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METHODOLOGY OF ANALYTICAL ESTIMATION OF UNCERTAINTY OF COORDINATE MEASUREMENTS

Jakubiec W.
University of Bielsko-Biala, Willowa 2, 43-309 Bielsko-Biala, Poland, wjakubiec@ath.bielsko.pl

Abstract: *The paper presents the methodology of analytical estimation of coordinate measurement uncertainty. The underlying assumption of this methodology is, that a measuring task is an indirect measurement, where measurands are not coordinates but differences of coordinates. The model used for analysis takes into account geometrical errors, kinematics and dimensions of the machine. The author defined the functions expressing maximum values occurring for differences of particular geometrical errors. The paper also demonstrates the models of several measuring tasks carried out on 2D machines with similar design to classical 3D machines.*
Key words: *metrology, coordinate measuring machine, uncertainty, distance, feature*

1. INTRODUCTION

The analytical models for the estimation of measurement uncertainty (Hernla, 2000; Pressel, 1997) proposed so far are not widely used. The objective of this paper is to show, that if a measuring task for coordinate metrology is treated as indirect measurement where measurands measured directly are distances along measuring axes (i.e. differences of coordinates) then a measuring task model can be built with a view to finding an effective estimate of measurement uncertainty by means of analytical techniques.
Moreover, just like for simulation technique, the model used for analysis of coordinate measurement uncertainty has to take into account geometrical errors, kinematics and dimensions of the machine (Forbes & Harris, 2000). Important is also the necessity of defining functions expressing maximum values occurring for differences of particular geometrical errors.
The following chapters present the models of several simple measuring tasks realised on 2D machines (of similar design to classical 3D machines). 2D models are simple enough for using analytical calculations of partial derivatives when calculating sensitivity coefficients. The proposed methodology can be generalised and used for tasks carried out on 3D machines.

2. DISTANCE BETWEEN TWO POINTS

The model of the measurement of the distance between two points by means of 2D measuring machine (with a design similar to classical 3D machines) is shown on Fig. 1.
The distance l between points $A(x_A, y_A)$ and $B(x_B, y_B)$ can be calculated by the following formula:

$$l = \sqrt{(x_{BA})^2 + (y_{BA})^2} \qquad (1)$$

where: $x_{BA} = x_B - x_A$, $y_{BA} = y_B - y_A$ (analogous indication in indexes are used in further designations of coordinates' differences)
According to the generality principle (Jakubiec & Malinowski, 2004) developed by the author, for the estimation of measurement uncertainty one has to assume that measurands are not coordinates but the differences of coordinates $(x_B - x_A)$ and $(y_B - y_A)$ measured directly by length measuring systems. The assumption that measurands are coordinates x_A, x_B, y_A and y_B is formally correct but does not fulfil the generality requirement and leads to the overestimation of measurement uncertainty and therefore is of no use.

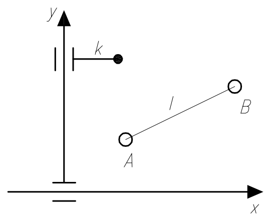

Fig. 1. Measurement of the distance between two points

The standard combine uncertainty can be calculated according to the following formula

$$u_l = \sqrt{\left(\frac{\partial l}{\partial x_{BA}} \cdot u_{xBA}\right)^2 + \left(\frac{\partial l}{\partial y_{BA}} \cdot u_{yBA}\right)^2} \qquad (2)$$

Sensitivity coefficients (partial derivatives) can be expressed as functions of measured length l:

$$\frac{\partial l}{\partial x_{BA}} = \frac{x_{BA}}{l}; \ \frac{\partial l}{\partial y_{BA}} = \frac{y_{BA}}{l} \qquad (3)$$

It can be seen that for the kinematical model of measuring machine shown on Fig. 1, the following geometrical errors of the machine occur: $xpx(x)$, $xty(x)$, $xrz(x)$, $ypy(y)$, $ytx(y)$, $yrz(y)$ and xwy. The error xwy is a real number and the remaining errors are functions of indication of the respective length measuring system of the machine.
The errors cause indication error in the point $A(x_A, y_A)$ which can be expressed as:

$$e_{xA} = xpx(x_A) + ytx(y_A) - xrz(x_A) \cdot y_A - xwy \cdot y_A \qquad (4)$$

$$e_{yA} = ypy(y_A) + xty(x_A) + yrz(y_A) \cdot k \qquad (5)$$

In order to determine component standard uncertainties u_{xBA} and u_{yBA} one has to note that the respective measurement errors e_{xBA} and e_{yBA} are differences between indication errors in the points A and B

$$e_{xBA} = e_{xB} - e_{xA}; \ e_{yBA} = e_{yB} - e_{yA} \qquad (6)$$

therefore they depend on geometrical errors of the machine according to the following formulae:

$$e_{xBA} = xpx(x_B) - xpx(x_A) + ytx(y_B) - ytx(y_A) + \\ - (xrz(x_B) \cdot y_B - xrz(x_A) \cdot y_A) + \\ - (xwy \cdot y_B - xwy \cdot y_A) \tag{7}$$

$$e_{yBA} = ypy(y_B) - ypy(y_A) + xty(x_B) - xty(x_A) + \\ + yrz(y_B) \cdot k - yrz(y_A) \cdot k \tag{8}$$

In other words, one can assume that the errors e_{xBA} and e_{yBA} are also calculated on the basis of indirect measurement as sums of several components.

In order to estimate properly the component uncertainty e_{xBA} one has to distinguish four components of the error:

$$e_1 = xpx(x_B) - xpx(x_A) \tag{9}$$

$$e_2 = ytx(y_B) - ytx(y_A) \tag{10}$$

$$e_3 = xrz(x_B) \cdot y_B - xrz(x_A) \cdot y_A \tag{11}$$

$$e_4 = xwy \cdot y_{BA} \tag{12}$$

Similarly for the component e_{yBA} one has to distinguish further three components of the error:

$$e_5 = ypy(y_B) - ypy(y_A) \tag{13}$$

$$e_6 = xty(x_B) - xty(x_A) \tag{14}$$

$$e_7 = k \cdot yrz(y_B) - k \cdot yrz(y_A) \tag{15}$$

All the mentioned components of the error will be assumed as equal to zero and then, the uncertainty of such an assumption will be estimated. For the mentioned components of the error, the following applies:

$$u_{(xB-xA)} = \sqrt{u_1^2 + u_2^2 + u_3^2 + u_4^2} \tag{16}$$

$$u_{(yB-yA)} = \sqrt{u_5^2 + u_6^2 + u_7^2} \tag{17}$$

In order to estimate the uncertainty u_i, below are defined the functions xpx_M, xty_M i xry_M (which constitute a certain representation of the respective functions *xpx*, *xty* and *xry*) expressing the maximum values that the differences of particular geometrical errors (position, straightness and rotational) can assume depending on the value of a measured distance (coordinate difference). The definition of these functions is an important element of the methodology of analytical estimation of coordinate measurement uncertainty.

In the model there occur two position errors, two straightness errors and two rotational errors. The maximum values which the differences of these errors can assume are defined as functions of differences of coordinates:

$$xpx_M(l) = \max_x (|xpx(x) - xpx(x+l)|) \tag{18}$$

$$ytx_M(l) = \max_y (|ytx(y) - ytx(y+l)|) \tag{19}$$

$$ypy_M(l) = \max_y (|ypy(y) - ypy(y+l)|) \tag{20}$$

$$xty_M(l) = \max_x (|xty(x) - xty(x+l)|) \tag{21}$$

$$xrz_M(l) = \max_x (|xrz(x) - xrz(x+l)|) \tag{22}$$

$$yrz_M(l) = \max_y (|yrz(y) - yrz(y+l)|) \tag{23}$$

Moreover, the maximum value xwy_M that the absolute value of error of the angle between axes *x* and *y* can assume was defined:

$$xwy_M = \max(|xwy|) \tag{24}$$

With the help of the above defined functions, the limiting values of 6 out of 7 of previously mentioned components of errors e_{xBA} and e_{yBA} can be estimated directly:

$$|xpx(x_B) - xpx(x_A)| \leq xpx_M(|x_{BA}|) \tag{25}$$

$$|ytx(y_B) - ytx(y_A)| \leq ytx_M(|y_{BA}|) \tag{26}$$

$$|xwy \cdot (y_B - y_A)| \leq xwy_M \cdot |y_{BA}| \tag{27}$$

$$|ypy(y_B) - ypy(y_A)| \leq ypy_M(|y_{BA}|) \tag{28}$$

$$|xty(x_B) - xty(x_A)| \leq xty_M(|x_{BA}|) \tag{29}$$

$$|k \cdot yrz(y_B) - k \cdot yrz(y_A)| \leq k \cdot yrz_M(|y_{BA}|) \tag{30}$$

Next, according to the logic of uncertainty estimation by means of the method B, for these 6 components of uncertainty, the respective components of standard uncertainty can be expressed as products of extreme value of errors and coefficient k_i resulting from the probability distribution of the error:

$$u_1 = k_1 \cdot xpx_M(|x_{BA}|) \tag{31}$$

$$u_2 = k_2 \cdot ytx_M(|y_{BA}|) \tag{32}$$

$$u_4 = k_4 \cdot xwy_M \cdot |y_{BA}| \tag{33}$$

$$u_5 = k_5 \cdot ypy_M(|y_{BA}|) \tag{34}$$

$$u_6 = k_6 \cdot xty_M(|x_{BA}|) \tag{35}$$

$$u_7 = k_7 \cdot k \cdot yrz_M(|y_{BA}|) \tag{36}$$

In one case, in order to find the proper estimate it is necessary to make e rearrangement allowing for the expressing of the component uncertainty as the function of xrz_M.

The maximum value that the component e_3 assumes can be estimated in two ways:

$$|xrz(x_B) \cdot y_B - xrz(x_A) \cdot y_A| = \\ = |y_A \cdot (xrz(x_B) - xrz(x_A)) + xrz(x_B) \cdot y_{BA}| \leq \\ \leq xrz_M(|x_{BA}|) \cdot y_A + xrz_{MAX}(x_B) \cdot |y_{BA}| \tag{37}$$

$$|xrz(x_B) \cdot y_B - xrz(x_A) \cdot y_A| = \\ = |y_B \cdot (xrz(x_B) - xrz(x_A)) + xrz(x_A) \cdot y_{BA}| \leq \\ \leq xrz_M(|x_{BA}|) \cdot y_B + xrz_M(x_A) \cdot |y_{BA}| \tag{38}$$

Since both estimates are true, the estimate giving smaller value should be used for the calculation of measurement uncertainty. Which formula gives smaller value it results from the position of the points *A* and *B* in the machine coordinate system. Therefore, one should always calculate both values and assume the smaller value to be the component of standard uncertainty.

The above estimates consider the fact that the analysed component of uncertainty depends on the measured difference of coordinates. If this fact were not considered, then as a third option, the estimate resulting directly from the formula (11) could be taken. Finally, one should use the following formulae:

$$u_{31} = k_3 \cdot [(xrz_M(|x_{BA}|)) \cdot y_A + (xrz_M(x_B)) \cdot |y_{BA}|] \\ u_{32} = k_3 \cdot [(xrz_M(|x_{BA}|)) \cdot y_B + (xrz_M(x_A)) \cdot |y_{BA}|] \\ u_{33} = k_3 \sqrt{y_B^2 \cdot (xrz_M(x_B))^2 + y_A^2 \cdot (xrz_M(x_A))^2} \\ u_3 = \min\{u_{31}, u_{32}, u_{33}\} \tag{39}$$

When calculating the components of uncertainty caused by rotational errors there is always a possibility of finding two or more different estimates. One should find these estimates and use the estimate with smallest value for the calculation of measurement uncertainty. The above statement is a key element of presented methodology.

3. MAXIMUM DIFFERENCES OF ERRORS

In order to determine the functions expressing the maximum values of particular errors' differences one needs to know the functions expressing the geometrical errors of the machine.

The PTB methodology used at the stage of data gathering for estimation of measurement uncertainty by simulation (Trapet et al., 1999) states that particular geometrical errors are identified on the basis of the results of repeated measurement of ball or hole plate. As a direct result of the procedure of identification for single measurement of the ball or hole plate one obtains the discrete values of functions expressing geometrical errors, with the digitising step arising from ball or hole plate pitch (Fig. 2a). Mean values of the results gathered from all repeated measurements are used for the determination of the systematic component of geometrical errors, and the standard deviation is a measure of the random component (Fig. 2b). During the simulation, values in the digitisation point are calculated as sums of systematic component and generated values of random component. Values in other points are interpolated.

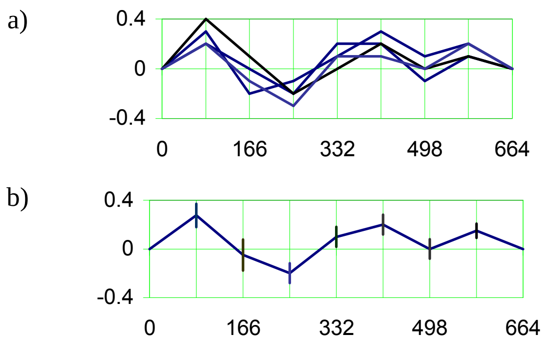

Fig. 2. Illustration of the PTB methodology: a) geometrical errors identified during repeated measurements of the ball or hole plate, b) data for simulation are systematic and random components

The proposed methodology has analogies with the PTB methodology and is described in the following way. The function expressing maximum values of errors' differences (Fig. 3a) is determined for each direct result of the identification procedure. For example, for *i*-th position error $xpx(x)$ the function $xpx_M(i,l)$ is calculated. Next, from the functions of all repeated measurements, the common function $xpx_M(l)$ is calculated according to the following formula (Fig. 3b):

$$xpx_M(l) = \max_i \{xpx_M(i,l)\} \tag{40}$$

For the purpose of this research theoretical models of geometrical errors were used. First of all it was assumed that the CAA procedure was carried out for the machine. Next assumption is that the typical cause of position errors is temperature error, thus functions $xpx(x)$ and $ypy(y)$ are linear without constant component (Fig. 4a). The typical reason of straightness and rotational errors are deformations of the machine guides caused by temperature gradients (Dorp van B. at al., 2001). So, the second degree polynomial was used as the model of straightness errors (Fig. 5a) and a linear function as the model of rotational errors (Fig. 6a).

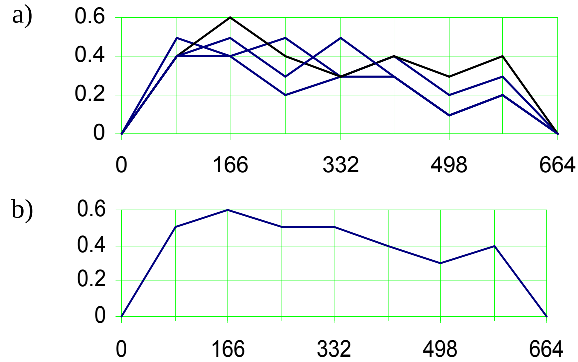

Fig. 3. Illustration of methodology proposed by the author: a) for each geometrical error from Fig. 2a a function expressing maximum values of errors’ differences is determined, b) from the functions from Fig. 3a the common function is determined

It turned out that for the position errors modelled by linear function without constant component the function expressing maximum errors' difference is a linear function with the slope equal to absolute value of the slope of the error function. For example for the function (Fig. 4a)

$$xpx(x) = ax \tag{41}$$

respective function expressing maximum errors is as follows (Fig. 4b)

$$xpx_M(l) = |a| \cdot l \tag{42}$$

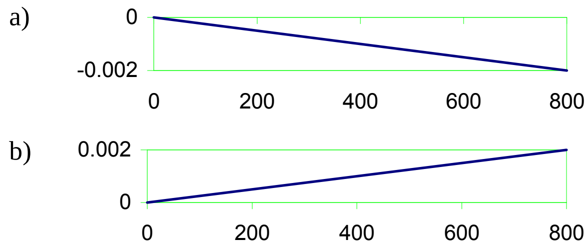

Fig. 4. Function expressing maximum effects of position error: a) exemplary chart of position error, b) chart of respective maximum errors' difference

Therefore one function with the slope *a* can be used as an estimate of maximum errors' difference for the whole family of functions of geometrical errors with the slope equal or less than absolute value of *a*.

It also turned out that for rotational errors modelled by the linear function with a constant (Fig. 5a) the function expressing maximum errors' difference is linear without constant with the slope equal to absolute value of the slope of the error function.

For example for the function

$$xry(x) = ax + b \tag{43}$$

respective function expressing maximum errors' difference is as follows (Fig. 5b)

$$xry_M(l) = |a| \cdot l \tag{44}$$

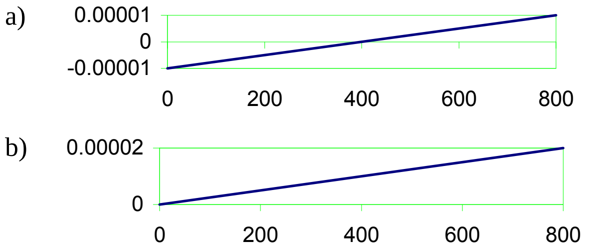

Fig. 5. Function expressing maximum effects of rotational error: a) exemplary chart of rotational error, b) chart of respective maximum errors' difference

Therefore, one linear function with the slope *a* can be used as an estimate of maximum errors' difference for the whole family of linear functions describing geometrical errors with the slope equal or less than absolute value of *a*.
Taking advantage of the purposefully developed software, respective functions expressing maximum errors' difference for straightness error in a form of an arc were obtained. It turned out that the function expressing the maximum errors' difference is also an arc with the identical deflection. For example, the function (Fig. 6a)

$$xty(x) = \frac{4s}{l^2}\left(x - \frac{l}{2}\right)^2 - s \qquad (45)$$

yields the function expressing maximum errors shown on Fig. 6b.

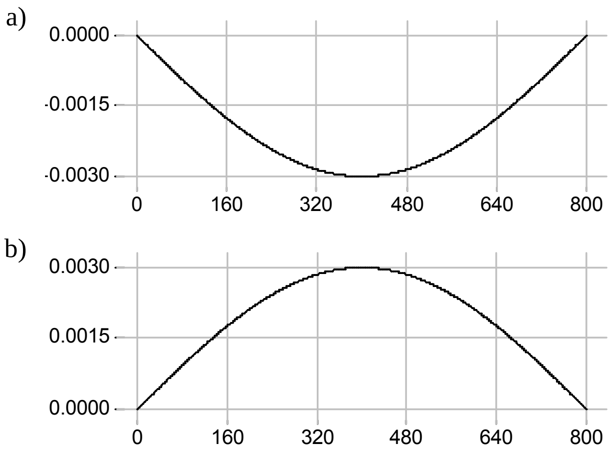

Fig. 6. Function expressing maximum effects of straightness error: a) exemplary chart of straightness error, b) chart of respective maximum error

Similarly, one function can be used as an estimate of maximum errors for the whole family of functions of geometrical errors, which have the form of an arc with deflection equal or less than *s*.
The proposed model was tested on the example of measurement of distance between two points for the following input data (only systematic errors are included):

$xpx_M = ypy_M = 0{,}002l/800$

$xty_M = ytx_M$ – arc according to (45), $s = 0{,}001$; $l = 800$

$xrz_M = 0{,}000006l$

$xwy_M = 0{,}000004l$

The resulting estimates of standard measurement uncertainty is presented on Fig. 7. The estimates concern four different lengths (30, 100, 300 i 600 mm). Three different orientations of the measured length were analysed. In each case the measured lengths are distributed in 27 different places in the measuring volume.

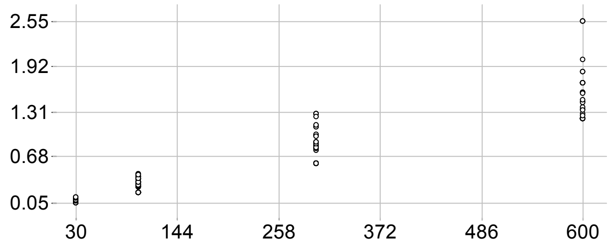

Fig. 7. The standard measurement uncertainty of distance between two points

4. DISTANCE BETWEEN POINT AND LINE

Points belonging to the line are marked as $A(x_A, y_A)$ and $B(x_B, y_B)$, and the point whose distance to the line *AB* is to be measured is $S(x_S, y_S)$ (Fig. 8).
The distance between the point *S* and the line *AB* is calculated as

$$l = |v \cdot r| \qquad (46)$$

where *v* – vector perpendicular to line *AB*, *r* – vector connecting any point of the line *AB* and the point *S*.

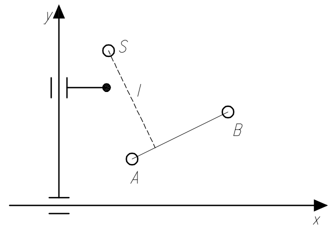

Fig. 8. Measurement of distance between point and line

Vector *v* is calculated as:

$$v = \left(\frac{y_{BA}}{\sqrt{x_{BA}^2 + y_{BA}^2}} ; \frac{-x_{BA}}{\sqrt{x_{BA}^2 + y_{BA}^2}} \right) \qquad (47)$$

With regard to the vector *r* two options are considered. In the first one, the point *A* is used for the definition of the vector *r*, and in the second one it is point *B*.
As a result the distance between the point and the line will be a function of difference of coordinates of the points *S* and *A* (first option) or the points *S* and *B* (second option).

$$r = \left((x_{AS}, y_{AS}) \text{ or } (x_{BS}, y_{BS}) \right) \qquad (48)$$

The distance *l* between the point and the line is

$$l = \left(\frac{|x_{AS} \cdot y_{BA} - x_{BA} \cdot y_{AS}|}{\sqrt{x_{BA}^2 + y_{BA}^2}} \text{ or } \frac{|x_{BS} \cdot y_{BA} - x_{BA} \cdot y_{BS}|}{\sqrt{x_{BA}^2 + y_{BA}^2}} \right) \qquad (49)$$

For the estimation of measurement uncertainty one has to assume that the measurands are distances x_{BA} and y_{BA} as well as x_{AS} and y_{AS} or x_{BS} and y_{BS}, depending on the option.So, the distance *l* is a function of four measured directly distances

$$l = \left(l(x_{BA}, y_{BA}, x_{AS}, y_{AS}) \text{ or } l(x_{BA}, y_{BA}, x_{BS}, y_{BS}) \right) \qquad (50)$$

Standard uncertainty of the measurement of the distance *l* is calculated as the minimum of 2 estimates:

$$u_l = \min\{u_{l1}, u_{l2}\} \qquad (51)$$

$$u_{l1}=\left\{\sqrt{\begin{aligned}&\left(\frac{\partial l}{\partial x_{BA}}\cdot u_{xBA}\right)^2+\left(\frac{\partial l}{\partial y_{BA}}\cdot u_{yBA}\right)^2+\\&+\left(\frac{\partial l}{\partial x_{AS}}\cdot u_{xAS}\right)^2+\left(\frac{\partial l}{\partial y_{AS}}\cdot u_{yAS}\right)^2\end{aligned}}\right. \tag{52}$$

$$u_{l2}=\left\{\sqrt{\begin{aligned}&\left(\frac{\partial l}{\partial x_{BA}}\cdot u_{xBA}\right)^2+\left(\frac{\partial l}{\partial y_{BA}}\cdot u_{yBA}\right)^2+\\&+\left(\frac{\partial l}{\partial x_{BS}}\cdot u_{xBS}\right)^2+\left(\frac{\partial l}{\partial y_{BS}}\cdot u_{yBS}\right)^2\end{aligned}}\right. \tag{53}$$

Sensitivity coefficients (partial derivatives) are respectively the following:

$$\frac{\partial l}{\partial x_{BA}}=\begin{cases}\dfrac{y_{BA}\cdot\left(y_{BA}\cdot y_{AS}+x_{BA}\cdot x_{AS}\right)}{\left(x_{BA}^2+y_{BA}^2\right)^{3/2}}\\[2ex]\dfrac{y_{BA}\left(y_{BA}\cdot y_{BS}+x_{BA}\cdot x_{BS}\right)}{\left(x_{BA}^2+y_{BA}^2\right)^{3/2}}\end{cases} \tag{54}$$

$$\frac{\partial l}{\partial y_{BA}}=\begin{cases}-\dfrac{x_{BA}\cdot\left(x_{BA}\cdot x_{AS}+y_{BA}\cdot y_{AS}\right)}{\left(x_{BA}^2+y_{BA}^2\right)^{3/2}}\\[2ex]-\dfrac{x_{BA}\left(x_{BA}\cdot x_{BS}+y_{BA}\cdot y_{BS}\right)}{\left(x_{BA}^2+y_{BA}^2\right)^{3/2}}\end{cases} \tag{55}$$

$$\frac{\partial l}{\partial x_{AS}}=\frac{\partial l}{\partial x_{BS}}=\frac{-y_{BA}}{\sqrt{x_{BA}^2+y_{BA}^2}} \tag{56}$$

$$\frac{\partial l}{\partial y_{AS}}=\frac{\partial l}{\partial y_{BS}}=\frac{x_{BA}}{\sqrt{x_{BA}^2+y_{BA}^2}} \tag{57}$$

To determine the component standard uncertainties u_{xBA} and u_{yBA} , u_{xAS} and u_{yAS} as well as u_{xBS} and u_{yBS} one has to notice that respective measuring errors depend on the geometrical errors of the machine according to functions analogical to (7) i (8). Consequently, the relationships (16) and (17) are in force as well as the estimates of standard uncertainties (31) to (36) and (39).

In a special case, when the line *AB* is parallel to the *x* axis ($y_A = y_B$), the sensitivity coefficients assume the following values:

$$\frac{\partial l}{\partial x_{BA}}=\frac{\partial l}{\partial x_{AS}}=\frac{\partial l}{\partial x_{BS}}=0 \tag{58}$$

$$\frac{\partial l}{\partial y_{AS}}=\frac{\partial l}{\partial y_{BS}}=1 \tag{59}$$

Then:

$$u_{l1}=\left\{\sqrt{\left(\frac{\partial l}{\partial y_{BA}}\cdot u_{yBA}\right)^2+u_{yAS}^2}\right. \tag{60}$$

$$u_{l2}=\left\{\sqrt{\left(\frac{\partial l}{\partial y_{BA}}\cdot u_{yBA}\right)^2+u_{yBS}^2}\right. \tag{61}$$

The analysis was performed for the same data as in chapter 3 and assuming that the distance *AB* is 100 mm and is oriented along *x*-axis ($x_A = 200$, $y_A = 200$, $x_B = 300$, $y_B = 200$).

The chart (Fig. 9) shows estimate of the standard measurement uncertainty of distance between point *S* and a straight line *AB* with different locations of point *S*:

$$x_S=\frac{x_A+x_B}{2}+t\cdot\left(x_B-x_A\right);t=-1{,}5\ldots1{,}5$$

$$y_S=250$$

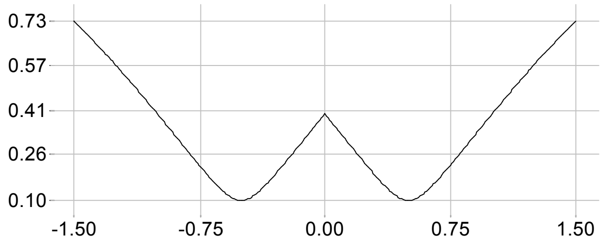

Fig. 9. The standard measurement uncertainty of the distance between point and a straight line for different locations of the point

The Fig. 10 shows an estimate of the standard measurement uncertainty of the distance between point and a straight line in a relation to orientation of the straight line *AB* and the point *S* in the coordinate system.

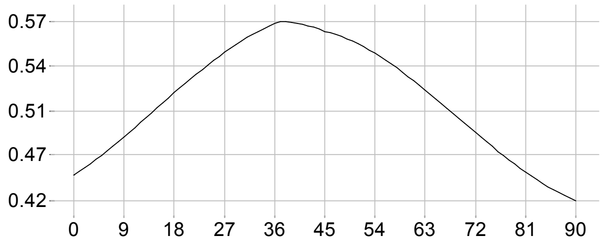

Fig. 10. The standard uncertainty of measurement of the distance between a point and a straight line in a relation to orientation of the straight line *AB* and the point *S* in the coordinate system.

5. PERPENDICULARITY

The straight line being datum is determined by two points $A(x_A, y_A)$ and $B(x_B, y_B)$. The straight line for which the deviation is to be calculated is determined by the points $C(x_C, y_C)$ and $D(x_D, y_D)$ (Fig. 11).

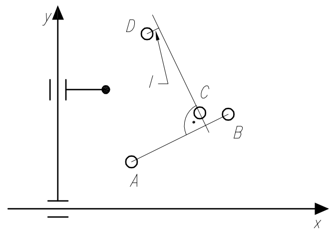

Fig. 11. Measurement of perpendicularity deviation of two straight lines

The deviation of perpendicularity is calculated as the distance between the point *D* and the straight line perpendicular to the straight line *AB* and going through point *C*.

The distance between the point *D* and the straight line can be calculated as

$$l=\left|v\cdot r\right| \tag{62}$$

where: *v* – vector parallel to straight line *AB*, *r* – vector CD

$$v = \left(\frac{x_{BA}}{\sqrt{x_{BA}^2 + y_{BA}^2}}; \frac{y_{BA}}{\sqrt{x_{BA}^2 + y_{BA}^2}} \right) \quad (63)$$

$$r = (x_{DC}, y_{DC}) \quad (64)$$

$$l = \frac{x_{BA} \cdot x_{DC} + y_{BA} \cdot y_{DC}}{\sqrt{x_{BA}^2 + y_{BA}^2}} \quad (65)$$

For the estimation of the measurement uncertainty one has to assume that the deviation of perpendicularity is the function of four differences of coordinates measured directly .

$$l = l(x_{BA}, y_{BA}, x_{DC}, y_{DC}) \quad (66)$$

Respective partial derivatives

$$\frac{\partial l}{\partial x_{BA}} = \frac{y_{BA}(y_{BA} \cdot x_{DC} - x_{BA} \cdot y_{DC})}{(x_{BA}^2 + y_{BA}^2)^{3/2}} \quad (67)$$

$$\frac{\partial l}{\partial y_{BA}} = \frac{x_{BA}(x_{BA} \cdot y_{DC} - y_{BA} \cdot x_{DC})}{(x_{BA}^2 + y_{BA}^2)^{3/2}} \quad (68)$$

$$\frac{\partial l}{\partial x_{DC}} = \frac{x_{BA}}{\sqrt{x_{BA}^2 + y_{BA}^2}} \quad (69)$$

$$\frac{\partial l}{\partial y_{DC}} = \frac{y_{BA}}{\sqrt{x_{BA}^2 + y_{BA}^2}} \quad (70)$$

For the above data, the analysis of measurement uncertainty of perpendicularity deviation is performed. The Fig. 12 shows the change of standard uncertainty in relation to the length of datum *AB*. The length of the line *CD* is 100 mm.

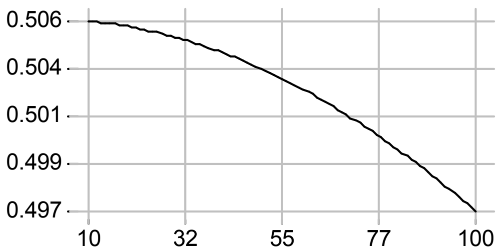

Fig. 12. The standard measurement uncertainty of perpendicularity deviation in relation to the length of datum line *AB*

6. CONCLUSIONS

The proper definition of the measurement model is the fundamental element enabling the use of analytical technique. The significantly better estimate of uncertainty is reached thanks to the assumption that the measurands are not coordinates but differences of coordinates.

Another crucial element of the proposed methodology is the definition of the functions of maximum values of the errors' differences, with arguments as measured differences of coordinates.

The proposed methodology is in accordance with the methodology of estimation of measurement uncertainty described in GUM and the commonly accepted methodology proposed by the PTB for virtual CMM (Trapet et al., 1999). It uses the same input data and the same models for description of geometrical errors. The proposed methodology has similar limitations, e.g. the quality of the result is dependent on the quality of input data.

Compared to the virtual CMM it has the following advantages.

- The generalization of the results is easier — estimated uncertainty barely depends on the location of the workpiece in the measuring volume.
- The functions expressing the uncertainty in the relation to the orientation of the workpiece in the coordinate system can be presented graphically, while VCMM gives the uncertainty for the particular position of the workpiece.
- It is possible to derive formulae valid for particular measuring tasks, which can be used directly by users of CMMs of similar type. Therefore the methodology should be easier to implement in industrial conditions.

The research on the connection between the type and value of geometrical errors, and formulae expressing the maximum values of the differences of the errors should indicate the areas for improvement of CMMs.

The proposed methodology was described on the example of 2D measuring machines but it has already been extended to 3D machines.

7. REFERENCES

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