14th IMEKO TC10 Workshop on Technical Diagnostics
New Perspectives in Measurements, Tools and Techniques
for systems reliability, maintainability and safety
Milan, Italy, June 27-28, 2016

METHOD FOR TEMPERATURE MEASURING INSIDE A CYLINDRICAL BODY BASED ON SURFACE MEASUREMENTS

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***Abstract* -** The article is devoted to the measurement challenge associated with thermal monitoring that occurs during the heat treatment of products. We propose a method for calculating temperature values at internal points of a cylindrical body based on measuring the temperature functions on the surface of the body. The method for calculating temperature values involves the solving an inverse problem for a nonlinear parabolic equation with unknown initial conditions. The computational scheme is based on the use of finite-difference equations and a regularization technique. The reliability and efficiency of the method was confirmed by computational results. The experimental results are also presented in this paper.

Keywords: measurement challenge, heat treatment of the product, accuracy of measurement, computational procedure, inverse problem

1. INTRODUCTION

The reliability and useful life of equipment, steel structures and pipelines are directly dependent on the quality of their constituent elements, parts and units. One of the processes that has a significant impact on the reliability of operation of the products, as well as on the structure and properties of the metals and alloys from which the product is made, is a heat treatment process. Heat treatment is used to eliminate internal stresses in metals and alloys, casting defects and deformed structures in semi-finished and finished products, to improve machinability by cutting parts or pressure, to enhance the quality of finished products and to prepare them for final processing. The efficiency of heat treatment depends on the selected conditions and the control modes. The problems of development a monitoring system and optimum modes are considered, for example in the research works [1]–[4].

Monitoring of condition and maintaining the optimum mode strongly depend on the possibility of making the temperature measurements in the whole body, including at internal points. For example, Najafi, Woodbury, Beck, and Keltner considered the challenge of continual monitoring of the furnace during operation in [5]. Aguado, Huerta, Chinesta and Cueto [6] present a simple technique for real-time monitoring of thermal processes based on classical harmonic analysis and recent model reduction techniques. However, we can measure the temperature only on the outer surface during heat treatment of a massive body. On other hand, the differences in the values of the temperature on the surface and within the part arise due to the large dimensions of the workpiece and sluggishness of heat transfer. Thus, it is required to solve the measurement challenge associated with thermal monitoring that allow us to obtain the temperature values at internal points of a body based on measuring the temperature functions on the surface of the body. The problem of calculating the temperature in internal points of the body based on boundary measurements of temperature functions is called the inverse boundary problem and it is required to use mathematical results for solving this problem.

In recent years the methods that were developed using the results of mathematical studies have been widely applied. The use of mathematical tools for the development of methods for data processing allows to solve the measurement challenges at a qualitatively higher level. The studies [6]–[10] represent an example of such research.

We propose a new method for determining the required temperature at internal points of the body. The obtained results provided the basis for solving the measurement challenge associated with thermal monitoring. This method is based on a regularization technique. To evaluate the accuracy of numerical results a computational experiment was carry out. Computational results for some the test data are presented in this paper.

2. SETTING OF THE PROBLEM

We consider the mathematical aspect of the measurement challenge associated with thermal monitoring that occurs during the heat treatment of a cylindrical body without phase transitions in material. The background of measurement challenge is formulated as follows.

2.1. Basic Assumptions

The body has a cylindrical shape with insulated bases. Heat transfer is carried out only through its lateral surface. The main characteristics of the material from which the body is made, such as specific heat, thermal conductivity and material density, depend on the temperature at each point of the body. We can control the heat flow from the surface inside the body by regulating the intensity of its heating or cooling. In addition, we known that the heating process has smooth variation.

In order to ensure stability in calculating the temperature values at internal points of body we must know both the temperature on the axis of the cylinder and on the surface as well as the initial temperature in the body. However, in our case we can use only temperature and rate of heat flow which are measured on the cylinder surface and we cannot directly measure the temperature inside and on the cylinder axis neither in time. This situation occur when the body exposed to prior thermal action in the recent past, for example during complex heat treating.

Based on this background we can represent the mathematical model of the measurement challenge as follows.

2.2. Mathematical foundation

We consider a cylindrical body in a polar coordinate system such that the cylinder axis passes through the origin. Let R be the cylinder radius, r be the current radius, $r \in [0, R]$, and t be the current time, $t \in [0, T]$ and $u(r, t)$ is the temperature function corresponding to the temperature values at each point of the body. Denote $Q_T = (0, R) \times (0, T)$ and $\overline{Q_T} = [0, R] \times [0, T]$. According to the measurement requirements, the mathematical model describing the dependence of temperature on surface measurements can be represented as follows:

$$c(u)\rho(u)\frac{\partial u}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda\left(u\right)r\frac{\partial u}{\partial r}\right) \quad (r,t) \in Q_T \qquad (1)$$

$$u\left(R,t\right) = g\left(R,t\right), \quad \lambda\frac{\partial u}{\partial r}|_{(R,t)} = q\left(t\right) \quad t \in [0,T], \qquad (2)$$

where the function $c\left(u\right)$ is specific heat, $\lambda\left(u\right)$ is thermal conductivity and $\rho\left(u\right)$ is material density. The function $g\left(t\right)$ corresponds to the temperature measured on the surface and the function $q\left(u\right)$ describes the heat flow density on the surface of the body.

The important aspect of calculating the temperature value lies in the fact that when measuring boundary temperature functions $g\left(t\right)$ and $q\left(u\right)$, different errors inevitably occur. Therefore, in this study, we can presume that instead of the exact $g\left(t\right)$ and $q\left(u\right)$ we know the values with noise $g_\delta\left(t\right)$ and $q_\delta\left(u\right)$ and the allowable error estimate δ. In this problem, it is required to find the temperature value on the cylinder axis describing the function:

$$u\left(0,t\right) = \phi\left(t\right), \quad t \in [0,T]. \qquad (3)$$

The existence of a solution to the mathematical problem (1) – (3) was proved in [11]. It is well known that this problem is ill-posed and we use a regularization technique to solve this problem. According to the heating characteristics we known that this process has smooth variation. Therefore, we consider that $c\left(u\right)$, $\lambda\left(u\right)$ and $\rho\left(u\right)$ are continuously differentiable functions of second order and there exist constants $\Phi, \beta > 0$ such that the inequality

$$\max\left|u\left(r,t\right)\right| \le \Phi e^{\beta(r+t)} \qquad (4)$$

hold in each $(r, t) \in \overline{Q_T}$ together with the following relation

$$\max\left\{\max_{(r,t)\in Q_T}|\partial_t^2 u|, \max_{(r,t)\in Q_T}|\partial_r^3 u|\right\} \le C. \qquad (5)$$

Taking into account (1), (2), (4) and (5), we propose the discrete regularization method for calculating temperature values in internal points of body.

3. METHOD FOR CALCULATING TEMPERATURE VALUES

The main idea of the discrete regularization method is as follows. Consider a finite difference grid G in $\overline{Q_T} = [0, R] \times [0, T]$ such that

$$G = \begin{cases} (r_i, t_j) : r = (i-1)h, t = (j-1)\tau, \\ h = R/N, \tau = T/M, i = \overline{1, N+1}, j = \overline{1, M+1}. \end{cases}$$

Using the discrete functions $v(r_i, t_j) = v_{i,j}$ and a finite difference analog of the partial derivatives, we replace the differential equation (1) by a finite-difference equation. Then equation (1) has the following form

$$c_{i,j}\rho_{i,j}\frac{v_{i,j+1} - v_{i,j}}{\tau} = \frac{\lambda_{i+1,j} - \lambda_{i,j}}{v_{i+1,j} - v_{i,j}}\frac{\left(v_{i+1,j} - v_{i,j}\right)^2}{h^2} + \\ + \frac{\lambda_{i,j}\left(v_{i+1,j} - v_{i,j}\right)}{(i-1)h^2} + \lambda_{i,j}\frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{h^2}. \qquad (6)$$

and we represent the boundary conditions (2) as

$$v_{2,j} = u_{1,j} = p_j, \; v_{2,j} = u_{1,j} + hq_j = p_j + hq_j. \qquad (7)$$

We must calculate the values $v_{i+1,j}$ from (6) It is well known, that the explicit scheme(7) is unstable. To improve it, we apply the discrete regularization method (DRM). The linear form of DRM was proposed in [12] for linear heating of an object when the basic characteristics c, ρ and λ are dependent only on time and spatial variables. As a result, the mathematical model of this process has been represented via a linear parabolic equation and we use the regularization functional in form of an additional linear term.

In this contribution, we develop this method for situation when the coefficients $c(u), \rho(u)$ and $\lambda(u)$ are dependent on temperature directly. Therefore the heating of the body is described by a non-linear parabolic equation and the regularization technique is applied as follows. We apply the Taylor expansion to the coefficients and enhance (6) via the stabilizing functional with some regularization parameter. Further, we choose the discretization step according to special conditions and calculate the values $v_{i+1,j}$ from (6). As a result, we achieve stability of the computational scheme.

4. ESTIMATION OF THE TEMPERATURE ERRORS

To estimate the temperature errors, we define parameters S, η, μ, λ^* and functions w_j as follows:

$$S = \max_j |q_j|, \quad \eta = \max_{i,j} \frac{c_{i,j}\rho_{i,j}(i-1)h^2}{(\lambda_{i+1,j}\rho_{i,j}(i-1)+\lambda_{i,j})\tau},$$
$$\lambda^* = \min_{i,j}\lambda_{i,j}, \quad \mu = \max_{i,j}\frac{\lambda_{i,j}(i-1)}{\lambda_{i,j}i - \alpha(i-1)|v_{i,j}|},$$

$$w_i = \max_{i,j}|v_{i,j} - u_{i,j}|,$$

where $v_{i,j}$ satisfy (6) with conditions (7) and $u_{i,j}$ satisfy (1) with conditions (2). Define the parameter Ψ by the formula

$$\Psi = \eta C\tau^2 + 3\mu Ch^2 + \left(\frac{k_\lambda}{\lambda^*}C^2 + \mu C\right)h^4.$$

Proceeding as it is always done in the regularization theory, see, e.g. [13] and taking into account (4) and (5), we estimate the maximal deviation of the calculated temperature from exact values as follows

$$w_{N+1} \le (1+2\mu)^{N-1}\left(\delta + h\delta + Ch + \frac{1}{4\mu^2}(\delta + \Psi)\right).$$

To evaluate the reliability of this method for calculating temperature, computational experiments were carried out. Some experimental results are presented in this paper.

5. COMPUTATIONAL RESULTS

The efficiency of the developed method was verified by comparing the numerical results for calculating the temperature value ϕ_δ on the cylinder axis with the test functions $u(0,t) = \phi_0(t)$ and the numerical solution u_δ to (1), (2) with test values $u(r,t) = u_0(r,t)$ respectively. We introduced additional noises via calculating values p_δ and q_δ by the formulas

$$p_\delta = u_0(r_1, t_k) + erp_k,$$
$$q_\delta = \frac{u_0(r_2,t_k) - u_0(r_1,t_k)}{r_2 - r_1} + erq_k$$

where the values erp_k and erq_k are simulated as evenly distributed random variables in $[-\delta, \delta]$. Further, we choose the discretization steps in keeping with the conditions that guarantee the stability of method. Then, the numerical solution to the problem (1)–(2) is calculated via proposed method. The required temperature values u_δ and ϕ_δ were calculated by using the relations (6)– (7). Then the calculated axis temperatures ϕ_δ were compared with test functions.

The computational results for some test functions are illustrated in Figures 1–6 and are given in Tab 1. The same notations were used in all the Figures. The one-dimensional Figures 1, 3, 5 illustrate the graphs of temperature functions ϕ_δ and $u(0,t)$ on the axes. The notation u_0 corresponds to the exact test function $u(0,t)$, and the designation u_δ^α corresponds to the numerical solution to the problem (1)-(3) obtained via discrete regularization method. The noise level of measured values is denoted as δ. The designation α corresponds to the regularization parameter.

The two-dimensional figure illustrates the surfaces, which correspond to the temperature values at internal points of the body. In the two-dimensional figures, the notation "Exact" corresponds to the exact temperature. The surfaces corresponding to the solution to the inverse problem (1) – (3), which are obtained at internal points of the cylindrical body are denoted as "Regularized Solution".

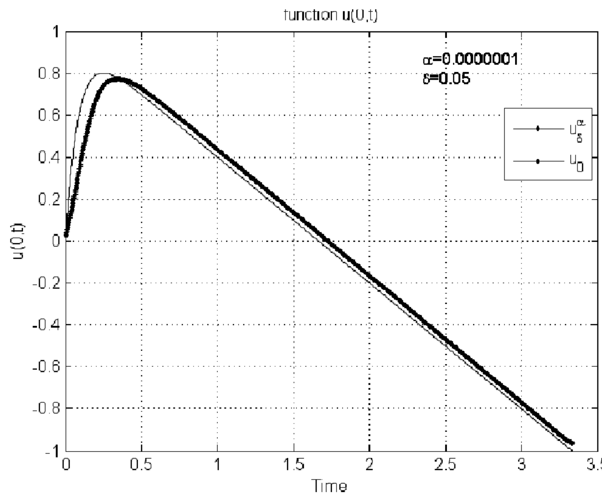

Fig. 1. Temperature functions on the axis. We use the function $\phi_0 = u(0,t) = \dfrac{1-e^{-t/\gamma}}{1-e^{-1/\gamma}} - t$ with $\gamma = 0.05$ as the test values.

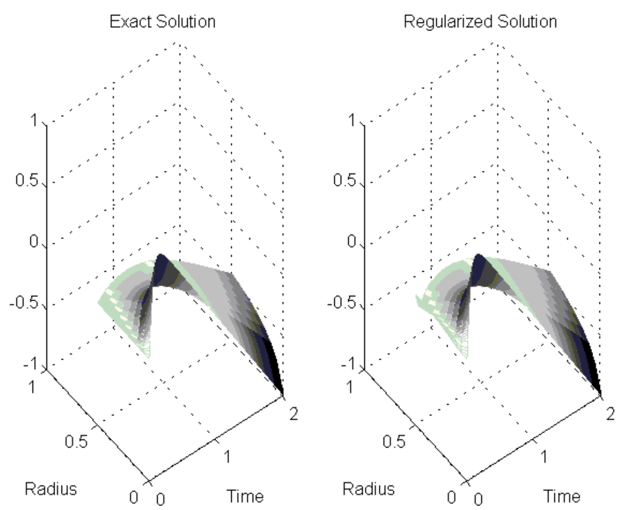

Fig. 2. The temperature values at internal points of the cylinder obtained for the test function $u(0,t) = \dfrac{1-e^{-t/0.05}}{1-e^{-1/0.05}} - t.$

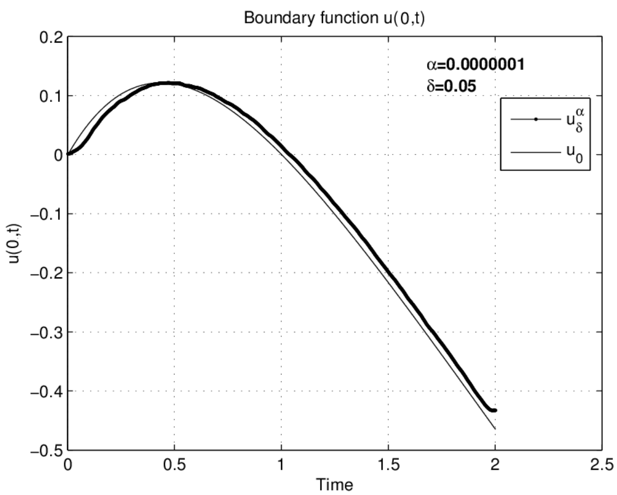

Fig. 3. Temperature functions on the axis. We use the function $\phi_0 = u(0,t) = te^{-t}$ as the test values.

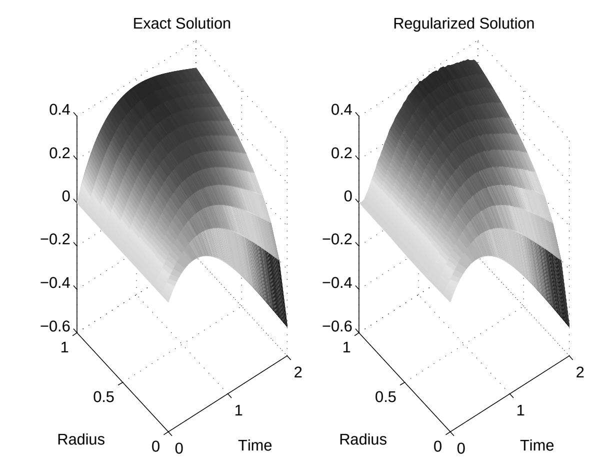

Fig. 4. The temperature values at internal points of the cylinder obtained for the test function $u(0,t) = te^{-t}$.

To evaluate the stability of the method and to obtain experimental error estimates of the temperature deviations, we calculated the quantities

$$\Delta = \max_{(r,t)\in D} |\phi_0 - \phi_\delta|$$

where $D = (\varepsilon, T - \varepsilon)$. In Table 1, we give some average values of these quantities.

The results of the experiments lead to the following conclusions. The approach based on discrete regularization method provides solution to measurement challenge with sufficient accuracy. The stability of the numerical solutions highly depend on the error level and discretization steps. This method allows to calculate the temperature values in internal points of body and to estimate expected temperature errors.

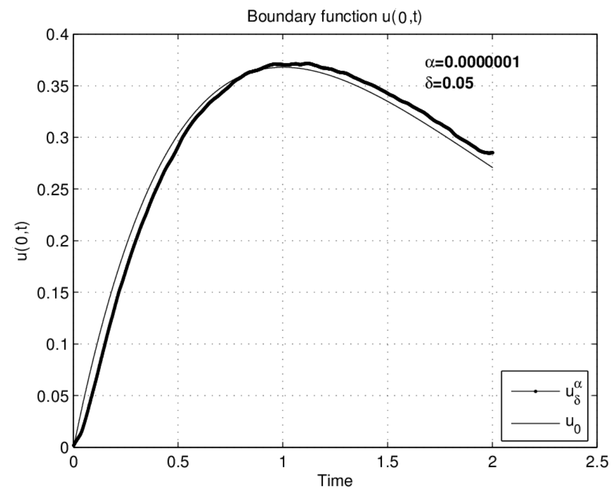

Fig. 5. Temperature functions on the axis. We use the function $\phi_0 = u(0,t) = t(e^{-t} - e^{-1})$ as the test values.

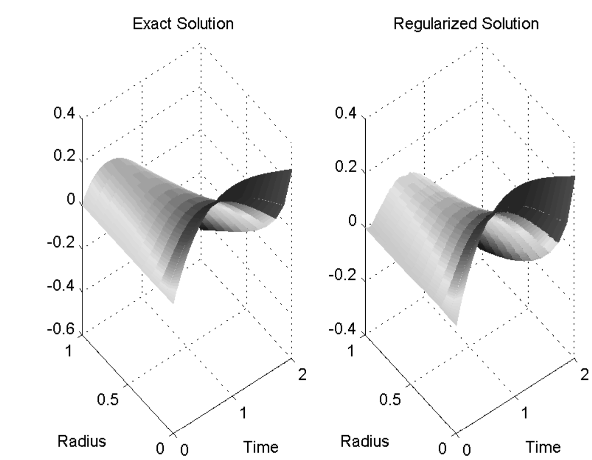

Fig. 6. The temperature values at internal points of the cylinder obtained for the test function $\phi_0 = u(0,t) = t(e^{-t} - e^{-1})$

6. CONCLUSION

In this article, the measurement challenge associated with thermal monitoring that occurs during heat treatment of products is considered. Explicit finite difference scheme based on a discrete regularization method was conditionally stable in some domain and allows to determine the thermal field at internal points of the cylindrical body under unknown initial temperatures with sufficient accuracy. The stability of scheme is confirmed by the results of the computational experiments. The obtained results use for the thermal monitoring of the complex heat treating and provide the basis for creation of the optimum modes.

Table 1. Experimental estimates of temperature deviations.

Test function	δ	Δ
$u(0,t) = \dfrac{1 - e^{-t/0.05}}{1 - e^{-1/0.05}} - t$	0.01	0.0184
	0.03	0.0347
	0.05	0.0565
$u(0,t) = te^{-t}$	0.01	0.0151
	0.03	0.0282
	0.05	0.0377
$u(0,t) = t(e^{-t} - e^{-1})$	0.01	0.0193
	0.03	0.0319
	0.05	0.0561

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