Failure rates sensitivity analysis using Monte Carlo simulation

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***Abstract*- This paper is focused on sensitivity analysis achieved on system reliability: sensitivity assessment allows putting in discussion nominal values taken from a generic database and focus on uncertainty quantification and propagation of uncertainty from input to output. There are many sources of uncertainty that can affect input data and this uncertainty imposes a limit on our confidence in the response of the model.**
In this paper sensitivity analysis is achieved on GE Oil&Gas mineral lube oil console, a system forced to endure extreme process and environmental conditions.
In order to have a wide sample survey Monte Carlo simulation is used supposing two different probability distributions (uniform and normal) for failure rates coming from OREDA Offshore Reliability Data Handbook.

Keywords: Sensitivity analysis, Reliability assessment, Reliability Block Diagram (RBD), Oil & Gas applications, Gas turbine auxiliary systems, Monte Carlo simulation, Uniform distribution, Normal distribution

I. Introduction

Modern technologies and business requirements lead to a growth in manufacturing product complexity and miniaturization of components: this trend increased number and variety of failures and for this reason the interest in RAMS (Reliability, Availability, Maintainability and Safety) and diagnostics parameters is growing in many different manufacturing fields, in particular for *Oil&Gas* applications where products are forced to endure extreme process and environmental conditions.
This paper is focused on *GE Oil&Gas* gas turbine auxiliary systems with the aim of assess sensitivity analysis of reliability outcomes: reliability parameters are achieved with a dedicated tool named *RBDesigner* that semi-automatically generates a Reliability Block Diagram (RBD) starting from the *P&ID schemes* (sketch of thermal-hydraulic systems) and provides the most important reliability parameters such as reliability vs. time, hazard rate vs. time and MTTF [1, 2].
Sensitivity analysis, instead, is achieved using Crystal Ball® software and Monte Carlo simulation with failure rates from OREDA Offshore Reliability Data Handbook [3].

II. Miner lube oil console

One of the most important gas turbine auxiliary systems is mineral lube oil console: mineral oil is used to reduce friction and fatigue between moving surfaces (e.g. bearings); for this reason the efficiency of the oil console is critical for the proper workability of the whole gas turbine. The lube oil console RBD is shown in Figure 1. This diagram contains all the items required for the proper operation of the system and it contains both mechanical and electronic items. Failure of any block of the diagram produces entire system fault [1, 2]
The sub-system that mostly affects system reliability is the pumps section: there are two branches containing one pump each (main and auxiliary pump) and each pump is supplied by two electrical motors (main and standby motor).
These motors are in cold standby architecture because only the main one is in use; the other motor is disconnected from power supply until main failure occurs.
The pump branches, instead, can be considered in hot standby architecture; the response-time required in case of failure of the main pump is very short and for this reason the standby branch can be considered always operative.
Reliability function used for cold standby architectures is shown below [1, 2], [4-9]:

$$R_S(t) = R_1(t) + (1-p)\cdot\int_0^t f_1(x)\cdot R_{2,a}(t-x)\cdot dx \quad (1)$$

where we have R_s reliability of the system, R_1 reliability of the active component (main) p switch failure probability, f_1 pdf of the active component, $R_{2,a}$ reliability of the standby component in active mode and x time of main failure and further standby activation.

Reliability prediction for the system under analysis was achieved using *RBDesigner*, a tool developed to achieve reliability prediction in the early product design stages: this tool gives a reliability feedback to design engineers to reduce re-design costs and time for improvements using failure rates coming from OREDA Offshore Reliability Data Handbook [3].
Starting from the sketches of the thermal-hydraulic system the user can build a Reliability Block Diagram and calculate reliability parameters: these outcomes are used to assess sensitivity analysis on failure rates coming from the chosen database.

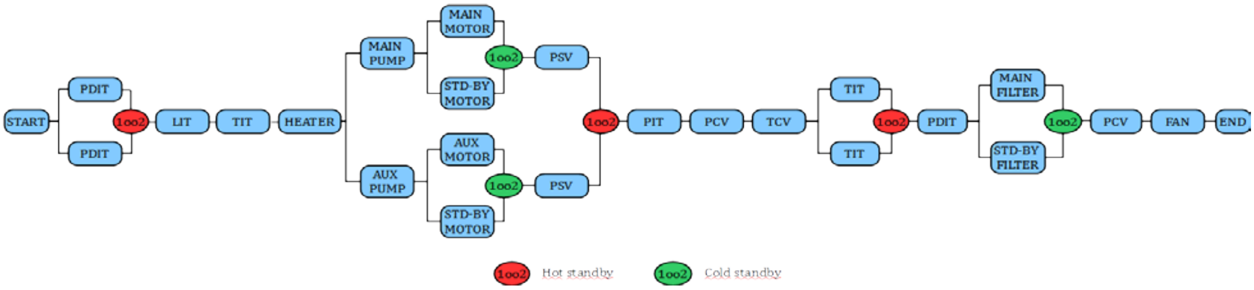

Figure 1. Mineral lube oil console RBD

III. Sensitivity Analysis

Sensitivity analysis allows project engineers to put in discussion nominal values taken from a generic database and focus on uncertainty quantification and propagation of uncertainty from input to output.
There are many sources of uncertainty that can affect input data including errors of measurement, absence of information and poor or partial understanding of the driving forces and mechanisms. This uncertainty imposes a limit on our confidence in the response or output of the model.
Good modelling practice requires an evaluation of the confidence in the model, so two steps are necessary: first step is the quantification of uncertainty (*uncertainty analysis*) and the second is evaluation of how much each input is contributing to the output uncertainty (*sensitivity analysis*); this is necessary to order by importance the strength and relevance of the inputs in determining the variation in the output.

Monte Carlo method is used in this paper for repeated random sampling to obtain numerical results; simulations are ran many times over in order to obtain the distribution of the unknown probabilistic entity under analysis.
In this study the use of Monte Carlo method is necessary because the reliability function under analysis is too complex for standard analytical computation so random sampling is achieved using Crystal Ball® software.

Two definitions are essential to go further in this study: confidence interval (CI) and confidence level (CL).
Confidence interval is a range of a population parameter and it is used to indicate the reliability of an estimate. In other words, CI consists of a range of values that act as good estimates of an unknown population parameter; quite often none of these values may cover the value of the parameter. *Confidence level* indicates the probability that the confidence range captures the true population parameter given a distribution of samples. Level of confidence indicates how frequently the observed interval contains the parameter and this value is represented by a percentage: usually CL is the complement of respective level of significance, in this paper 90% confidence interval reflects a significance level of 0.1.

The set of nominal failure rates taken from OREDA Handbook is supposed to follow two different probability distributions: uniform and normal.
The *continuous uniform distribution* (or rectangular distribution) is a symmetric probability distribution in which all values in the distribution's interval are equally probable. The interval is defined by the two parameters, “a” and “b”, which are its minimum and maximum value; in this study the width of the interval was set at ±10% of nominal value.

$$\lambda_i \in [0.9\lambda_{nom} \div 1.1\lambda_{nom}] \quad (2) \qquad f(\lambda) = \frac{1}{b-a}, \ \lambda \in [a;b] \quad (3) \qquad \mu = \frac{a+b}{2} = \lambda_{nom} \quad (4)$$

The *normal distribution* (or Gaussian distribution) is a continuous probability distribution with single central peak at the mean value; the shape of the curve is described as bell-shaped with the graph falling off evenly on either side of the mean.

$$\lambda_i \approx N\left(\mu_i, \sigma_i^2\right) \quad (5) \qquad f(\lambda) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\lambda-\mu)^2}{2\sigma^2}}, \ \lambda \in \Re^+ \quad (6) \qquad \mu = \frac{a+b}{2} \quad (7) \qquad \sigma = \sqrt{\frac{(b-a)^2}{12}} \quad (8)$$

IV. Simulation

The simulation is made on a sample survey of 50000 units: every simulation allocates new failure rates to any equipment in the system depending on the probability distribution considered.
At the end of simulation a sample of reliability function with yearly cadence (until sixth year) is carried out in order to evaluate the progression of the confidence interval: analysis is interrupted at the sixth year (48000h) because this is the standard time between two consecutive gas turbine total reviews, supposing 2% annual unavailability (8000 service hours per year). This process renews the whole system and is not possible to carry on reliability assessment before and after that operation.
Crystal Ball® output is shown in fig. 2 (A):

- ✓ Top left chart: probability of reliability values at a settled time;
- ✓ Bottom left chart: cumulative distribution function (reliability value less than or equal to a established value);
- ✓ Top right table: most important statistical parameter (mean value, variance and standard deviation);
- ✓ Bottom right table: percentile indicating the value below which a given percentage of observations in a group fall.

Confidence level in this study can be considered as the risk estimation to assess reliability incorrectly, in other words it is the risk assessment of reliability system accuracy.
In fig. 2 (B) is shown Crystal Ball® output with confidence level fixed at 90%: software shows two values that define the highlighted portion of histogram. 90% of cases system reliability is inside the range centered on mean value and it corresponds to 90% of area of the region bounded by the graph.

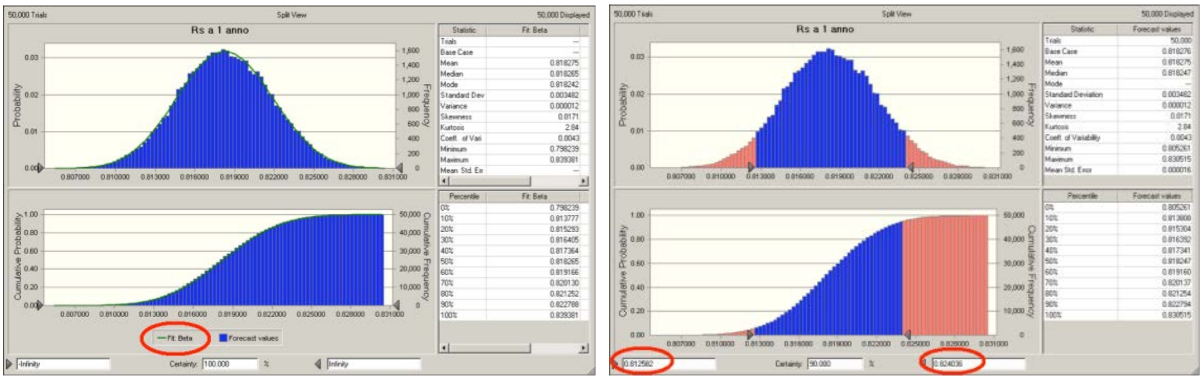

Figure 2. Crystal Ball® standard output (A); Crystal Ball® output 90% level of confidence (B)

V. Uniform and normal distribution results

Best-fitting distribution in the six cases under analysis considering uniform distribution is beta distribution:

$$f(x) = \frac{x^{(\alpha-1)}(1-x)^{(\beta-1)}}{B(\alpha,\beta)}, \qquad B(\alpha,\beta) = \int_0^1 x^{(\alpha-1)}(1-x)^{(\beta-1)}dx \quad (9)$$

In table 2 the following physical quantities and parameters are shown: system reliability mean value (*mean value*), confidence interval lower/upper boundary associated to 90% confidence level (*inf value, sup value*) and relative percentage deviation (*Dev. rel. inf %* e *Dev. rel. sup %*).

$$Dev.rel.\inf \% = \frac{\inf - mean}{mean} \cdot 100 \quad (10) \qquad Dev.rel.\sup \% = \frac{\sup - mean}{mean} \cdot 100 \quad (11)$$

Resulting intervals have good symmetry at both ends; upper boundary values are quite scattered but difference is very little (hundredth percentage points) and can be ignored.

Table 2. System reliability - uniform distribution, confidence interval 90%

Year	R_s(t) mean value	R_s(t) inf value	R_s(t) sup value	Dev. rel. inf %	Dev. rel. sup %
0	1	1	1	0	0
1	0,8182	0,8125	0,8240	-0,70	0,70
2	0,6677	0,6584	0,6772	-1,40	1,42
3	0,5435	0,5321	0,5553	-2,11	2,16
4	0,4414	0,4289	0,4543	-2,84	2,91
5	0,3577	0,3449	0,3708	-3,57	3,68
6	0,2892	0,2767	0,3021	-4,31	4,46

In order to give readers right interpretation of table 2 an example is shown: failure rates are affected by 10% uncertainty on nominal values (uniform distribution) and it causes 0,70% uncertainty on system reliability (with confidence level settled at 90%). E.g. first year system reliability can be written as follow:

$$R_S(1year) = (0{,}8182 \pm 0{,}0057) \quad (12)$$

In Figure 3 is possible to notice the confidence interval increase from year “zero” to sixth and the corresponding data scattering growth.

Best-fitting in the six cases under analysis considering normal distribution are lognormal (13) and gamma (14) distributions:

$$f(x) = \frac{e^{\frac{-(\log x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2 x}}, \quad x > 0 \quad (13) \qquad f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \text{ con } \Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt \quad (14)$$

In table 3 the following physical quantities and parameters are shown: system reliability mean value (*mean value*), confidence interval lower/upper boundary associated to 90% confidence level (*inf value, sup value*) and relative percentage deviation (*Dev. rel. inf %* e *Dev. rel. sup %*).
Also in this analysis resulting intervals have good symmetry at both ends, upper boundary values are quite scattered but difference is very little and can be ignored.

Table 3. System reliability - normal distribution, confidence interval 90%

Year	R_s(t) mean value	R_s(t) inf value	R_s(t) sup value	Dev. rel. inf %	Dev. rel. sup %
0	1	1	1	0	0
1	0,8182	0,8125	0,8240	-0,70	0,71
2	0,6677	0,6582	0,6772	-1,42	1,43
3	0,5435	0,5319	0,5553	-2,14	2,16
4	0,4414	0,4287	0,4542	-2,87	2,92
5	0,3576	0,3447	0,3708	-3,62	3,69
6	0,2891	0,2765	0,3021	-4,36	4,47

Confidence intervals have the same size than uniform distribution but mean values are quite different because data refers to two different simulations (different set of samples). The confidence interval increase from “zero” year to sixth and the corresponding data scattering growth has the same trend than previously.

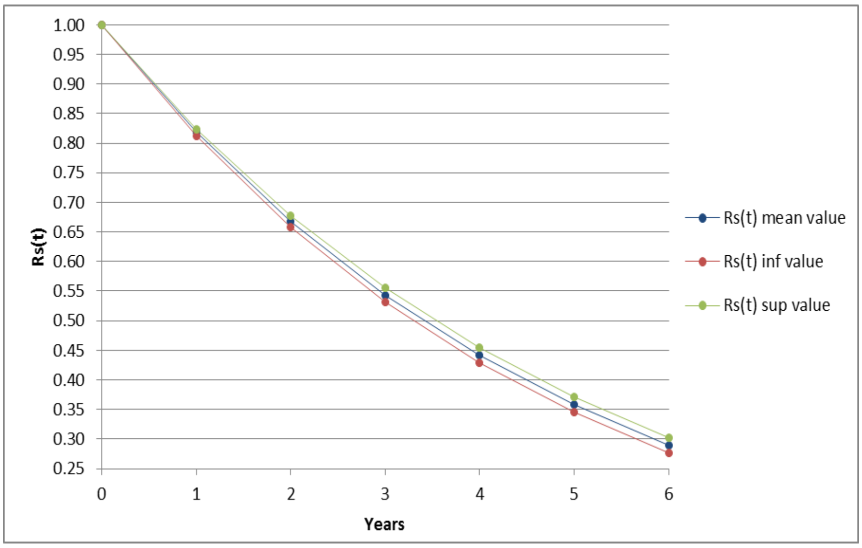

Figure 3. System reliability and corresponding confidence interval

VI. Conclusion

The supposed distribution (uniform or normal) for the set of failure rates coming from OREDA Handbook do not affect the size of confidence interval of system reliability: this trend is expected since the distributions are "comparable" supposing same mean value and variance. This approach allows also to obtain more detailed safety analysis [10-13].

Confidence intervals achieved supposing 90% level of confidence are very narrow cause of the reduced variability of input failure rates (10% nominal value): however if customer has particular requirement or data source is not trustworthy it will be necessary to consider a wider variability.

Future developments are the comparison of reliability parameters achieved using different data sources [14] and sensitivity assessment supposing different probability functions.

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