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ON GEOMETRIC STRUCTURE OF VISUAL SPACE

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Abstract: *The stages of visual space forming in human mind are considered. Relationships between distances in Euclidean and hyperbolic visual spaces as well as in Klein and Poincare models have been found. A variant of anisotropic visual space is presented.*
Key words: *Visual perception, visual space, Klein and Poincare models, anisotropy, moon illusion.*

1. INTRODUCTION

The image of surroundings produced by a camera equipped with conventional objective lens is well known to differ from natural visual perception. It has been shown (Kovalev, 2004) that the distortion of object's scales and space depths may range up to a hundred percent on both close and distant ranges. The errors occur due to the fact that the camera lens builds *a two-dimensional* (2D) image using fixed-focusing distance whereas the human visual system produces *three-dimensional* (3D) space using variable focus (Kovalev, 2006a). The intrinsic geometry of visual space formed in human mind during this process is a hyperbolic Lobachevsky geometry (Kovalev, 2006b; Kleĭn, 1939).

2. VISUAL SPACE CONSTRUCTION

One can depicture visual space as the result of several processes: physical, physiological, psychophysical and cognitive (Kovalev, 2006b). Viewing can be conventionally divided into three stages: 1) data acquisition by a sensory subsystem; 2) building the sensory model; 3) forming the visual space.

At the first stage, the sensory subsystem builds a 2D retinal image of the observed space by scanning a space of objects using a concentrated sight (Kovalev, 2006a). Then it estimates distances to objects using direct and indirect depth cues. Direct depth cues stimulate an accommodation of eyes, their convergence and a 3D disparity. Indirect depth cues include occlusions, a reduction of object sizes and their displacement towards the horizon with the distance, lowering of a contrast and a decrease in a gradient of a texture with the distance, motion parallax. According to experiments, a sensory subsystem truly estimates distances to objects with a precision sufficient for the visual system's needs.

At the second stage of the visual space construction, a retinal image, according to Emmert's psychophysical law, is "stretched" in size and depth proportional to perceived distances to objects. As it is shown in (Kovalev, 2006a, 2006b), these distances are defined by a focal power of a reduced accommodating eye presented in the form of a thin lens

$$1/f = 1/r + 1/d, \tag{1}$$

where f is a focal length; r is a distance to an object; d is a hyperfocal distance, else the "beginning" of infinity. The value of d is a personal constant of an observer, varying from 3 to 6 m. Taking into account a fixation of an eye at different points of space using polar coordinate system (r, θ, φ) with the origin coinciding with a center of projection, one can get the following transformation (Kovalev, 2006b)

$$q = fm = \frac{rc}{r+d},\ \theta_q = \theta,\ \varphi_q = \varphi, \tag{2}$$

where q is a perceived distance; m is a scale factor; $c=dm$; θ_q, φ_q are angular coordinates. So, at the second stage by means of (2) the spatial image of an environment, else the projective sensory model, is constructed.

3. ISOTROPIC MODEL

There was found (Kovalev, 2006a, 2006b) an agreement between the sensory model and Klein model of Lobachevsky space regarding definitions of a plane, a line and a movement (Kleĭn, 1939). The identity of models gives a basis to consider that a metric on human visual space is non-Euclidean. It has been found that there is one-to-one correspondence between Euclidean and Lobachevsky spaces through an image on the model. Thus, the transformation (2) converts the Euclidean space to the space of Klein model. Under $r \to \infty$ a perceived distance $q \to c$, and consequently the space of Klein model constitutes an interior of a sphere of a radius c.

The Klein model could be easily transformed to the Poincare model by means of stereographic projections (Kleĭn, 1939). Thus a conformal mapping in the form of

$$p = \frac{2q}{1+\sqrt{1-(q/c)^2}},\ \theta_p=\theta_q,\ \varphi_p=\varphi_q \qquad (3)$$

appears. Under $r \to \infty$, $q \to c$ $p \to 2c$, and consequently the space of Poincare model constitutes an interior of a sphere of a radius $2c$.

Finally, it is possible to convert from Klein and Poincare models to Lobachevsky's hyperbolic space (Kovalev, 2006b)

$$\rho = c\tanh^{-1}(q/c) = 2c\tanh^{-1}(p/2c) =$$
$$= \frac{c}{2}\ln\left(1+2\frac{r}{d}\right),$$
$$\theta_\rho = \theta_q = \theta_p = \theta,\ \varphi_\rho = \varphi_q = \varphi_p = \varphi. \qquad (4)$$

Hence, at the third stage of vision process the visual space possessing a non-Euclidean intrinsic geometry is formed. The total Gaussian curvature of the visual space is a constant and equals to $K=1/(ic)^2$, where ic is an imaginary radius. The visual space is a metric space that is locally Euclidean. Despite coordinate surfaces are imaginary, a distance between two points along a straight line or geodetic is a real value. Equations (2, 3, 4) define a distance to a point along a radial direction from the point of origin, else the optical center of an eye. That is enough for construction and research of a spatial image of a three-dimensional scene.

4. ANISOTROPIC MODEL AND THE MOON ILLUSION

The further specification of considered models is related to a problem of the «moon illusion». This problem has interested scientists for more than 20 centuries and still remains unresolved. In a structure of isotropic visual space infinitely far objects such as stars, the sun and the moon are situated on a surface which is close to a limit sphere, *horosphere*, so that under Emmert's law sizes are independent of an elevation over the horizon. However, it's not true in practice. It is well known that at the rise or setting the moon and the sun appear greater in size than that in an average part of the sky or in zenith. It is noted in (Kovalev, 2006b) that anisotropy of visual space increases depending on an angle of elevation, or latitude θ, and this was shown in Kaufman's experiments involving artificial moons (Kaufman & Kaufman, 2000). Under θ=45° the average distance to the limiting surface is d=3.65 m, that is 1.5 times less than under θ=1.5° (d=5.47 m). Unfortunately, it is impossible to determine an exact type of the limiting surface using the only two points from Kaufman's experiments. Though it is possible to assume that it is not a sphere, but a sphere uniformly compressed towards the vertical axis, else a spheroid.

In Kovalev (2006c), the model of anisotropic visual space constituting an internal area of a spheroid is considered. Let a compressed ellipsoid of revolution (spheroid) be given in rectangular Cartesian coordinate system XYZ specified by an initial equation in the form of

$$(x^2+z^2)/a^2 + y^2/b^2 = 1 \qquad (5)$$

where a is a major semiaxis; b is a minor semiaxis. In a polar system of coordinates where the point of origin coincides with the optical center of an eye, the spheroid can be specified by a hyperfocal distance d_θ, a latitude θ and a longitude φ

$$d_\theta = \frac{ab}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}},$$
$$\theta = arctg\left(\frac{y}{\sqrt{x^2+z^2}}\right),\ \varphi = arctg\left(\frac{z}{x}\right). \qquad (6)$$

The mapping of the Euclidean point (r, θ, φ) to the interiors of spheroid looks as follows:

$$g = \frac{rd_\theta}{r+d_\theta},\ \theta_g=\theta,\ \varphi_g=\varphi, \qquad (7)$$

where g is a radial distance from an observer to an image of a point. Obviously, the distance g cannot exceed the hyperfocal distance d_θ since under $r \to \infty$ $g \to d_\theta$. Having all visible points of the Euclidean space projected toward the spheroid (6) we get, on the one hand, an image of an environment, and on another one - an anisotropic model of the visual space that agrees with the model of Gilbert's hyperbolic space. In all horizontal hyperplanes which are parallel to a plane of visioning $\theta g=\theta=0$, the model becomes isotropic and agrees with the Klein model for Lobachevsky's planes. With an appropriate choice of affine coordinates $\chi=x/a$, $\upsilon=y/b$, $\zeta=z/a$ the equation of the spheroid (5) can be represented in the form of $\chi^2+\upsilon^2+\zeta^2=1$. Then in any meridian section of the spheroid, an ellipse can be considered as a unit circumference in the Euclidean metric. Thus, accurate within an affine transformation the Gilbert's space coincides with Lobachevsky's space. This means that alongside with the projective model according to (7) it is possible to construct a conformal Poincare's mapping in the form of

$$p = 2c\frac{g/d_\theta}{1+\sqrt{1-(g/d_\theta)^2}},\ \theta_p = arctg\left(\frac{a}{b}tg\theta\right),$$
$$\varphi_p = \varphi, \qquad (8)$$

where c>0 is a radius of Gaussian curvature defined above. Fig. 1(a, b) shows two different models of global structure of anisotropic visual space (Kovalev, 2006c). Coordinate origin is associated with center of the "self" and Z-axis coincides with the principal line of sight.

The distinctive feature of the visual space is that it is isomorphic and even isometric to Euclidean space in a vicinity of an observer.

Therefore, on small distances (up to 2-4 m) objects are perceived nearly as of a natural size. On greater distances (more than 10 m) objects are 2 to 4 times greater than "optical" ones.

If an object is the moon or the sun, their sizes are proportional to the hyperfocal distance d_θ. From Eq. (6) it follows that a relative increase (in relation to position in zenith) in the sensory model equals to:

$$V_\theta = \frac{d_\theta}{b} = \frac{1}{\sqrt{1-\varepsilon^2\cos^2\theta}}, \tag{9}$$

where $\varepsilon = \sqrt{1-b^2/a^2}$ is an eccentricity of the spheroid.

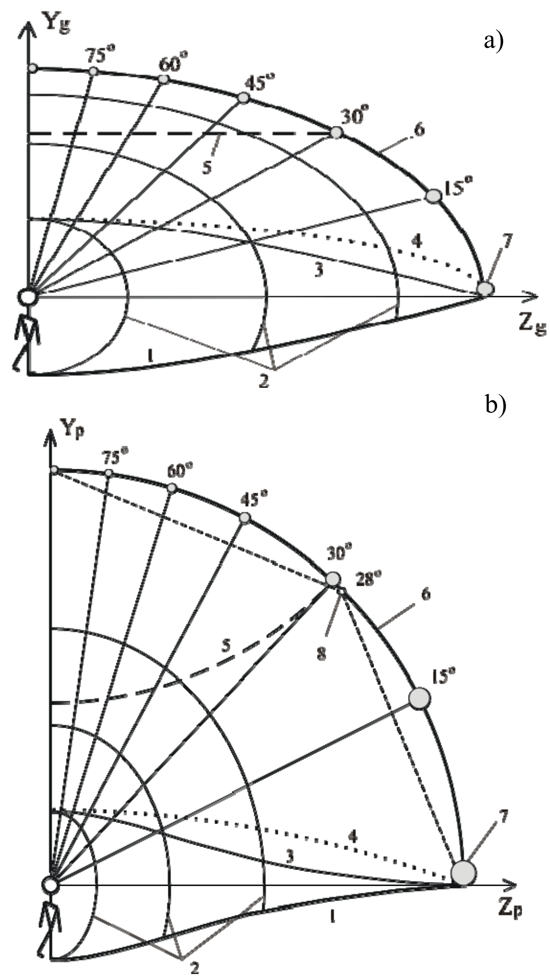

Fig. 1. Anisotropic visual space: a – projective model; b – conformal model. 1 – ground; 2 – circles; 3 – Euclid's parallel to Z-axis; 4 – equidistant curve; 5 – geodesic; 6 – sky dome; 7 – moon at horizon; 8 – sky dome bisection point.

In view of the conformal mapping, there appears an angular amplification that equals to V_θ^2 (Kovalev, 2006c). The moon (the sun) at the horizon is 1.87-3.5 times greater than that to the zenith moon. In calculations the spheroid of semi-axis a=2.927 m and b=5.475 m was used. These values were obtained using averaged data of the Kaufman's experiments (Kaufman & Kaufman, 2000).

5. CONCLUSION

The further specification of considered models is connected to experimental measurements of a shape of the limiting surface of the visual space that, in essence, is a personal function of an observer. In addition, it is required to research features of a binocular sight on distances less than 1 m where the effect of an inverse perspective appears.

The essential application areas of the obtained results are the following:

1. The general perspective theory as a technique of mapping 3D space to 2D plane so it's perceived closer to natural vision. Elaboration of groups that contain both linear and nonlinear perspectives including ones for computer graphics and 3D-displays with the limited visual depth.
2. Convergence of the technical and cognitive vision. Elaboration of the cognitive vision systems (in both the wide and narrow senses). Cognitive vision in its narrow sense is a reliable, natural and comfortable visual perception for robust recognition of visual situation by human operator in solving visual monitoring and (remote-) control tasks. Cognitive vision in its wide sense is a technical visual perception and pattern recognition, learning and embedding into various artificial intelligence systems.

6. REFERENCES

Kaufman, L. & Kaufman, J.H. (2000). Explaining the moon illusion. *Proceedings of the National academy of Sciences of the USA,* Vol. 97, No. 1. pp. 500-505

Kleĭn, F. *Geometry*, 3rd ed. (Springer-Verlag, Berlin, 1925; Dover Publ., New York, 1939)

Kovalev, A.M. (2004). Estimation of object distortions while mapping the perceptional space onto picture plane. *Optoelectronics, Instrumentation and Data Processing*. Vol. 40, No. 6. pp. 72-83

Kovalev, A.M. (2006a). On Models of the Visual Space. *Optics and Spectroscopy*, Vol. 100, No. 1, pp. 131–138, © Pleiades Publishing, Inc.

Kovalev, A.M. (2006b). Visual space description in Klein and Poincare models. *Optoelectronics, Instrumentation and Data Processing*. Vol. 42, No. 4, pp. 57-66

Kovalev, A.M. (2006c). On anisotropic visual space model. *Optoelectronics, Instrumentation and Data Processing,* Vol. 42, No. 6, pp. 47-54