EVALUATION OF FORM ERRORS IN CYLINDRICAL FEATURES USING COORDINATE MEASUREMENT DATA

Ravindra.K and Chittaranjan Das.V
R.V.R. & J.C. College of Engineering, Guntur-522019, A.P. E-mail: ravindra_kom@rediffmail.com

Madar Valli. P
College of Engineering, GITAM, Visakhapatnam, A.P.

Nageswara Rao. D
College of Engineering, Andhra University, Visakhapatnam, A.P.

Abstract: *The geometric features of the manufactured components are inspected for their acceptability. Most of the machine parts consist of cylindrical form, which should possess the required geometrical tolerance and should be checked for their acceptance. At the present scenario, the Coordinate Measuring Machines (CMMs) are considered to be reliable and capable to perform automated inspection on the manufactured parts. In CMMs, the measured data is being processed using Least Squares Method to assess the geometric features of the part. The error values assessed by CMM are not the minimum, there by resulting in the rejection of good parts. In this paper a technique is presented to assess the cylindricity, which gives the minimum form error using the CMM data. The proposed method is simple and the results depict the validity of the assessed cylindricity at par with that of the other techniques.*
Keywords: *Form errors, Circularity, Cylindricity, Coordinate Measuring Machine*

1. INTRODUCTION

In the present manufacturing scenario, the parts are aught to be manufactured with good quality and specified geometric tolerances. Some times these tolerances are very close. Parts are inspected for compliance with the tolerance specifications. There are different inspection procedures proposed by researchers to assess the dimensional tolerarances of the geometrical features of the manufactured parts. With the advancement of computerized technology, a high precision machine, the Co-ordinate Measuring Machine (CMM), was developed for inspection purposes. Current CMM verification algorithms are based on the Least Squares solution, which minimizes the sum of squared errors. This method has a generalized structure that can be applied to various geometric features, including a sculptured surface; but it yields an approximate solution that does not guaranty the minimum zone value (T.S.R. Murthy et.al.,1980); and there by results in a possible over estimation of the form tolerance. There fore, the CMM algorithms may successfully reject some good parts (Rao.U. et.al.,1994).

2. LITERATURE REVIEW

The form errors of three dimensional surfaces like cylindrical surfaces are important features from the point of their function to be performed in fundamental mechanical products such as transmission systems, revolving devices, assembly parts, injection molds and precision gauges. Various factors in the manufacturing processes may cause a cylindrical feature to depart from its ideal shape. The measurement of circularity of cylindrical components in a particular section gives limited information, which is sufficient for general engineering applications. In case of precision machines and instruments, these components have to satisfy stringent geometrical and form tolerances. Since most of the functional components are cylindrical, the measurement and evaluation of cylindrical components are needed.

The ISO and DIN7184, BS308 Standards give the following definitions:

1. The radial separation of two co-axial cylinders fitted to the total surface under test such that their radial separation is a minimum.
2. The surface of the component is required to lie between two co-axial cylindrical surfaces having a radial separation of the specified tolerance.

The geometric interpretation of cylindricity is given in ANSI Y14.5M (1994), in which "the Cylindricity is defined as a combination of Circularity, Straightness and Taper."

Goto, M. and Izuka, K. (1978) proposed a Method of Orthogonal Polynomials by representing the cylindricity error by orthogonal functions consisting of Fourier series and orthogonal polynomials. Kakino, Y. and Kitazava, J.(1978) developed Multi Stylus Method for in process measurement of cylindricity. T.S.R. Murthy (1982) developed an orthogonal polynomial technique and the surface development method to evaluate the cylindricity of a work part. The technique uses orthogonal functions, including Fourier series and orthogonal polynomials to represent the cylindricity. Tsukada and Kanada (1988) presented an optimal search method for solving the cylindricity problem. The method is used to search for the direction of cylindrical axis and the best fit circle in various distances from the cylindrical axis in such a way that the minimum zone solution can be reached.

Shunmugam, M.S.(1987) found the minimum average deviation of a cylinder by using Simplex Method. The minimum average deviation minimizes the function, instead of a minimum zone solution. The result obtained appears to be closer than that obtained by the Least Squares solution. Dean J.W. Dawson (1992) analyzed the limitations of current instrumentation and measurement techniques with respect to the cylindricity. Four types of reference cylinders that fit to the measured data are analyzed each with its relevant application criteria:

(i) the minimum zone cylinder is used where running fits are used,
(ii) the minimum circumscribed cylinder is used in components where the surface of the outside diameter is important, such as plug gauges for the inspection of internal bores,
(iii) the maximum inscribed cylinder is used where the inside diameter of a component is important such as ring gauges, and
(iv) the least squares cylinder (LSC) is used in many general applications when measuring cylindricity.

The traditional hard gauge, as opposed to the soft gauge in the form of software, can only provide a pass or fail answer for the assessment ie.. a go and no-go inspection method. Due to the great variation among the cylindrical features in the modern products, different types of gauges are required, which limits the applicability of specialized hard gauges.

Roy, U. and Yaoxian, Xu. (1995) Proposed computational geometry based techniques that are used to generate a pair of concentric cylinders for checking the cylindricity tolerance and a center axis for the verification of orientation tolerances of cylindrical feature in a computer aided automatic inspection environment. This method of checking the cylindricity tolerance is more efficient and accurate than the traditional Least Squares Method. Lai and Chen (1996) used a nonlinear transformation method to convert a cylinder into a plane and then applied the CPRS (Control Plane Rotation Scheme) flatness evaluation scheme to obtain appropriate control points, there by a series of inverse transformation procedures are employed to compute the cylindrical parameters.

Radhakrishnan et. al. (1998) proposed a linear iterative cyclic co-ordinate search technique to obtain the optimal solution for cylindricity evaluation. The distribution of characteristic data points that provides the minimum zone cylindricity is discussed. Chen and Wu (1999) applied a mathematical model for the minimum circumscribe cylinder to evaluate the accuracy of the spindle of a CNC machine. Choi and Kurfess (1999) proposed a general zone-fitting method that can be applied to characterize various geometric features, including nonlinear parametric surfaces and multiple combined surfaces. Shuo-Yan Chou and Chung-Wei Sun (2000) proposed an approach which provides a more accurate method for assessing cylindricity for cylindrical features in a complex environment. The mathematical model derives a fine tuned axis of the reference cylinder. The global optimum of the model yields the assessment of cylindricity by using the Simulated Annealing Algorithm, namely Hide and Seek. Hsin-Yi Lai et. al. (2000) proposed a heuristic approach to model form errors for cylindricity evaluation using Genetic Algorithms (GAs). Numerical examples indicated that the proposed GA method provides better accuracy on cylindrical evaluation compared to least squares method. This method shows good flexibility and excellent performance in evaluating the engineering surfaces using co-ordinate measurement data. Lao, Y.Z. et al. (2003) proposed a simple procedure for appropriately initializing the estimation of the axis. This is relevant to the three-dimensional straightness evaluation. Axis estimation and the hyperboloid technique constitute an integrated methodology for cylindricity evaluation. Later Devillers, O. et al. (2003) showed that constructing the zone cylinder through six points in space involves the solution of a system of six degree – four equations. But it requires sophisticated algebraic geometry tools. Hossein Cheraghi et al. (2003) presented efficient procedures for the evaluation of cylindricity error and straightness errors of medial line accurately and quickly, as both are having similar structures. The advantage of the proposed cylindricity procedure is that it allows different methods for the evaluation of cylindricity error such as MCC, MIC, MRS or LSCs.

3. CYLINDRICITY

Many machine parts possess cylindrical features both internal and external. These parts should meet the required tolerances for proper functioning in an assembly. The form error of three-dimensional surfaces like cylindrical surfaces should be precisely tested and evaluated. Two basic geometric characteristics that are used to control form and function of cylindrical features are Cylindricity and Straightness of an axis.

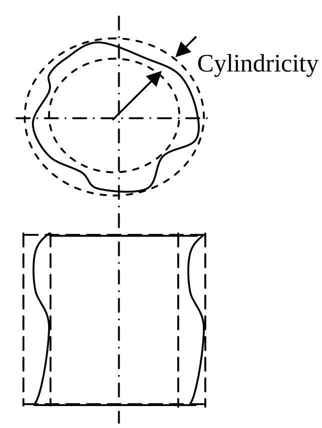

Fig: 1 Geometric Interpretation of Cylindricity

Fig: 1 shows a cylindrical feature with the form / contour bounded by two coaxial cylinders. The radial deviation between the two cylinders is called ‘Cylindricity’ error.

The Cylindrical form errors are estimated by taking measurements at different sections perpendicular to the axis. As the measurements are confined to each section plane, Circularity errors and centers are evaluated for each plane. The circularity traces of different planes are combined with the straightness of the generators of the cylinder. Generally, the methods for the evaluation of form errors can be divided into the Intrinsic datum methods and Extrinsic datum methods.

3.1 Intrinsic Datum Method

In this method, points on the surface of the parts are used as datum. The most commonly used intrinsic techniques for cylindricity evaluation are diametrical measurement, V-block measurement and bench center measurement. These methods are generally inaccurate due to existence of multiple sources of error associated with the measurement process.

3.2 Extrinsic Datum Method

An external member is used as the datum surface. The measurement data on the cylindrical surface is collected and used in a computer program to calculate the geometric form error. These methods are more time consuming to apply, but results in more accurate evaluation. With the existence of Co-ordinate Measuring Machines, more accurate co-ordinate information is available and there by minimizes the time consumption. The cylindricity and straightness errors are evaluated by several criteria based on the Reference Cylinders. A reference cylinder is first obtained by using the data measured from the surface of the feature. The measured points are then compared with the reference cylinder. There are four common methods to assess the reference cylinders:

1. Least Squares Cylinder (LSC) method
2. Maximum Inscribed Cylinder (MIC) method
3. Minimum Circumscribed Cylinder (MCC) method
4. Minimum Zone Cylinder (MZC) method

3.2.1 Least Squares Cylinder Method

The least squares Cylinder represents the average of all peaks and valleys of the surface contour, and can be defined mathematically “the sum of the squares of a sufficient number of equally spaced radial ordinates measured from the reference cylinder to the profile has minimum value”. The axis of the least square cylinder is used to fit the smallest circumscribed and the largest inscribed cylinders to the profile. The radial difference between these two cylinders is used as the measure of Cylindricity error.

Fig:2 shows the cylindrical contour contained by the co-axial inner and outer cylinders, whose axis is the axis of the Least Squares Reference cylinder. The Least Squares fitting technique is the commonly used method for Cylindricity evaluation. The least Squares technique is efficient in computation and is widely used in Co-ordinate Measurement Machines.

For a cylindrical feature having measurement data as P_i (x_i, y_i, z_i), the deviation of the point ‘P_i’ from the least squares axis (a, b, c) is represented by the following equation:

$$\mathbf{e = R - r} \qquad \text{--------} \quad \{ 1 \}$$

Where r = minimum of r_c, R = maximum of r_c, r_c = normal radial location of the point from the LSC axis

This method is the most popular approach for computing the cylindricity form error approximately because of its ease of computation. Earlier standards accepted this method for assessing cylindricity. But, in the case of high precision measurement, assessment of Cylindricity error by the least squares method results in unnecessary waste of materials due to lower acceptance rate.

4. PROPOSED METHODOLOGY

Cylindrical features are characterized by its form error ‘Cylindricity’ and ‘Straightness’. In this proposed methodology, computational geometry based technique is used to generate a pair of concentric cylinders to check the cylindricity error and Straightness error for the verification of orientation errors of cylindrical feature. The co-ordinate positions of the points representing the contour of the cylindrical feature are processed to evaluate the cylindricity and straightness errors. The proposed methodology is explained below for the evaluation of Cylindricity and Straightness errors.

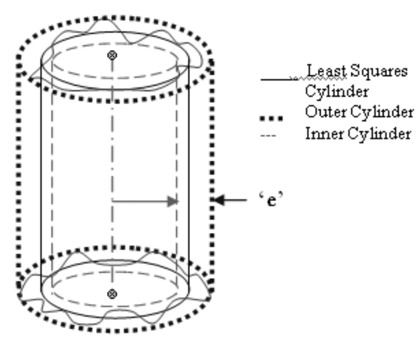

Fig: 2 Cylindricity in Least Squares Cylinder Method

4.1 Evaluation of Cylindricity

As shown in the Fig: 3 the cylindrical surface is divided into several cross-sections normal to the local Z-axis. The number of cross sections is dependent on the required accuracy of inspection. The data points $P_{i,j}$ are obtained by scanning the contour of the cylindrical feature on Coordinate Measuring machine, where ‘i’ is the number of points on each plane (parallel to X-Y plane) and ‘j’ is the position of the plane normal to the cylinder Z-axis.

The data on each plane is processed to evaluate the centres using the methodology proposed by Ravindra.K et.al. (2004). It processes the given co-ordinate positions of the points representing the contour of the circular feature. In this methodology, the centre of the reference circle is evaluated for which the radial deviation between the nearest and the farthest points lying on the contour is minimum.

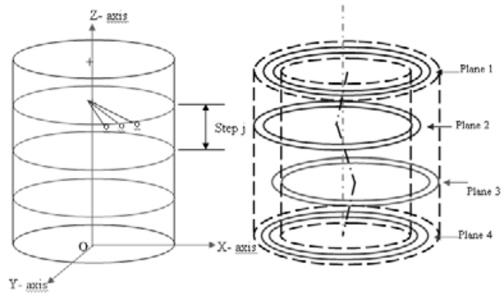

Fig: 3 Division into lateral cross sections and their circularity errors with evaluated centers

Fig: 4.a shows the scattered positions of the measured points of the contour. In each iteration process, it is assumed that a circle is passing through the randomly selected set having three points. Using the concept of three point circle method, the centre of the circle is evaluated.

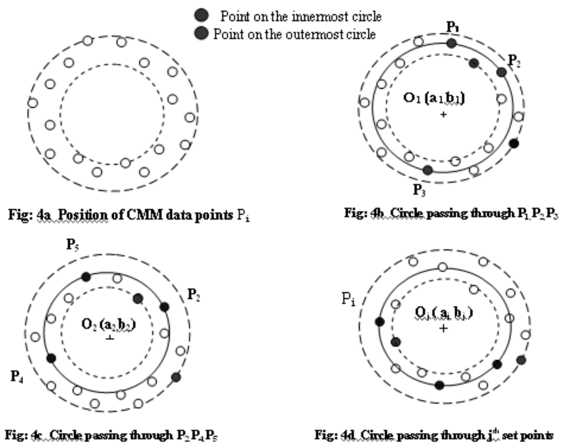

Fig: 4a Position of CMM data points P_i

Fig: 4b Circle passing through $P_1 P_2 P_3$

Fig: 4c Circle passing through $P_2 P_4 P_5$

Fig: 4d Circle passing through j^{th} set points

Fig: 4.b shows the circle which is passing through the selected points P_1, P_2 & P_3 and O_1 (a_1, b_1). The radial deviation (e_1) between the farthest and the nearest points from this center is evaluated. In the next step, another set of three points are selected. The new centre of the circle and the corresponding radial deviation between the farthest and nearest points is evaluated. Fig: 4c shows the new center $O_2(a_2, b_2)$ of the circle which is passing through the second set of points.

Fig: 4d shows the selected three points from the data set P_i (x_i, y_i), for which the center of the assessing circle as O_j (a_j, b_j). The radial deviation between the farthest and nearest points is 'e_j'. In this manner, the centers (O_1, O_2, --- O_j) and the corresponding radial deviations (e_1, e_2 ---- e_j) are evaluated for all possible combinations of point sets. From these centers, the center with minimum radial deviation is considered. The radial deviation between the farthest and nearest points is the minimum radial deviation. This is the minimum 'Circularity or Form error' that can be assessed from the co-ordinate measurement data.

The two concentric circles with minimum radial separation for the data points $P_i(x_i, y_i, z_i)$ and their center point $O_j(a_j, b_j, c_j)$ are evaluated. The axis of the cylindrical feature is assessed from the evaluated centers (O_j) of each cross-sectional point sets.

The best fit axis through these centers is evaluated from the centers $O_j(a_j, b_j, c_j)$. To evaluate the inner and outer cylinders to retain the entire data points of the cylindrical feature, the best fit axis to these centers is evaluated. With reference to the axis, the normal radial distances of all the points P_i from the axis are evaluated. The radial deviation between the farthest and nearest points gives the form error 'Cylindricity'.

4.2 Evaluation of Straightness

Straightness error of the median line (best fit line joining the evaluated centers in each plane) is evaluated by transforming to a 2-D minimum circumscribed circle problem. The center points of each plane representing the median line are projected on to X-Y plane. The diameter of the smallest circumscribed circle containing these projected centers is evaluated as 'Straightness error'.

5. MATHEMATICAL MODELLING

CMM measurement data points $P_i(x_i, y_i)$ of a circular feature in each plane are processed to evaluate the center of the best fit circle and the minimum circularity error. Fig:5 shows the scattered points P_i of the circular feature. Let $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$ be the points through which the assessing circle passes and O_j (a_j, b_j) be the centre of the circle.

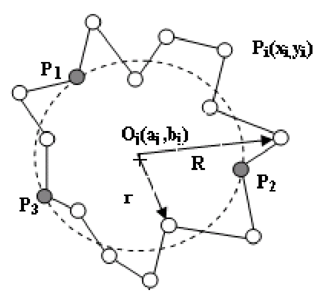

Fig: 5 Scattered points on the part feature

The radial location of each point of P_1, P_2 and P3 is 'R' from the center Oj (a_j, b_j) for j = 1 and 'R' is expressed as:

$$[(x_1 - a_j)^2 + (y_1 - b_j)^2]^2 = R^2 \quad \text{----- (2)}$$

$$[(x_2 - a_j)^2 + (y_2 - b_j)^2]^2 = R^2 \quad \text{----- (3)}$$

$$[(x_3 - a_j)^2 + (y_3 - b_j)^2]^2 = R^2 \quad \text{----- (4)}$$

By solving the above equations, the centre of the circle O_j (a_j, b_j) is evaluated. The radial distance of the points from the center O_j (a_j, b_j) is

$$r_i = [(x_i - a_j)^2 + (y_i - b_j)^2]^{1/2} \quad \text{---- (5)}$$

for i = 1 to n

$$\text{Deviation} = e_j = R_j - r_j \quad \text{----- (6)}$$

for j = 1 to $^nc_3$

Where $R_j = \max [r_i]$ and $r_j = \min [r_i]$

$$\text{Circularity} = e = \text{Min} [e_j] \quad \text{----- (7)}$$

Eq. 7 gives the 'Circularity' error.

As shown in the Fig: 3, the measurement data generated at each plane are P_i (x_i, y_i, z_i) on the cylindrical surface. The number of planes scanned is ‘j’. The center and circularity errors are evaluated using the data points of each plane using the above explained method. The evaluated centers of each plane of measurement are O_j(a_j, b_j, c_j).

Table 1 : CMM Data collected from Cheraghi, S.H. et al..(2003)

No.	X – Coord.	Y – Coord.	Z – Coord.	No.	X – Coord.	Y – Coord.	Z– Coord.
01	11.0943	0.4522	65.2328	13	10.815	0.5918	85.2304
02	5.0940	10.8450	65.0765	14	4.8148	10.9846	85.0740
03	-6.9063	10.8439	65.0089	15	-7.1855	10.9835	85.0641
04	-12.9065	0.4498	65.0897	16	-13.1858	0.5894	84.8952
05	-6.9063	-9.9429	65.0540	17	-7.1855	-9.8033	85.0516
06	5.0940	-9.9418	65.2216	18	4.8149	-9.8022	85.2171
07	10.9546	0.5220	75.2316	19	10.6754	0.6616	95.2291
08	4.9544	10.9148	75.0752	20	4.6752	11.0544	95.0728
09	-7.0459	10.9137	75.0770	21	-7.3253	11.0533	94.9077
10	-13.0461	0.5196	74.8964	22	-13.3254	0.6592	95.0940
11	-7.0459	-9.8731	75.0528	23	-7.3252	-9.7335	95,0504
12	4.95447	-9.8720	75.2204	24	4.6752	-9.7323	95.2179

The axis is assumed to pass through two of these centers O_1 and O_2. The axis is expressed as:

$$\frac{x-a_1}{a_2-a_1}=\frac{y-b_1}{b_2-b_1}=\frac{z-c_1}{c_2-c_1} \qquad \text{-----} \quad (8)$$

The normal radial distance from this axis to any point P_i (x_i, y_i, z_i) on the cylindrical surface is expressed as:

$$r_i = \left[\left[(x_i-a_1)^2+(y_i-b_1)^2+(z_i-c_1)^2 \right] - \left[l(a_1-x_i)+m(b_1-y_i)+n(c_1-z_i) \right]^2 \right]^{1/2}$$

for I = 1 to n ----- (9)

where *l* , *m* and *n* are the direction cosines of the axis

$$l = \frac{a_2-a_1}{\sqrt{(a_2-a_1)^2+(b_2-b_1)^2+(c_2-c_1)^2}}$$

$$m = \frac{b_2-b_1}{\sqrt{(a_2-a_1)^2+(b_2-b_1)^2+(c_2-c_1)^2}}$$

$$n = \frac{c_2-c_1}{\sqrt{(a_2-a_1)^2+(b_2-b_1)^2+(c_2-c_1)^2}}$$

Radial locations of all the points are evaluated from the axis. The radial deviation between the nearest and farthest points is given by:

Radial deviation = e_k = Max [r_i] – Min [r_i] -- (10)

Like this for all possible combinations (for k = 1 to jC_2) of center points through which the axis passes, radial deviations e_1, e_2, - - - - - e_k are evaluated. Out of these, minimum value is the cylindrity error of the surface. The corresponding axis is the best fit axis to which the cylindricity error is minimum.

Cylindricity error = Min [e_k] ----- (11)

6. ALGORITHM

The computational procedure based on the proposed strategy has been developed in the following algorithm:

Step 1: Reads the number of planes and the measurement data Pi (x_i, y_i, z_i) from a defined text file

Step 2: Processes the data of each plane using the algorithm that is explained above to evaluate the center and the circularity errors.

Step 3: Evaluates the best fit axis by iteration process.

Step 4: Selects two center points through which the axis is assumed to pass through.

Step 5: Evaluates the radial distances min [r_i] and max [r_i]of the nearest and farthest points of all the data set from this axis

Step 6: Evaluates the radial deviation ‘e_j’ between the max [r_i] and min [r_i]

Step 7: The axis is assumed to pass through the possible combination of two point sets; the steps 4 to 6 are repeated.

Step 8: Out of these radial deviations, the minimum radial deviation is the ‘Cylindricity error’ and the corresponding axis is the axis of the cylindrical features.

Step 6: Evaluates the straightness error, which is the diameter of the smallest circumscribed cylinder containing the evaluated centers of each plane

7. PERFORMANCE EVALUATION

To verify the performance of the above algorithm, a computer program was developed in C++ language. The measurement data set is collected from the previous literature of Cheraghi, S.H. et al..(2003) to evaluate the cylindricity error of a component and is presented in Table:1. The above point's data set is stored in a text file. The program retrieves the number of planes of measurement and the respective data.

The methodology that is proposed by Ravindra.K et.al. [2004] is employed to evaluate the centers in each plane. The evaluated centers are presented in Table: 2

Based on the evaluated centers, the program estimates the best fit axis and the radii of outer and inner cylinders from the axis. Then the radial deviation between these two cylinders is evaluated, which is the 'Cylindricity error'. The diameter of smallest circumscribed cylinder containing the axis is evaluated as 'Straightness error'.

Table 2. Centers evaluated in each plane

Plane	Center		Max. Radius In mm	Min. Radius In mm
	X- coord	Y- coord		
1	-0.9061	0.4509	12.0196	12.0004
2	-1.0460	0.5213	12.0027	12.0011
3	-1.1854	0.5904	12.0004	12.0023
4	-1.3250	0.6604	12.0004	12.0016

8. CONCLUSIONS

When tested on the sample data , the proposed method evaluates the Cylindricity error of the cylindrical feature as 2.8 µm. Where as, the Least Squares Cylinder method evaluates as 3.7 µm. The cylindricity error evaluated by the proposed method is lesser than the value obtained from the Least Squares Cylinder method. The evaluated straightness error is 1.4 mm

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