9th INTERNATIONAL SYMPOSIUM ON MEASUREMENT AND QUALITY CONTROL (9th ISMQC)
November 21 – 24, 2007, IIT Madras

INFLUENCE OF COVERAGE FACTOR *k* IN EVALUATING THE EXPANDED MEASUREMENT UNCERTAINTY

[a]Vedran M., [b]Sanjin M., [c]Biserka R., [d]Srđan M. and [e]Gorana B.
Faculties of Mechanical Engineering and Naval Architecture, Zagreb, Croatia,
[a] vedran.mudronja@fsb.hr [b] sanjin.mahovic@fsb.hr [c] biserka.runje@fsb.hr
[d] srdjan.medic@fsb.hr [e] gorana.balic@fsb.hr

Abstract: *The usual estimation of the coverage factor k, using the Guide to the Expression of Uncertainty in Measurement - GUM method is based on the Central Limit Theorem and directly related to the Gaussian or t-distribution of probability of the measured value whose conditions in practice do not always have to be satisfied. The work studies especially the cases when one or several input values have a significantly higher contribution of uncertainty in relation to other input values. In such cases it is not clear whether the GUM method conditions have been met or not, which may sometimes lead to incorrect evaluation of the expanded measurement uncertainty.*

Keywords: *Expanded measurement uncertainty, GUM method, MCS method, coverage factor k*

1. INTRODUCTION

The expanded uncertainty is a value which determines the interval around the measurement result which may be expected to encompass the major part of the probability distributions that could reasonably be assigned to the measured value. The expanded uncertainty is obtained by multiplying the combine standard uncertainty $u_c(y)$ by coverage factor *k*, and is denoted with *U*. The value of the coverage factor *k* can only be found if there is wide knowledge on the probability distribution of every input value and if these distributions are combined in order to obtain the distribution of the output value. The evaluations of x_i input values and their standard uncertainty $u(x_i)$ are not themselves suitable for this purpose. In accordance with GUM method the value of coverage factor *k* is determined on the basis of the required probability *P*, for interval from *y-U* to *y+U* and the effective number of degrees of freedom ν_{eff} according to expression (1).

$$k = t_p(\nu_{eff}) \qquad (1)$$

The effective number of degrees of freedom ν_{eff} is calculated from the known *Welch-Satterthwaite* formula. In order to determine the coverage factor *k*, apart from the GUM method, the alternative method of Monte Carlo simulation (MCS method) has also been used in the work. The MCS method is based on generating random numbers from the probability density functions for each input value x_i and creation of respective value of the output value *y*, combining various distributions which define the input values. In the work, the coverage factor *k* is directly determined from the experimental probability density function obtained by combining different probability density functions of input values.

2. EXPANDED MEASUREMENT UNCERTAINTY AND COVERAGE FACTOR

The evaluation of the measurement uncertainty was performed for the calibration procedure of the micrometer setting rod and vernier calliper.

2. 1. Evaluation of measurement uncertainty for calibration of setting rod

<u>Mathematical measurement model:</u>

$$L_X = L_{ix} + \delta L_{ix} + \delta L_T + \delta L_E + \delta L_A + \delta L_P \qquad (2)$$

L_X	actual (corrected) length of the setting rod
L_{ix}	measured length of the setting rod
δL_{ix}	influence of the maximum permissible error
δL_T	influence of temperature
δL_E	influence of elastic deformation
δL_A	influence of Abbe error
δL_P	influence of misalignment of measuring probes

The uncertainty elements and the expanded uncertainties, evaluated by the GUM method are presented in Table 1. It may be noted from Table 1. that the uncertainty element $u(\delta L_{ix})$, especially in case of longer setting rods, will have substantially greater contribution to uncertainty compared to others. In other words, for a certain length of the setting rod the conditions of the Central Limit Theorem cease to be valid. In applying the GUM method it is difficult to predict at which moment the required conditions of Central Limit Theorem are not met any more. For the mentioned example the measurement uncertainty was also calculated by applying the MCS method.

Table 1. Uncertainty budget for the calibration of the setting rod

Input Value	Source of Uncertainty	Standard Uncertainty	Probability Distribution	Sensitivity Coefficient	Uncertainty Contribution, μm L in m
L_{ix}	Repeatability	0,17	normal	1	0,17
δL_{ix}	Maximum permissible error	$0{,}29 + 5{,}8 \cdot L$	rectangular	1	$0{,}29 + 5{,}8 \cdot L$
δL_T	Temperature correction	$0{,}20 + 0{,}7 \cdot L$	normal	1	$0{,}20 + 0{,}7 \cdot L$
δL_E	Elastic deformation	0,021	normal	1	0,021
δL_A	Abbe's error	0,020	rectangular	1	0,020
δL_P	Misalignment	0,062	rectangular	1	0,062
Combined standard measurement uncertainty Linearized expanded measurement uncertainty U; P= 95%, k=2				$u_c(L_x) = \sqrt{(0{,}39)^2 + 34{,}13L^2}$ $U = (0{,}60 + 11{,}5L)$ μm, L in m	

Table 2. Input values and probability density functions in simulation of output value L_x

Input value x_i		Probability density function $g(x_i)$
measured length of the setting rod	L_{ix}	Normal distribution (M, 0; 0,17μm)
influence of the maximum permissible error	δL_{ix}	Rectangular distribution (M; $-0{,}29 + 5{,}8 \cdot L$; $0{,}29 + 5{,}8 \cdot L$)
influence of temperature	δL_T	Normal distribution (M; 0 ; $0{,}20 + 0{,}7 \cdot L$ μm)
influence of elastic deformation	δL_E	Normal distribution (M; 0 μm; 0,21 μm)
influence of Abbe error	δL_A	Rectangular distribution (M;-0,03 μm; 0,03 μm)
influence of misalignment of measuring probes	δL_P	Rectangular distribution (M;-0,11 μm; 0,11 μm)

The input values x_i and probability density functions (pdf) for the setting rod are presented in Table 2. The probability density function of the output value L_x has been simulated by the MCS method with M = 100000 simulations. By applying the MCS method it has been determined that, depending on the length of the setting rod, the output distributions change their appearance from the normal-like to trapezoidal-like distributions as seen in figures 1a, 1b and 1c. For setting rods of nominal lengths 25 mm, 100 mm and. 500 mm the coverage interval and coverage factor are directly determined from the experimental probability density function. By MCS method it has been determined that the value of the coverage factor k changes regarding the length of setting rod which is not the case when applying the GUM method. The coverage interval and coverage factor are directly determined from the experimental probability density function obtained by combining different probability density functions of input values. In measuring the length, the expanded measurement uncertainty is very often expressed in dependence on the length, as presented in Table 1. This example indicates the problems that may result in expressing the expanded measurement uncertainty.

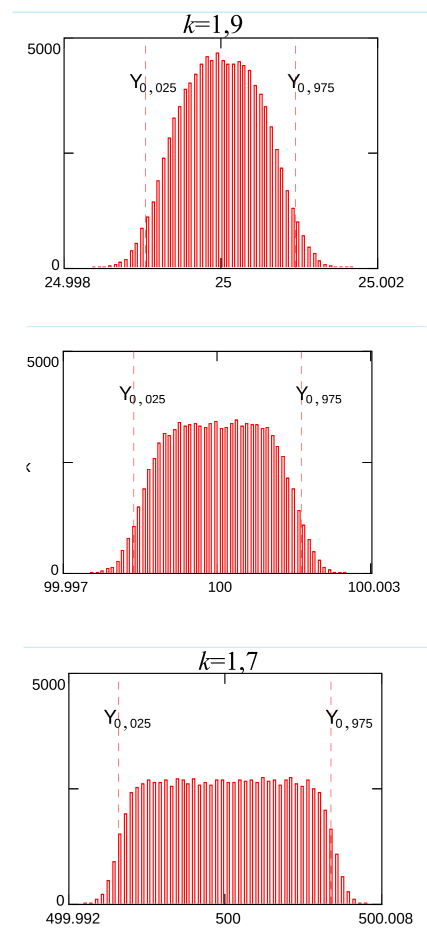

Fig. 1(a), (b) and (c). PDF for 25 mm setting rod, 100 mm setting rod and 500 mm setting rod

The following example, evaluation of measurement uncertainty for the calibration of the vernier calliper, given in document EA 4/02 also shows that the conditions necessary for the application of GUM method are not always fulfilled. The influencing values and their contributions in the calibration procedure of vernier calliper are presented in Table 3.

2.1 Evaluation of Measurement Uncertainty for the Calibration of Vernier Calliper

Mathematical measurement model:

$$E_X = l_{iX} + l_S + L_S \cdot \overline{\alpha} \Delta t + \delta l_{iX} + \delta l_M \qquad (3)$$

l_{iX} - indication of the calliper

l_S - length of the actual gauge block

L_S - nominal length of the actual gauge block

$\overline{\alpha}$ - average thermal expansion coefficient of the calliper and the gauge block

Δt - difference in temperature between the calliper and gauge block

δl_{iX} - correction due to the finite resolution of the calliper

δl_M - correction due to mechanical effects, such as applied measurement force, Abbe error, flatness and parallelism errors of the measurement forces.

Table 3. Uncertainty budget for 150 mm vernier calliper

Quantity	Estimate	Standard Uncertainty	Probability Distribution	Sensitivity Coefficient	Uncertainty Contribution, µm and L in m
l_{iX}	150,10 mm	-	-	-	-
l_S	150,00 m	-0,46 µm	rectangular	1	-0,46 µm
Δt	0	1,15 K	rectangular	1,7 µm K^{-1}	2,0 µm
δl_{iX}	0	15 µm	rectangular	1	15 µm
δl_M	0	29 µm	rectangular	1	29 µm
E_X	0,10 mm				33 µm

The method used for calculating the coverage factor is clearly related to the fact that uncertainty of measurement associated with the result is dominated by two influences: the mechanical effects and the finite resolution of the vernier scale. Thus the assumption of a normal distribution for the output quantity is not justified. [EA 4/02]

This case brings to the fore the advantages of Monte Carlo method. The input values x_i and probability density functions (pdf) for the 150 mm length vernier calliper are presented in Table 4.

Table 4. Input values and probability density functions in simulation of value E_x

Input value x_i		Probability density function $g(x_i)$
indication of the calliper	l_{iX}	-
length of the actual gauge block	l_S	Rectangular distribution (M; $-0{,}8 \cdot 10^{-3}$ µm; $0{,}8 \cdot 10^{-3}$ µm)
difference in temperature between the calliper and gauge block	Δt	Rectangular distribution (M; -2 K; 2 K)
correction due to the finite resolution of the calliper	δl_{iX}	Rectangular distribution (M; $-26 \cdot 10^{-3}$ µm; $26 \cdot 10^{-3}$ µm)
correction due to mechanical effects (Abbe error, flatness and parallelism errors)	δl_M	Rectangular distribution (M; -50 µm; 50 µm)

The probability density function of the output value *E*x is shown in Fig. 2.

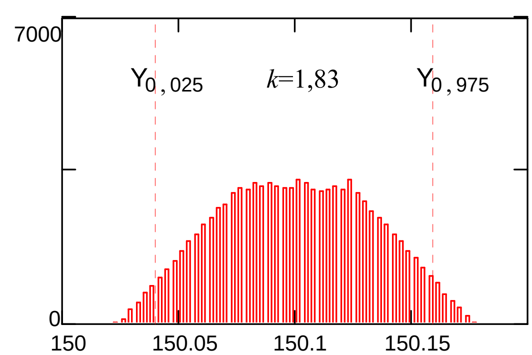

Fig. 2. Probability density function $g(E_x)$

The estimated standard deviation of the output value $g(E_x)$ amounts to 0,033 µm. The output value E_x is within the interval: $[(E_x)_{0,025}$ = 150,04 mm; $(E_x)_{0,975}$ = 150,16 mm] with P = 95%.

By applying the MCS method the form of output distribution and coverage interval are very easily obtained. By varying the input values, the output distributions change their form as presented in Fig. 3 and 4.

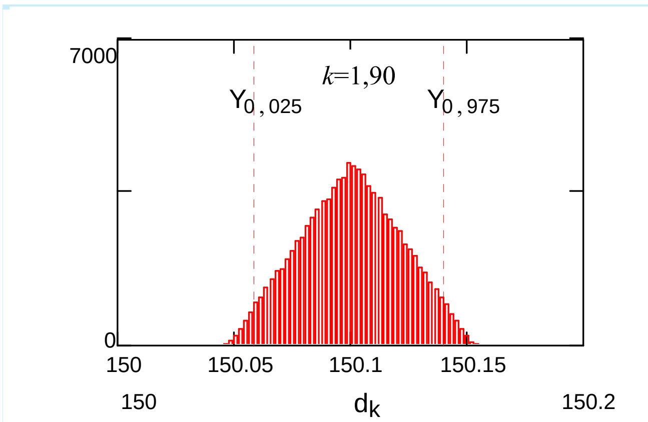

Fig. 3. Probability density function $g(E_x)$
(two elements of uncertainty dominant and equal regarding amount)

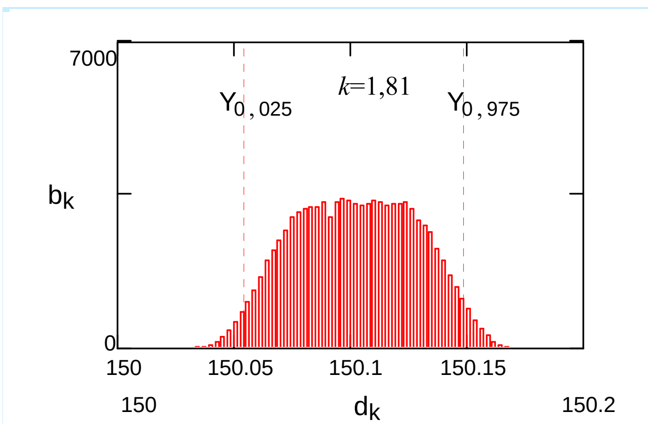

Fig. 4. Probability density function $g(E_x)$
(two elements of uncertainty dominant and equal regarding amount, major influence of third element)

5. CONCLUSION

Based on the example of evaluating the measurement uncertainty in the procedure of calibrating the micrometre setting rod it has been shown that the conditions of GUM method in practice are not always satisfied which may lead to incorrect evaluation of the expanded measurement uncertainty. In our case, the incorrect evaluation of the coverage factor *k*, has substantially increased the value of the expanded measurement uncertainty, especially in case of longer setting rods. The evaluation of measurement uncertainty for the calibration of vernier calliper shows that in the same measurement model the change in the relation of uncertainty elements changes the output distribution. The change in the output distribution results in the change of the coverage interval which is very difficult to observe when GUM method is applied. The MCS method leads fast and simple to the information about the coverage interval which is very significant for the calculation and interpretation of the measurement uncertainty.

6. REFERENCES

BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML (1995) *Guide to the expression of uncertainty in measurement.* International Organization for Standardization, Geneva.

EA 4/02 (1999). *Expression of the Uncertainty of Measurement of Calibration.*

JCGM (2006). *Evaluation of measurement data – Supplement 1 to the Guide to the expression of uncertainty in measurement.* Propagation of distribution using Monte Carlo method. Joint Committee for Guides in Metrology

Cox, M. G. & Harris, P. M. (2001) *Measurement Uncertainty and the Propagation of Distributions*, 10th International Metrology Congress, Saint-Louis, France, 22-25 October 2001.

Cox, M. G. & Harris, P. M., (2001). *The planned supplemental guide to the GUM: Numerical methods for propagation distributions*, Euromet Length Workshop, October 2001.

Coleman, W. H. & Steele, W. G. (2000) *Experimentation and Uncertainty Analysis for Engineers,* A Wiley - Interscience Publication, John Wiley & Sons, Inc, USA.