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SAMPLING STRATEGIES FOR VERIFICATION OF FREEFORM PROFILES USING COORDINATE MEASURING MACHINES

Rajamohan, G., Shunmugam, M. S.* and Samuel, G. L.
Manufacturing Engineering Section, Department of Mechanical Engineering
Indian Institute of Technology Madras, Chennai – 600 036
* Corresponding author. E-mail: shun@iitm.ac.in

Abstract: *Verification of engineering components having freeform profiles on a coordinate measuring machine (CMM) requires accurate measurement of sufficient number of sample points. While the measurement accuracy increases with increased sample size, it is often limited by cost and time considerations. Thus, for a given sample size, the locations of the measurement points are to be determined such that the actual shape may be effectively characterized. Several attempts are reported in the literature. A simple algorithm based on dominant points is proposed in this paper. Simulation studies have been carried out on a freeform profile. Comparison of the results with those obtained from uniform spacing and equi-parameter sampling methods reveals that the proposed method performs effectively.*
Keywords: *B-splines, sampling, sample size, freeform profiles*

1. INTRODUCTION

Freeform features are widely used in the design and manufacturing of dies and moulds, patterns and models, plastic products, etc. in many fields ranging from automotive and aerospace applications to biomedical, entertainment and geographical data processing applications. The verification of these features using a CMM requires accurate measurement of a number of sample points. These sample points are used to create a geometric model, called the substitute geometry, for the feature being measured. The substitute geometry is then compared with the design intent to determine conformance. This is because the deviations of the sampled points may satisfy the given tolerance while some of the non-sampled points may still be out of the tolerance range. It is assumed that the set of sample points is representative of the feature being inspected to ensure the validity of the evaluation process. While the measurement accuracy increases with increased sample size, it is often limited by cost and time constraints. Thus, for a given sample size, it is often required to determine the locations of the measurement points such that the actual shape may be effectively characterized.

Sampling of freeform features is a complex task, performed by experienced personnel. The common practice is to distribute the sample points in a uniform pattern. Though the method is simple, it may result in inadequate sampling when there are sharp changes in curvatures and unnecessarily more sampling at relatively flat regions, both of which are undesirable in the measurement process. A number of research efforts to overcome this problem have been reported in the literature. Weckenmann, et al. (1995) suggest that a good sampling strategy should lead to accurate determination of the parameters of the substitute feature sought from the data points sensed, in a minimum time, using a suitable evaluation criterion, with sufficient reliability and within a predetermined confidence range. Cho and Kim (1995) proposed a sampling strategy using the mean surface curvature and a factor called *region selection ratio*. Their method divides the surface into sub-regions and ranks them according to their mean curvature and the sampling points are distributed over the surface based on this ranking and the region selection ratio. Pahk, et al. (1995) presented three sampling methods, viz. uniform distribution, curvature dependent distribution and hybrid distribution. The first two methods are not very effective in distributing the sample points according to profile complexities. The hybrid method, which is a combination of the uniform and curvature dependent methods, attempts to neutralize this drawback. Edgeworth and Wilhelm (1999) proposed an iterative sampling process, based on the surface normal measurement data to develop an interpolating curve between sample points. The interpolant identifies areas where further samples may be required for a complete measurement. This method requires the surface normal for any point on the surface that may be supplied as a file with a dense array of nominal coordinates and surface normals from which the program could select the closest match to the ideal target. Ainsworth, et al. (2000) proposed selection of the measurement point based on parameterization based adaptive subdivision sampling of the design model. ElKott, et al. (2002) proposed four heuristic algorithms based on the surface features, viz. equi-parametric sampling, patch size based sampling, patch mean Gaussian curvature based sampling, and combined patch size and patch mean Gaussian curvature based sampling. They also have presented a genetic algorithm based optimisation method.

The methods reported in the literature are found to be not fulfilling all the requirements of a good sampling strategy. Some of the methods are inefficient in handling features with rapidly changing curvatures, while many methods rely on user judgement of certain parameters that are used in deciding the locations of sampling points. Most importantly, the accuracy of the substitute geometry has not been considered while evaluating the sampling strategies. Unless the accuracy of the substitute geometry is ensured, the interpretation of the results becomes meaningless. In the present work, a new sampling strategy based on dominant points is proposed. To bring out the salient features of the proposed strategy, freeform profile is considered. Simulation is carried out using uniform spacing, equi-parameter and dominant points samplings. The results show that the proposed strategy effectively handles curvature variations.

2. B-SPLINE REPRESENTATION OF FREEFORM PROFILES

The B-spline curve, shown in Fig. 1, is defined by *(n+1)* control points (or de Boor points) P_i. The parametric form of the curve is given by (Zeid, 2005),

$$P(u) = \sum_{i=0}^{n} P_i N_{i,k}(u); \qquad 0 \le u \le u_{\max} \qquad (1)$$

Here, $N_{i,k}(u)$ is the B-spline function. The term u is called the parameter of the B-spline and k is the order (*degree* of the curve will be *one less* than the order) of the B-spline. The control points form the vertices of the control (or de Boor) polygon. The degree of the B-spline curve is independent of the number of control points. Usually, value of u_{max} is taken as 1.

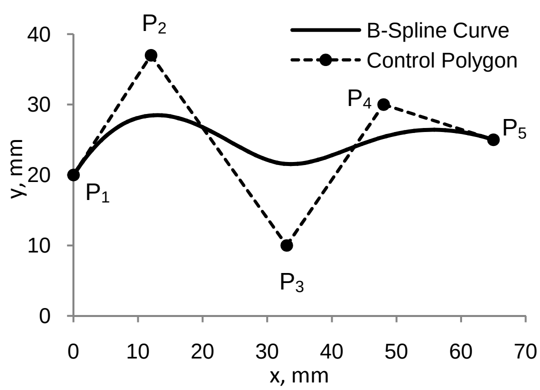

Fig. 1 B-spline curve and control points

The value of $N_{i,k}(u)$ can be estimated using the following recursive relations, choosing 0/0 = 0, if the denominators in the equation (2) becomes zero.

$$N_{i,k}(u) = \frac{u - u_i}{u_{i+k} - u_i} N_{i,k-1}(u) + \frac{u_{i+k+1} - u}{u_{i+k+1} - u_{i+1}} N_{i+1,k-1}(u) \qquad (2)$$

where,

$$N_{i,1}(u) = \begin{cases} 1 & ; u_i \le u < u_{i+1} \\ 0 & ; \text{otherwise} \end{cases} \qquad (3)$$

The terms u_i in equations (2) and (3) are called *parametric knots* or *knot values*. These values form a sequence of non-decreasing numbers called the *knot vector*.

The B-spline representation of curves has the capability to introduce local changes to the curve through changing the locations of the control points. The local control comes from the segmentation of the curve. Segmentation or curve splitting is the process of replacing an existing curve by two or more curve segments of the same curve type. It is achieved by subdividing the parameter range into spans using the knot vectors. Each span is controlled by k adjacent control points only. Recombining these segments, composite curve with a shape identical to that of the original curve is obtained. The major advantage of B-splines is that it provides a flexible and powerful tool for designers to represent a wide variety of shapes, from the very simple curves to even very complex freeform profiles.

3. SAMPLING METHODS

From metrological viewpoint, it is undesirable to have sample points at the edges, as these points cannot be measured precisely. In this study, the start and end points are set at 0.01 and 0.99 respectively in parametric space.

3. 1. Uniform spacing method

In the uniform spacing method (Fig. 2), the sampling points are distributed with equal spacing along the *x*-axis. Values of x_{min} and x_{max} corresponding to 0.01 and 0.99 respectively must be fixed. Then, the values of intermediate points are obtained from equation (4).

$$x_i = x_{\min} + \frac{x_{\max} - x_{\min}}{(K-1)} \qquad (4)$$

where, K is the sample size.

Since the B-spline curve is defined in parametric space, the Cartesian coordinate data must be transformed into equivalent parametric coordinates. This is carried out by equating the right hand side of equation (1) to x_i-value and solving the resultant cubic equation for the value of u.

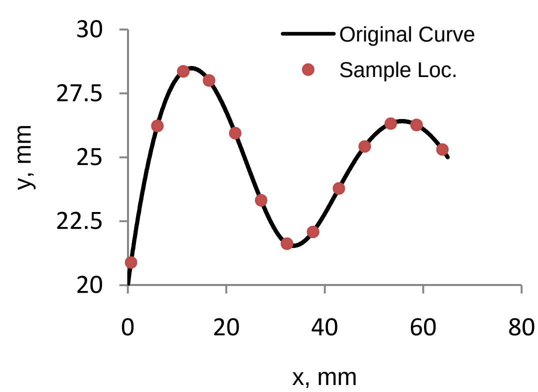

Fig. 2 Uniform spacing (along x-axis) sampling

3. 2 Equi-parameter method (ElKott, et al., 2002)

In the equi-parameter sampling method (Fig. 2), the sample positions (s_j) can be determined as:

$$s_j = 0.01 + 0.98\frac{j}{(K-1)}; j = 0, 1, ..., (K - 1) \quad (5)$$

where, K is the sample size.

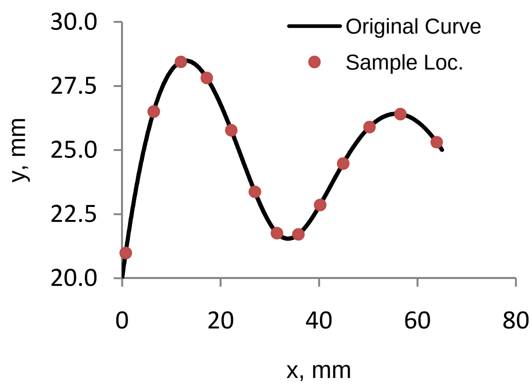

Fig. 3 Equi-parameter sampling

3. 3 Proposed sampling method

A B-spline curve can be approximated using a reasonable number of *key* points that reflect certain characteristics of the curve, such as the curvature (Razdan, 1999). The dominant points are those points wherein the key geometrical characteristics of the B-spline curves are available (Ansari and Delp, 1991; Marji and Siy, 2003; Wu, 2003; Park and Lee, 2007). The points with maximum local curvature and inflection points are considered here as the dominant points (Fig. 4). Apart from these points, two more points defining the start (0.01) and end (0.99) points of the substitute geometry (profile) are taken to be the sample points to satisfy the metrological requirement,

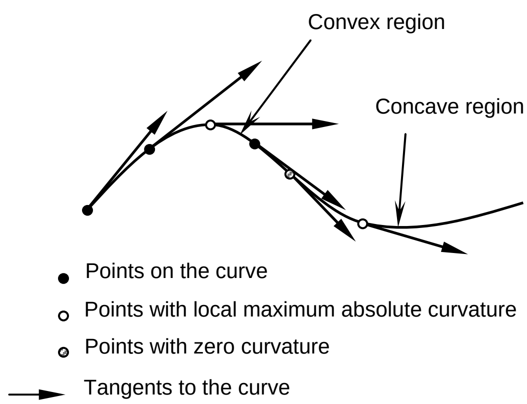

Fig. 4 Illustration of dominant points

without compromising on the completeness of measurement. The dominant points along with the start and end points constitute the initial set of sample points.

The initial set of sample points are used to form segments of the profile. Each segment can be refined by adding more points to obtain a smoother curve. These points can be added in a number of ways, for instance, by halving the curve segment. This halving approach results in worse results than by selecting points based on geometrical information (Park and Lee, 2007). Thus, the local curvature information in each segment is used here for selecting the appropriate sample point.

The additional points are added to the initial sample set one at a time. Starting with initial segments, one point is placed at first segment wherein the curvature is equal to the mean curvature of the segment and the profile is reconstructed (Reconstruction procedure is explained in section 4). For the reconstructed profile, the maximum normal deviation is estimated. This process is continued on all remaining segments and the corresponding maximum normal errors are found. The segment which has the lowest value for the maximum normal error gets an additional sample point at a location where the curvature is equal to the mean curvature of the segment. This point is added to the initial sample set and the procedure is repeated with the newly formed sample set. New sample points are added until the maximum normal error between the original and reconstructed profiles reaches the specified limit or sample size reaches a preset value (Fig. 5).

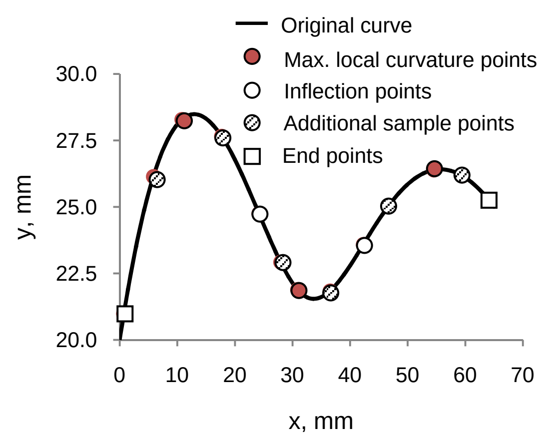

Fig. 5 Dominant points based sampling

4. SUBSTITUTE GEOMETRY

Interpolating cubic B-spline curves are used to obtain the reconstructed curve passing through the sample points. The steps involved in obtaining the control points of the interpolating curve are explained below.

Re-parameterization: There are three commonly used methods of re-parameterization (Zhongwei, et al., 2003) of knots, namely uniform spaced method, chord length method and centripetal method. The chord length method of re-parameterization is followed in this work. In the chord length method, the length of the curve (l) is computed first according to:

$$l = \sum_{j=1}^{n} \left\| s_j - s_{j-1} \right\| \quad (6)$$

The parameter (u_j) associated with the point s_j is computed as:

$$u_j = u_{j-1} + \left\| s_j - s_{j-1} \right\| \frac{u_{max}}{l} \quad (7)$$

where, $u_1 = 0.0$,

$u_{max} = 1.0$

$j = 1, 2, \ldots, K+1$.

To reflect the distribution of the parameter values, the knots are selected by averaging technique.

$$\left.\begin{aligned} &u_0 = \ldots = u_p = 0.0 \\ &u_{m-p} = \ldots = u_m = u_{\max} \\ &u_{j+p} = \frac{1}{p}\sum_{i=j}^{j+p+1} u_i\ ;\ j = 1, 2, \ldots, (n-p) \end{aligned}\right\} \quad (8)$$

where, p is the degree of the B-spline curve.

Determination of control points: It is intuitive that if the sample point lies on the B-spline curve, it must satisfy equation (1). Writing equation (1) for each of the sampling point yields (Toe and To, 2004):

$$\left.\begin{aligned} &s_1(u_1) = N_{1,k}(u_1)\, P_1 + \ldots + N_{n,k}(u_1)\, P_n \\ &s_2(u_2) = N_{1,k}(u_2)\, P_1 + \ldots + N_{n,k}(u_2)\, P_n \\ &s_3(u_3) = N_{1,k}(u_3)\, P_1 + \ldots + N_{n,k}(u_3)\, P_n \\ &\ldots \\ &s_n(u_n) = N_{1,k}(u_n)\, P_1 + \ldots + N_{n,k}(u_n)\, P_n \end{aligned}\right\} (9)$$

Here, $N_{i,\,k}(u)$ values are the basis function values evaluated using equations (2) and (3) and P_i's are the control points to be determined. By solving the system of equations in (9), the control points of the reconstructed curve can be determined. The number of control points of the reconstructed curve is the same as the number of sample points so that the resulting curve will be interpolating all the sample points.

5. SIMULATION OF SAMPLING ALGORITHMS

5. 1 Sample size and substitute profile

In first phase of the work, substitute profiles obtained using different sample size and strategies are studied. Following the procedure outlined in section 3, number of sampling points is increased from 8 to 25 and error between the original profile and substitute profile are computed. Fig. 6 shows the plot of errors clearly bringing out that the proposed method is able to construct the substitute geometry with an error of 0.0018 mm when the sample size is 25. For the subsequent simulation studies, this sample size was used for all the sampling strategies.

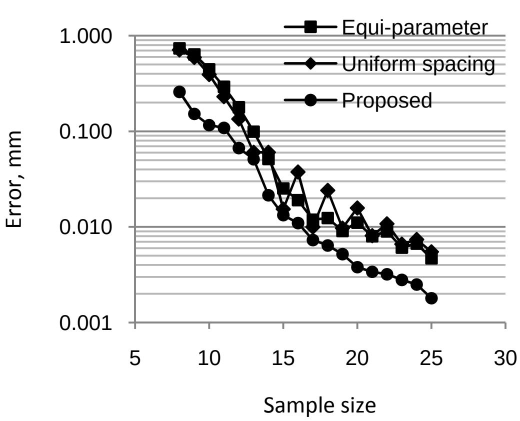

Fig. 6 Error in reconstructed B-spline from sample points

5. 2 Superimposing form and CMM errors

The simulated profile is obtained by superimposing on the original profile two error components, namely the form error, δ_f and the CMM measurement error, δ_m (ElKott, et al., 2002). About one thousand points are chosen along the entire profile and these errors are applied to each of them.

The form error is distributed over the profile using the following equation.

$$\delta_f^{'} = \left(\frac{\kappa - \kappa_{\min}}{\kappa_{\max} - \kappa_{\min}} - 0.5\right)\delta_f \qquad (10)$$

where, κ - curvature at the sample location

κ_{max} - maximum curvature of the profile; and

κ_{min} - minimum curvature of the profile

δ_f is taken to be *0.010 mm.* The CMM measurement error (δ_m) is assumed to vary according to normal distribution with mean as *0.0 mm* and standard deviation as *0.001 mm*. Thus, if $S_i\ (x_i,\ y_i)$ is any sample point on the profile, then its actual location due to the combined effect of form and measurement errors will be $S_i^{'}\ (x_i + \delta'_f + \delta_m,\ yi + \delta'_f + \delta_m)$.

6. RESULTS AND DISCUSSIONS

Cubic B-spline curve, defined by a set of control points P_i, are used to generate the freeform profile in this work (Fig. 1). This profile has 5 control points and these are: *(0, 20), (12, 37), (33, 10), (48, 30)* and *(65, 25)*. Uniform spacing method, equi-parameter sampling method and the proposed sampling methods are applied on this chosen profile. It can be seen from Fig. 6 that the error in the substitute geometry with reference to original profile decreases as the sample size increases. For a sample size of 25, the proposed sampling method is able to reconstruct with an error of 0.0018 mm.

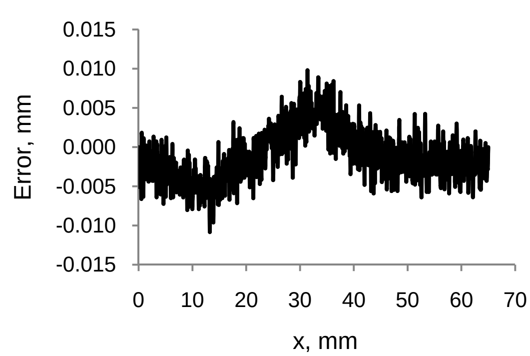

Fig. 7 Distribution of CMM and Form Errors

The form error component is added to the original profile on the basis of curvature values. This is in tune with the practical observation that machining of highly curved portions in a profile is likely to have more form error. The CMM error is assumed to follow normal distribution depending on the repeatability and accuracy capabilities. These two error components shown in Fig. 7 are added to the original profile to obtain the simulated profile. Different sampling strategies are implemented in two modes, namely the discrete mode and the fitting modes. In the discrete mode, the normal deviations of the sample points are calculated without reconstructing the profile, while in the fitting mode, the profile is reconstructed and compared with the original profile. The results of these studies are presented in Table 1.

Table 1. Profile error values obtained by different sampling methods (in mm)

Without form and CMM error					
Uniform spacing		Equi-parameter		Proposed	
Discrete	Fitted	Discrete	Fitted	Discrete	Fitted
0.0000	0.0055	0.0000	0.0047	0.0000	0.0018
With form error only					
Uniform spacing		Equi-parameter		Proposed	
Discrete	Fitted	Discrete	Fitted	Discrete	Fitted
0.0102	0.0130	0.0104	0.0116	0.0097	0.0117
With CMM error only					
Uniform spacing		Equi-parameter		Proposed	
Discrete	Fitted	Discrete	Fitted	Discrete	Fitted
0.0047	0.0050	0.0043	0.0057	0.0032	0.0042
With form and CMM errors					
Uniform spacing		Equi-parameter		Proposed	
Discrete	Fitted	Discrete	Fitted	Discrete	Fitted
0.0129	0.0146	0.0100	0.0112	0.0102	0.0109

With substitute geometry obtained by uniform spacing and equi-parameter sampling methods, normal error values are higher. The performance of the proposed method is therefore superior to the other sampling methods. Thus, for the same number of measuring points, the proposed method provides an effective sampling strategy. The error may also be obtained from the discrete mode (without reconstructing substitute geometry), but the value is always less than that obtained by reconstruction of substitute geometry.

7. CONCLUSIONS

Dominant points sampling of freeform profiles has been proposed to determine the locations of the sampling points. The algorithm has been tested using the simulated data over a variety of freeform profiles of varying shapes. Only the results corresponding to one profile is included in the paper. The results are compared with the uniform spacing and equi-parameter sampling approaches. The results indicate that, for a given sample size, the proposed method produces better results compared to the equi-parametric and uniform spacing methods. The proposed method distributes the sampling points such that the original curve is effectively characterized. It is evident from the observations that the method can cope up with curvature variations. The method is very simple to implement and can be easily extended to adaptive sampling of freeform features.

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