

SMOOTHING OF ISOCHROMATIC DATA OF STEREOLOGRAPHY BUILT SPLINE SHAFT

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Abstract: Digital Photoelasticity is an experimental method for determining stresses in 2D and 3D models. Recently Stereolithography, one of the Rapid Prototyping (RP) techniques, is being used widely to produce 3D photoelastic models of complicated shapes directly from Computer Aided Design (CAD) models. But with such models, the fringe data obtained is not continuous as in the case of conventional models. This is due to the porous nature of the stereolithography models attributed to the way in which the models are built. To address this issue, the use of various smoothing techniques is explored in this paper for better data estimation. The performance of different 2D smoothing techniques viz. averaging filter, median filter and Wiener filter on the quality of the fringe order data obtained for a slice cut from the stress frozen model of stereolithography built spline shaft is compared.

Key words: Digital photoelasticity, rapid prototyping, stereolithography, smoothing

1. INTRODUCTION

Digital Photoelasticity is a whole field experimental technique, which provides the information on the magnitude of difference in principal stresses (isochromatics) and direction (isoclinics) of principal stresses in the form of images known as phasemap (Ramesh, 2000). Photoelasticity is the only experimental technique available to study the stresses interior to the model. However, model making for 3D photoelastic analysis is difficult and time consuming. The recent advances in Rapid Prototyping (RP) have made it possible to produce 3D prototypes of very complicated shapes directly from Computer Aided Design (CAD) models (Ramesh, 2000). Among the various RP techniques available, Stereolithography has found wide acceptance among photoelasticians because of its ability to provide models that are photoelastically sensitive (Curtis et al., 2005, Karalekas & Ajelopoulos 2005). These 3D stereolithographic models can be stress frozen and then mechanically sliced for evaluating the stress field experimentally. One of the problems associated with the models made by stereolithographic process is porosity. This depends entirely on the choice of build parameters used in model making by stereolithography. With such porous models, the fringe data obtained is not continuous as in the case of conventional models. This issue needs to be addressed to evaluate the photoelastic data correctly from these models.

In this paper to address the issue of discontinuity of fringes, the use of smoothing techniques is explored. Recently, Ramji & Ramesh, 2007a emphasized the need of smoothing techniques even for collecting photoelastic data from conventional models. The focus of their study is to eliminate primarily the spikes in the phase values. They

used a one-dimensional robust outlier smoothing algorithm for smoothing the unwrapped isochromatic phase values. For smoothing two-dimensional data, one-dimensional smoothing algorithm is used by scanning along the horizontal direction followed by scanning along the vertical direction. Here in this paper, two-dimensional smoothing algorithms are considered for smoothing two-dimensional isochromatic data obtained for a slice cut from the stress frozen stereolithography built model. For completeness, the details of whole field photoelastic data acquisition are briefly summarized.

2. WHOLE FIELD PHOTOELASTIC DATA ACQUISITION

For evaluating the isochromatic values, a six-step phase shifting technique based on a circular polariscope arrangement (Ajovalasit et al., 1998) is used. The various optical arrangements necessary to get six phase shifted images are shown in Table 1. From the intensity equations of Table 1, the isoclinic (θ) and isochromatic parameters (δ) are obtained as (Ajovalasit et al., 1998)

$$\theta_c = \frac{1}{2} \tan^{-1} \left(\frac{I_5 - I_3}{I_4 - I_6} \right) \quad (1)$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{I_a \sin \delta \sin 2\theta}{I_a \sin \delta \cos 2\theta} \right) \quad \text{for } \sin \delta \neq 0$$

$$\delta_c = \tan^{-1} \left(\frac{(I_5 - I_3) \sin 2\theta_c + (I_4 - I_6) \cos 2\theta_c}{(I_1 - I_2)} \right) \quad (2)$$

$$= \tan^{-1} \left(\frac{I_a \sin \delta}{I_a \cos \delta} \right)$$

Where I_b is the background light intensity, I_a is the light intensity accounting for the amplitude of light, θ is the orientation of the principal stress direction with respect to the x -axis, δ is the fractional retardation in radians and the subscript c indicates that the principal values of the inverse trigonometric functions are used.

The isochromatic parameter obtained by Eq. (2) is usually represented in the form of an image known as phasemap. The phasemap is obtained using the following relations [Ramesh, 2000]

$$\delta_p = \begin{cases} \delta_c & \text{for } \delta_c > 0 \\ 2\pi + \delta_c & \text{for } \delta_c \leq 0 \end{cases} \quad (3)$$

A monochrome CCD camera (DC-700 SONY) having a resolution of 768×576 pixels is used to record the phase shifted images using sodium vapour lamp ($\lambda = 589.3$ nm) as light source.

3. ISOCHROMATIC PHASEMAP GENERATION AND PHASE UNWRAPPING

A slice cut from the stress frozen model of female spline shaft made of sterilithographic material is taken for the study. Figure 1 shows the dark field image of the slice recorded using white light source. The six phase shifted images corresponding to the sequence given in Table 1 are recorded using monochromatic light source which are shown in Fig. 2. The wrapped phasemap (Fig. 3) is obtained from the phase shifted images using Eqs. (1) to (3). On observing the phasemap one can see that there are many white dots appearing as noise which results in the discontinuity of fringes. This is due to the inherent nature of the stereolithography build process. Due to the symmetry of the problem, only one tooth is considered for unwrapping. Before unwrapping one has to correct the

Table 1. Optical arrangements for six-step phase shifting technique

| ξ | η | β | Intensity equation |
|----------|----------|---------|---|
| $3\pi/4$ | $\pi/4$ | $\pi/2$ | $I_1 = I_b + \frac{I_a}{2}(1 + \cos \delta)$ |
| $3\pi/4$ | $\pi/4$ | 0 | $I_2 = I_b + \frac{I_a}{2}(1 - \cos \delta)$ |
| $3\pi/4$ | 0 | 0 | $I_3 = I_b + \frac{I_a}{2}(1 - \sin 2\theta \sin \delta)$ |
| $3\pi/4$ | $\pi/4$ | $\pi/4$ | $I_4 = I_b + \frac{I_a}{2}(1 + \cos 2\theta \sin \delta)$ |
| $\pi/4$ | 0 | 0 | $I_5 = I_b + \frac{I_a}{2}(1 + \sin 2\theta \sin \delta)$ |
| $\pi/4$ | $3\pi/4$ | $\pi/4$ | $I_6 = I_b + \frac{I_a}{2}(1 - \cos 2\theta \sin \delta)$ |

phasemap for ambiguity which is indicated in Fig. 3 (Ramesh, 2000). The corrected phasemap after ambiguity removal (Ramesh, 2000) is shown in Fig. 4 and the unwrapped phase is represented as an image in Fig. 5. Figure 6 shows the fringe order data corresponding to the line AB shown in Fig. 5. The graph shows that there are many undulations along the line and hence some smoothing methodology is required to improve the results.

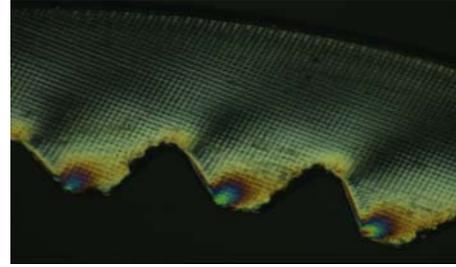


Fig.1 Dark field colour image of slice cut from the female spline shaft

4. SMOOTHING ALGORITHMS

The algorithm for smoothing is written in Matlab software. The built-in functions of 2D smoothing available in Image Processing Toolbox of Matlab software are used (Math Works Inc., 2005). Two-dimensional smoothing is done pixelwise by superposing a window of size $m \times n$ centered on the pixel considered. To restrict the smoothing process inside a specific domain of the image, the 'roipoly' command in Matlab is used which prompts the user to select the required polygonal boundary within the image. Then smoothing is carried out within the selected boundary based on the concept of masking. The smoothing algorithms considered in this work are explained next.

4.1 Averaging filter

Averaging filter is a low pass filter which is useful to remove unwanted noise from an image. In averaging filter each output pixel is set to an average of the pixel values in the $m \times n$ neighborhood of the corresponding input pixel.

4.2 Median filter

Median filter belongs to a class of spatial filters and is predominantly used in electronic speckle pattern interferometry where the noise is randomly spread over the model. In median filtering the value of an output pixel is determined by the median value of the $m \times n$ neighborhood pixels around the corresponding pixel in the input image.

4.3 Wiener filter

Wiener filter is a low pass filter which is generally used to filter an intensity image that has been degraded by constant power additive noise. Wiener filter uses a pixel wise adaptive Wiener method based on statistics estimated

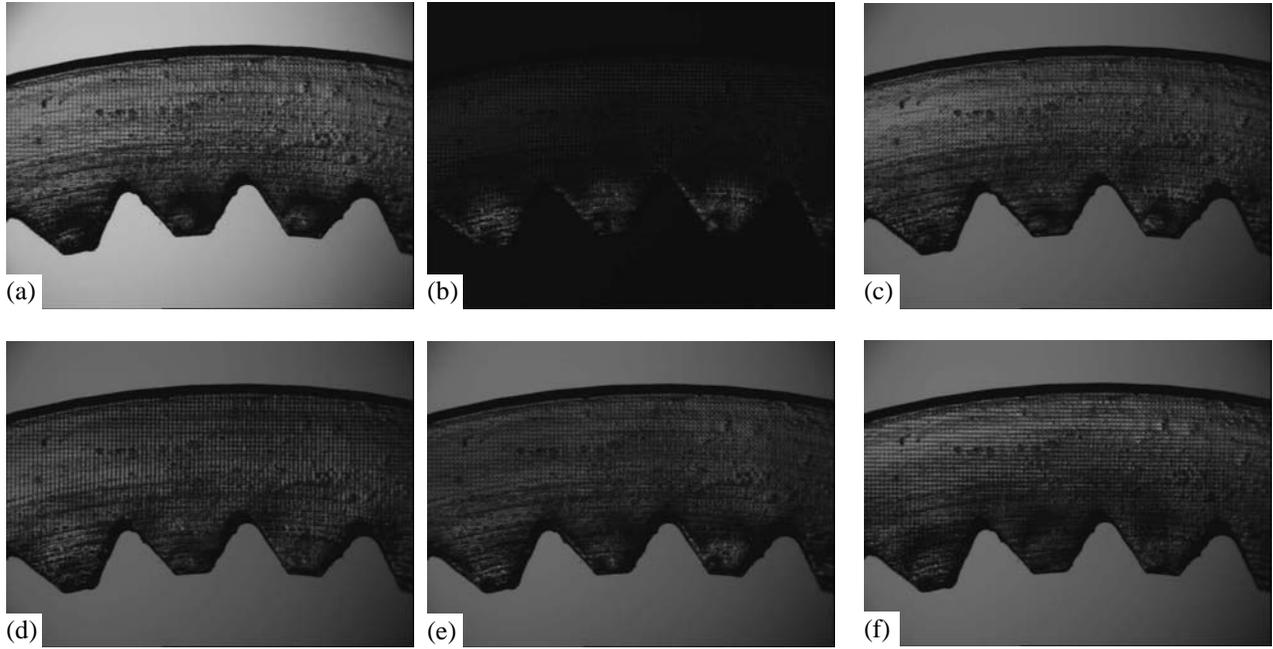


Fig. 2 Six phase shifted images corresponding to the optical arrangements given in Table 1

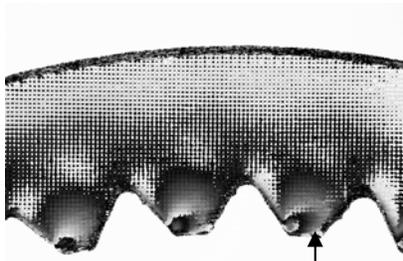


Fig. 3 Isochromatic phasemap

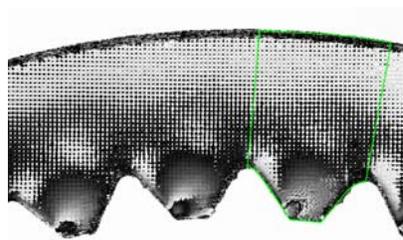


Fig. 4 Corrected phasemap

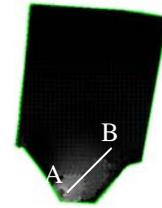


Fig. 5 Unwrapped phase

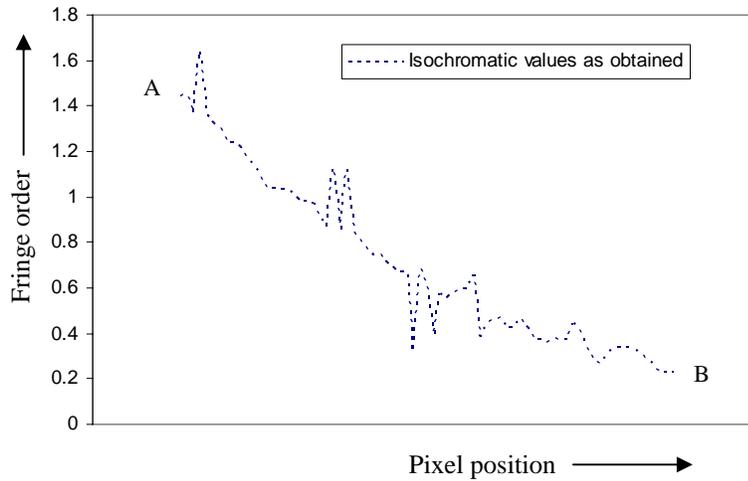


Fig. 6 Fringe order data corresponding to the line AB shown in Fig. 5

from a local neighborhood of each pixel. In this method, the local mean μ and variance σ^2 around each pixel in the $m \times n$ neighborhood are estimated. Using these estimates, a pixelwise Wiener filter is created as

$$b(m,n) = \mu + \frac{\sigma^2 - v^2}{\sigma^2} (a(m,n) - \mu) \quad (4)$$

Where, $a(m, n)$ is the input pixel, $b(m, n)$ is the output pixel and ν^2 is the noise variance determined by the average of all the local estimated variances.

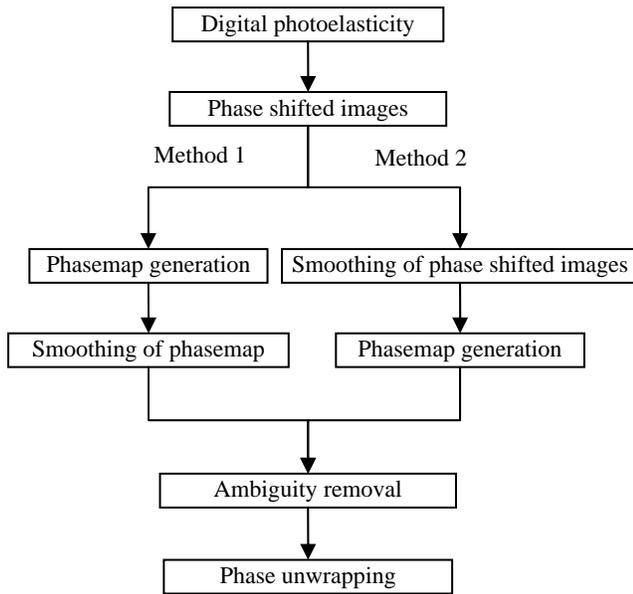


Fig. 7 The flowchart illustrating the two smoothing approaches used

5. PERFORMANCE OF THE SMOOTHING METHODOLOGIES

In order to smooth the isochromatic data, two different possibilities of smoothing methodologies are considered in this paper. In the first case, smoothing is applied directly on the final phasemap generated from the original phase shifted images. The smoothed phasemap obtained is then unwrapped to get continuous isochromatic values. In the second case, the smoothing procedure is applied on the individual phase shifted images and these smoothed images are used to obtain isochromatic phasemap which is then unwrapped for evaluating the continuous isochromatic values. The window size $m \times n$ considered for each smoothing algorithm is chosen in such a way that the best possible result is achieved by that method. The flowchart illustrating these two approaches is shown in Fig. 7.

5.1 Method 1: Smoothing applied on the phasemap

Figures 8a to 8c show the phasemaps with one of the tooth area (marked with green boundary line) smoothed by averaging filter, median filter and Wiener filter respectively. The window size used for each algorithm is as follows: averaging filter – 15×15 , median filter – 11×11 and Wiener filter – 21×21 .

5.2 Method 2: Smoothing applied on the phase shifted images

Rather than smoothing the phasemap directly, in this approach, smoothing is applied on the individual phase shifted images and these smoothed phase shifted images are used to obtain the isochromatic phasemap. Figure 9 shows the individual phase shifted images obtained with one of the tooth area (marked by a green boundary line in Fig. 9a) smoothed by averaging filter using a window size of 21×21 . The phasemap obtained is shown in Fig. 10a. Figure 10a shows that smoothing has helped in obtaining the phasemap better. The median filter and Wiener smoothing algorithms are also applied for the same tooth area and the resultant phasemaps obtained are shown in Figs. 10b & 10c. The window sizes used for median filter and Wiener filter are respectively 31×31 and 21×21 .

6. QUALITATIVE COMPARISON

Comparison of the smoothed tooth area of the phasemaps in Fig. 8 obtained by method 1 with the original phasemap (Fig. 3) shows that the original information in the phasemap itself is getting affected by the smoothing process. Also the phasemap is not improved much and still contains noise. However, the median filter performs better in smoothing compared to the other two methods. Comparing the smoothed portion of the phasemaps obtained in method 2, with the original phasemap (Fig. 3), one can observe that the averaging filter and median filter performs better than the Wiener filter algorithm in reducing the noise. On close scrutiny of the smoothed portion of the phase shifted images (Fig. 9) one can observe that the images have been significantly smoothed enough which makes the fringe contours continuous in the resultant phasemap.

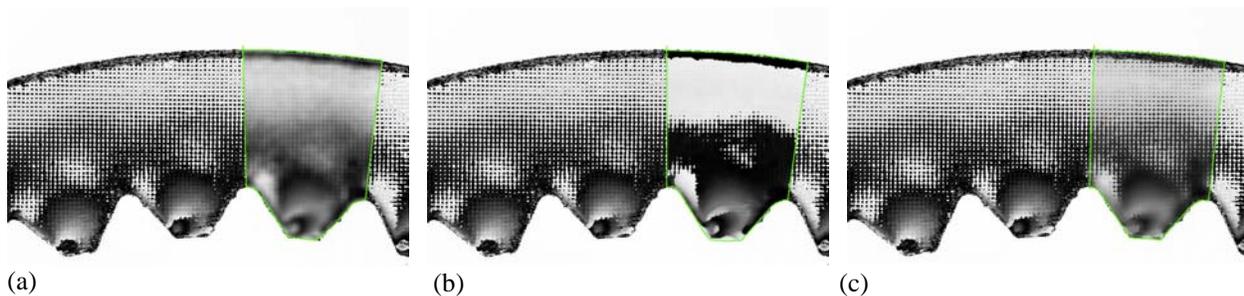


Fig. 8 Phasemap obtained after smoothing by (a) averaging filter (b) median filter (c) Wiener filter

7. QUANTITATIVE COMPARISON

The smoothed tooth area of the phasemaps obtained by the two methods using different smoothing algorithms are unwrapped after ambiguity removal by using a quality guided unwrapping algorithm (Ramji et al., 2007b). For quantitative comparison, only the phasemap obtained by median filter is considered for method 1 and for method 2, the phasemaps obtained by averaging filter and median filter are considered since these methods yield better results qualitatively. For illustration, the corrected phasemap and

the unwrapped phase obtained by averaging filter in method 2 is shown in Figs. 11 & 12 respectively. The isochromatic values obtained by these methods are then compared along the line AB in Fig. 12 with the primary isochromatic values which are shown in Fig. 13. Figure 13a shows the smoothed isochromatic data obtained by median filter in method 1. There are many undulations in the smoothed values. Figure 13b shows the smoothed isochromatic values obtained by median filter in method 2. Here the isochromatic values are smooth except a few spikes but at the starting of the line, the smoothed values

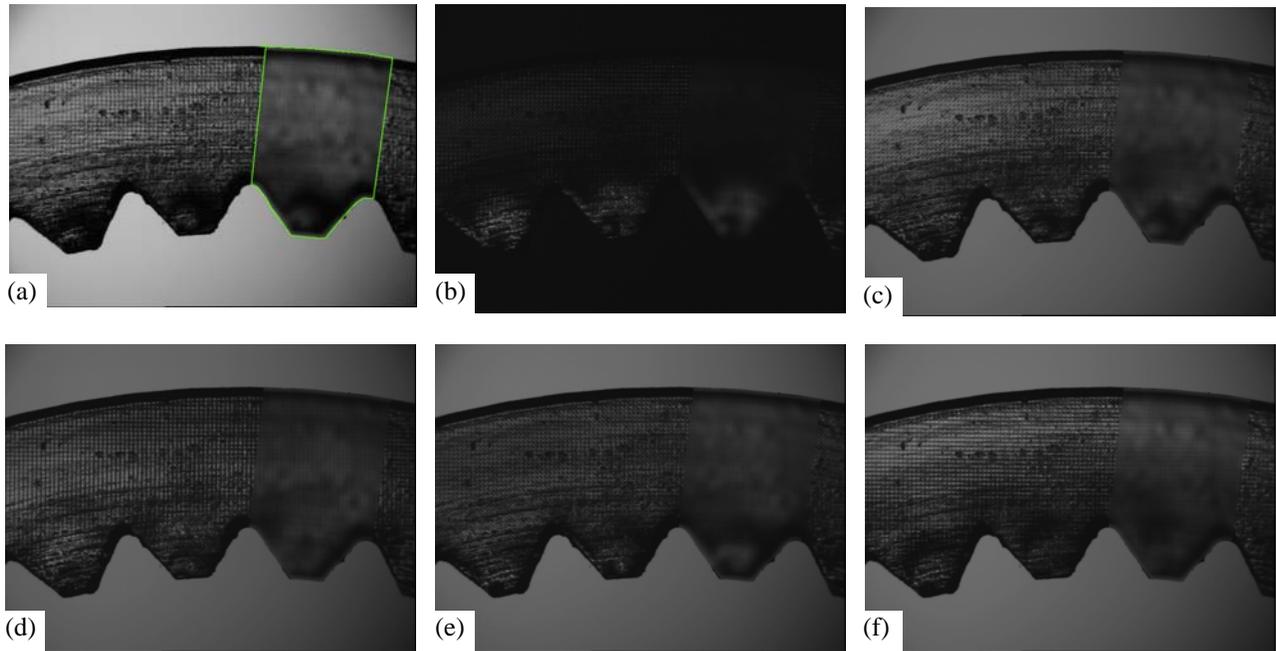


Fig. 9 Six phase shifted images after smoothing by averaging filter

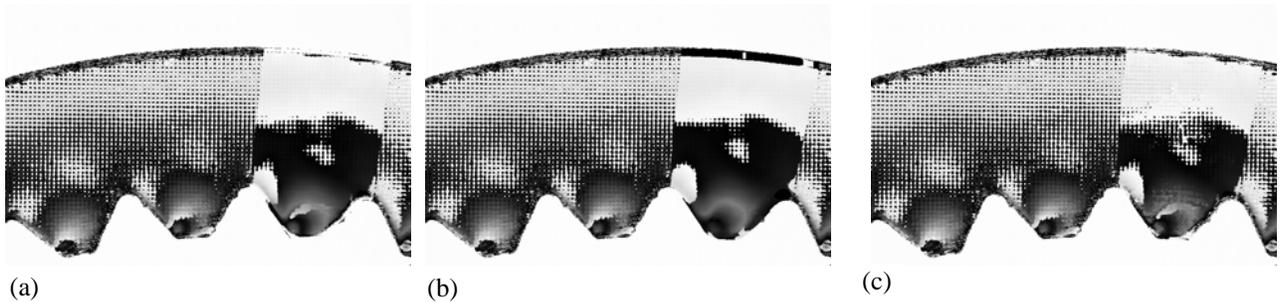


Fig. 10 Phasemap obtained from the smoothed phase shifted images by (a) averaging filter (b) median filter (c) Wiener filter

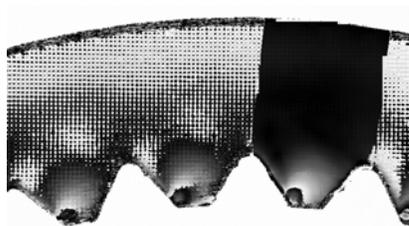


Fig. 11 Corrected phasemap obtained by averaging filter in method 2

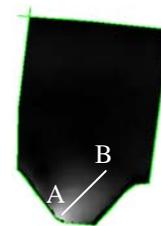


Fig. 12 Unwrapped phase

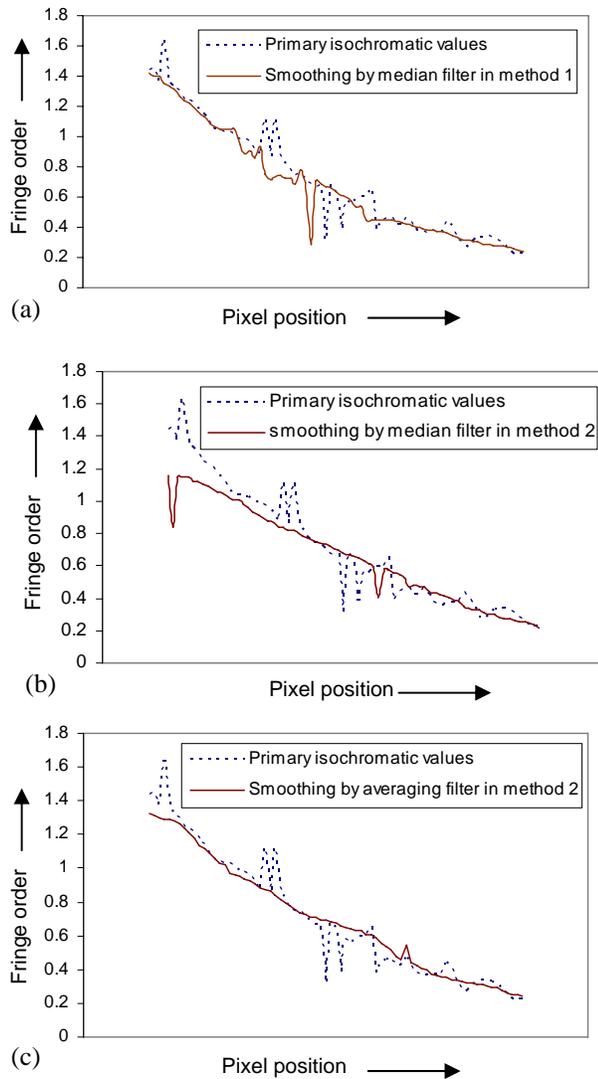


Fig. 13 Comparison of smoothed fringe order data with the primary isochromatic values along the line AB in Fig. 12 obtained by (a) median filter in method 1 (b) median filter in method 2 (c) averaging filter in method 2

are deviating much from the primary isochromatic values. Figure 13c shows the isochromatic data smoothed by averaging filter in method 2. The graph shows that the values are smooth and they follow the primary isochromatic values. Hence the averaging filter performs better in smoothing the isochromatic data for the phasemap obtained for the stereolithography built model.

8. CONCLUSION

The isochromatic phasemaps obtained from the models made by stereolithography process usually contains white dots as noise which results in discontinuity in the fringe contours. This is undesirable in digital photoelasticity. In this paper an attempt is made to remove this noise by using various 2D smoothing algorithms viz. median filtering, averaging filtering and Wiener filtering. Two different methods for smoothing is presented. The influence of the smoothing algorithms on the quality of the isochromatic results obtained are compared both qualitatively and quantitatively. Among these methods, it is found that the averaging filter applied on the phase shifted images has performed better. Hence it is recommended to smooth the phase shifted images to get isochromatic data from stereolithography built models.

9. REFERENCES

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