be estimated or measured for the specific material. Fig. 2. shows the estimated photo currents for the different altitude ranges. From the calculated results it is clearly seen that this effect is negligible under 60 km-s but above that it has a possibility that it will generate additive currents. However this is a very uncertain effect because we don't know exactly the power of the incoming radiation.

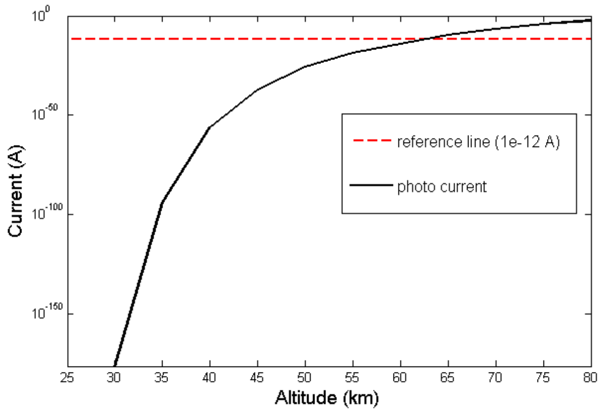

Figure 2. Generated photocurrent with altitude: The dashed line represents the minimal current that we can measure, therefore the photo current above that will distort the measurement.

B. Low frequency radiation

The inner electrode works as an antenna for electromagnetic waves so when electromagnetic wave reaches the inner electrode a current proportional to the amplitude of the radiation can be measured. The estimated current can be expressed analytically for a perpendicularly incident radiation with scattering to a cylinder of a small radius and thus can be calculated easily from Eq. 4.

$$I = -2\pi i E_0 R\sqrt{\frac{\varepsilon}{\mu}}\left(J_1(kR) - J_0(kR)\frac{H_1^{(1)}(kR)}{H_0^{(1)}(kR)}\right) \qquad (4)$$

where E_0 is the amplitude of the incident electric field, k is the wavenumber, R is the radius of the inner electrode, i is the imaginary unit, ε is the permittivity, μ is the permeability, J_i is the Bessel function of the first kind, and $H_0^{(1)}$, and $H_1^{(1)}$ are the Hankel functions of the first kind. The precision amplifiers cut off higher frequencies (above 160Hz), consequently the 2.5 GHz telemetry and the sun radiation do not contribute with measurable additional currents. However Extreme Low Frequency (ELF) waves are often formed in the middle atmosphere and because the ionosphere acts like a cavity resonator, this effect cannot be neglected.

C. Shock waves

Due to the rockets supersonic speed we have had to take into account the shockwaves and the turbulence effects in the flow. These can easily affect the measurement as the shock formation will push out the lighter ions and electrons. This effect is discussed in more details and also verified experimentally in one of the mentioned previous works. Mostly the electron density will be affected so the preliminary conclusion was that the measured negative current will be smaller due to the electron losses. We included this phenomenon in the probabilistic model because the effect can be uncertain due to the nonlinear behaviour of the flow. To estimate the electron loss numerical simulations were performed based on the Navier-Stokes equation. Fig. 3. shows the probability density function of the estimated electron loss.

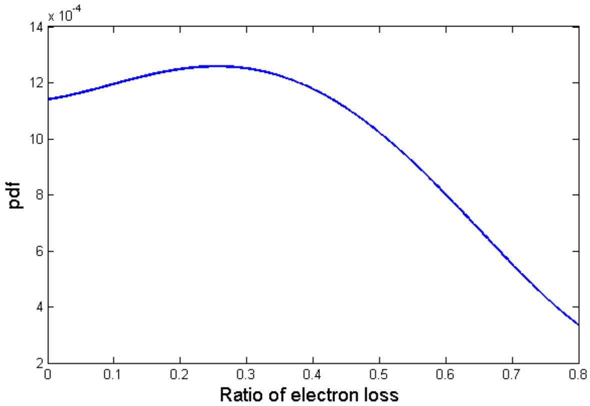

Figure 3. Probability density function of the estimated electron loss ratio due to shock waves

From this result we can calculate the number of electrons that will be lost during a measurement because we can also calculate the ratio of the flow loss in the pipe due to shock waves. From the estimated electron loss the estimated current drop can be calculated with pdf transformation.

D. Space charge effects

Particles with different charges are separated in the condenser due to their different mobility, so the electric field generated by them can distort the applied field for measurement. This can result in a voltage change when the separated particle number is high enough. It can be estimated if we solve the Poisson equation for a definite number of ions present. Considering that we use cylindrical geometry, only the radial dimension is of interest. Therefore:

$$V = V_0 \frac{\ln(r/a)}{\ln(b/a)} + (b^2 - a^2)\frac{N_i e}{4\varepsilon_0}\left(\frac{\ln(r/a)}{\ln(b/a)} - (r^2 - a^2)\right) \tag{5}$$

where V_0 is the applied voltage, r is the radial distance, b is the radius of the outer electrode, and a is the radius of the inner electrode, N_i is the density of ions, ε_0 is the vacuum permittivity and e is the elementary charge. This effect can affect the applied voltage, and thus the measured current. However we also measure the applied voltage so we will exactly know its value, apart from the uncertainties of the measuring method. We used the latter option so the errors of the voltage measurement will be discussed in more details in the next section.

E. Precision amplifier errors

Calculations showed that the measured currents are ca. in pA-nA range so very low noise precision amplifiers are needed. However it is not possible to construct a measuring device without some noise especially when we are measuring such low currents. The errors can be separated into four error sources: slope and bias error in the measured value of the adjusted voltage, stochastic noise on the currents, bias errors on the currents, and voltage and current drifts. The stochastic noise follows a normal distribution which is proven with hypothesis tests from several measurements. The nonlinear bias current can be considered a simple bias for a specific voltage level, assuming that the bias is changing only due to the voltage level, so only simple corrections are needed. These correction are different for each amplifier. We can avoid voltage error caused by the space charge effect if we measure also the voltage besides measuring the currents. The disadvantage is that it also has some uncertainty due mainly to a slope error. The pdf of the instrumentation noises are assumed normal which is again proven by hypothesis tests.

III. Error propagation model and confidence intervals

We used the GUM method to establish the error propagation model for the whole system including the environmental effects. Summarising we have the following distorting effects: photoelectric effect, shock waves, low frequency radiation, bias voltages, bias currents, voltage drift, current drift, voltage slope error and stochastic noise. The whole measured current for one ion type can be expressed as follows:

$$I = \frac{(n - n_{shock}) eL\mu 2\pi}{\ln(b/a)} \left[V(1+\beta) + \alpha_V \Delta T + V_{off} \right] + h_{rand} + \alpha_I \Delta T + I_{off} + I_{photo} + I_{rad} \qquad (6)$$

where n_{shock} is the number of particle loss, β is the slope error coefficient of the voltage measurement, α_V is the voltage measurements drift coefficient, α_I is the current measurements drift coefficient, I_{photo} is the current error due to photoelectric effect, I_{rad} is the current error due to the low frequency radiation, I_{off} is the current measurements offset, and h_{rand} is the stochastic noise. From this we can express the uncertainty of the current via. simple derivatives and a correlation analysis, where the latter was established by simulations. As our task is to determine the densities and the mobility spectra from the measured current the uncertainties of these can be expressed as follows:

$$I_{i,sat} = n_i evA \qquad \Rightarrow \qquad \Delta n_i = \frac{\Delta I_{i,sat}}{evA} \qquad (7)$$

$$G_i = \frac{2\pi n_i e L \mu_i}{\ln(b/a)} \qquad \Rightarrow \qquad \Delta\mu_i = \frac{\ln(b/a)}{2\pi eL} \left[\frac{1}{n_i} \Delta G_i - \frac{G_i}{n_i^2} \Delta n_i \right] \qquad (8)$$

where $I_{i,sat}$ is the saturation current, G_i is the conductance *(I/V)*, n_i is the ion concentration, *A* is the surface of the inner electrode, and *v* is the airflow speed at the inlet. The *i* subscript refers to the *i-th* kind of particle and the Δ means the uncertainty. The left side of Eq. 7. and Eq. 8. represents the quantities that we can get from the voltage-current characteristics because the saturation current is related to the break points, and the conductance is related to the slope of the curve. From the saturation current we can express the particle density and from the conductance we can get the mobilities of the particles. To calculate their uncertainties we must only know the uncertainties of the measured current and the measured voltage. The last step was to determine the confidence intervals. For this we had to estimate the probability density function of the whole measuring system for 12 different altitude ranges (or we must handle a non-stationary pdf which changes regarding to the altitude), which describes the total uncertainty of the system. To this purpose we used Monte Carlo simulations to get the pdf of the measured current taking into account all the measurement uncertainties. From the estimated pdf we can construct the confidence intervals for a given probability.

IV. Numerical simulations

To validate our model simulations with the presence of two different type of ions were made. The simulated results can be seen in Fig. 4. where the calculated confidence intervals (95%) are also shown around the measured currents.

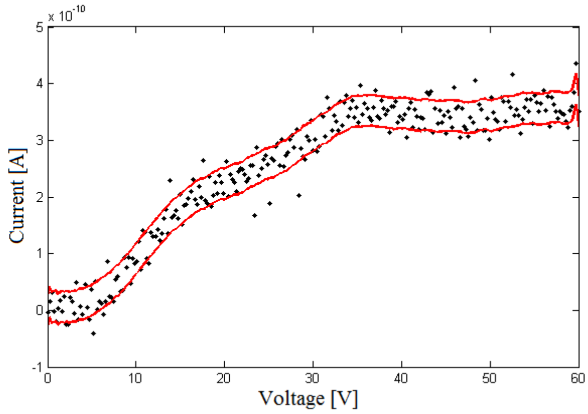

Figure 4. Simulated U-I characteristic and calculated confidence intervals with two ions present

The initial conditions and the results with the uncertainties are shown in Table 1, where n_{init} is the true ion density, μ_{init} is the true mobility, n_{est} and μ_{est} are the estimated values and Δn_{est}, $\Delta\mu_{est}$ are the uncertainties of the estimated values. A very good match was obtained with the initial conditions of the simulations.

	n_{init} [m^{-3}]	μ_{init} [m^2/Vs]	n_{est}	Δn_{est}	μ_{est}	$\Delta\mu_{est}$
Ion 1	7×10^{9}	1.3×10^{-2}	6.38×10^{9}	6.34×10^{8}	1.32×10^{-2}	3.94×10^{-5}
Ion 2	8.8×10^{9}	6.2×10^{-3}	8.79×10^{9}	6.02×10^{8}	6.2×10^{-3}	3.18×10^{-4}

Table 1. Calculated results with two ions present

The validation showed us that the model is reliable and thus it is good enough to express the measurement uncertainties however more detailed experiments are needed to fully validate our model. In 2015 the experimental setup is scheduled to fly again, with more sensors to make more accurate measurements.

V. Conclusions

In this paper we discussed a full error propagation model of a Gerdien condenser based rocket borne experiment for measuring air ion density at the altitudes between 25-85 km. The important environmental and system noises were discussed in detail and a full GUM uncertainty model was presented. The uncertainty model has been validated through numerical simulations. The importance of the results of this paper lies in that it provides an understanding of the physical system during a rocket flight and can be extended to other types of measurements where the environment is hardly accessible and a proper validation is not feasible.

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References

[1] Hashem Farrokh, *Design of a Simple Gerdien Condenser for Ionospheric D-region Charged Particle Density and Mobility Measurement*, Scientific report, The Pennsylvania State University, 1975

[2] Sherman. C, Bailey. A.D., Borghetti. J.A, Gerdien condenser system for measuring stratospheric charged particle densities, Instrumentation Papers, Oct. 1979 - Oct. 1983 Air Force Geophysics Lab., Hanscom AFB, MA., 02/1984

[3] D. A. Burt, *The Development of a Gerdien Condenser for Sounding Rockets*, Upper Air Research Laboratory, University of Utah, Scientific Report No. 8, May 1967

[4] Ion Losses in the Gerdien Condenser Intake System, Journal of Applied Meteorology, Volume 7, 456-458, 1968

[5] Balassa Gábor, Váradi Zsolt, Lőrinczi Ottó Botond, Bencze Pál, "*Rocket Borne Experiment for the Variation in Electric Conductivity with Altitude"*, 21st ESA Symposium on European Rocket & Balloon Programmes and Related Research, Thun, Switzerland, June 9-13, pp. 519-526, 2013.

[6] Beig G., Global change induced trends in ion composition of the troposhere to the lower thermosphere, Annales Geophysicae, 26. pp. 1181-1187, 2008

[7] Evaluation of measurement data - Guide to the expression of uncertainty in measurement, JCGM, 2008, http://www.bipm.org/utils/common/documents/jcgm/JCGM_100_2008_E.pdf

[8] J. D. J. Ingle and S. R. Crouch, *Spectrochemical Analysis*, Prentice Hall, New Jersey, 1988.

[9] Richards, P.G.Fennely,J.A. and Torr,D.G., EUVAC:A solar EUV flux model for aeronomic calculations. J.Geophys.Res., 99, 8981, 1994.

[10] U.S. Standard Atmosphere, U.S. Government Printing Office, Washington, D.C., 1976