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DETERMINATION OF YOUNG'S MODULUS OF CANTILEVER BEAM BY DIGITAL SPECKLE PATTERN INTERFEROMETRY

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Abstract: *Digital speckle pattern interferometry (DSPI) is becoming more and more popular for deformation analysis and testing due to its non-invasive nature. In the present work, out-of-plane static deformation method is used to determine Young's modulus (E) of cantilever beam made of Aluminium. A fringe pattern corresponding to deflection of cantilever beam due to point load at its free end was recorded. To improve the measurement accuracy from the fringe pattern, a filtering scheme based on combination of median/wiener and maximum filtering is used. This filtering scheme reduces noise, improves contrast and makes the fringe pattern more distinct. The deflection calculated by using fringe count of out-of-plane static deformation method at the distance of 90.58mm from fixed end is compared with the deflection at the same distance with respect to the corresponding measured value by Mitutoyo dial gauge at the free end. It is found that the values of deflection obtained by both the methods are in good agreement. The Young's modulus of elasticity was calculated using fringe count, load applied and dimension of the cantilever plate. The value comes to be 69.92GPa.The Young's modulus is also determined using UTM and is used to compare with the value calculated by out-of-plane displacement method of DSPI. The Young's modulus calculated using DSPI and that calculated using UTM are in good agreement.*

Keywords: *Digital speckle pattern interferometry, out-of-plane static deformation method, median/wiener and maximum filtering.*

1. INTRODUCTION

New material and components are being designed for aerospace's automobile and many more applications. In the design and development of new products, it is important that the material used in its parts is to satisfy its purpose that it does not break or deform under normal loads. To ensure this, all materials used are tested in various ways before the product is put into production. Stress analysis helps in designing component for a specific purpose and is also useful in failure analysis. Material's elastic properties are fundamentally important in engineering, physics and geophysics. Traditionally, elastic limit, yield strength, ultimate strength etc. are obtained by measuring strain as a function of stress. In real situation, analysis of components involving complex material properties, loading and boundary conditions, the engineer introduces necessary assumptions to make the analysis mathematically manageable, as it is not always possible to obtain rigorous mathematical solutions in such conditions. It is therefore easier to measure the displacement, stress and strain than to compute. For testing the material of a component on universal testing machine, one has to prepare the specimen of standard shape and size. If the geometry of the specimen is complicated and having slots and holes, it is difficult to analyze the material.

Speckle interferometric techniques such as electronic speckle interferometry (ESPI) and digital speckle pattern interferometry (DSPI) are very useful for determination of material properties and deformation analysis of the component without making any standard specimen. These methods have potential to provide the alternative and less intrusive for elastic modulus determination.

These are non-contact techniques and can be implemented directly in field. This technique find its important application in determining the elastic properties of composites, polymers etc.

In DSPI, illuminating the surface of the object with the help of laser light forms the speckle pattern. The object wave is imaged on the photosensitive part of the CCD camera where it is allowed to interfere with an in-line reference beam. The interferograms of two different states of the object are grabbed and subtracted. The speckle correlation fringes are thus displayed on computer monitor using digital techniques.

Availability of high-speed digital computers makes it easier to handle large amount of data very efficiently and almost in real time. These features make DSPI to emerge as a powerful non-contact type, full field inspection tool for testing the component in laboratory as well as in industrial environment.

1.1 Static deformation

1.1.1 Out-of-plane static deformation

When the deformation/displacement of an object takes place along the line of sight, then it is called out-of-plane deformation. For out-of-plane static deformation, the relationship between the displacement w_0 and fringe order (n), is given by $w_0 = \frac{n\lambda}{2}$ (1)

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where, n is the fringe number and λ is the wavelength of laser light used to illuminate the object.

1.2 Filtering

DSPI specklegram has inherent speckle noise. Filtering operation is used to sharpen the images, to selectively suppress image noise, to detect and enhance edges, or to alter the contrast of the image. In the present work the filters used are median filter, maximum filter and wiener filter.

2. ANALYSIS METHODOLOGY

The deflection curve for a cantilever beam, fixed at one end and loaded at the other end (Fig.1) is given by:

$$U_Z = \frac{F}{2EI}\left(lx^2 - \frac{x^3}{3}\right) \tag{2}$$

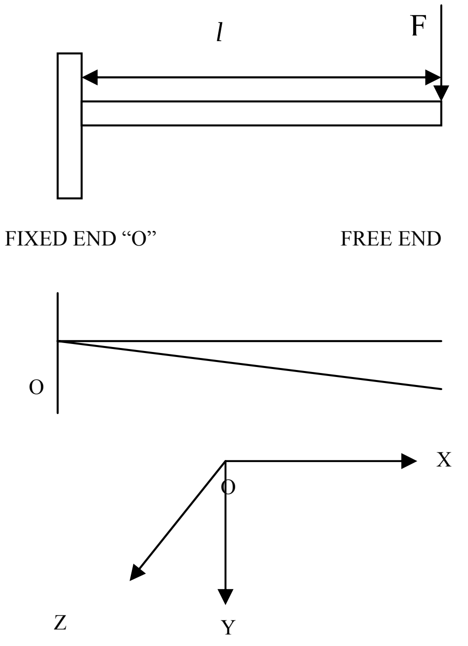


Fig.1: *Deflection in cantilever beam*

Where, F is the force causing the deflection, E is the Young’s modulus of the material of the beam, l is the distance of the point of load application on the beam and x is the distance from the fixed end to the point of deflection on the beam.

The moment of inertia I is dependent on the width and thickness of the beam and is given by:

$$I = \frac{bd^3}{12} \tag{3}$$

where, b is the width and d is the thickness of the beam.

For out-of-plane static deformation, the first state of the object is recorded when the rectangular plate fixed at one end (i.e. cantilever plate) is imaged without deformation. The second state of object is recorded after deformation in the cantilever plate. These two frames are subtracted and displayed on the monitor. Brightness of the fringes on the monitor is given by (9) can be given by:

$$B = 4KA_0A_r\left|\sin\left(\phi + \tfrac{\Delta}{2}\right)\sin\tfrac{\Delta}{2}\right| \tag{4}$$

Where, Ao and A_r are amplitude of object and reference wave fronts, respectively, ϕ is the phase difference between object and reference wave fronts before deformation, Δ is the phase difference introduced due to deformation in the cantilever plate, and K is the constant of proportionality. Brightness is zero when

$$\frac{\Delta}{2} = n\pi \quad or \quad \Delta = 2n\pi; \quad n = 0,\pm1,\pm2,. \tag{5}$$

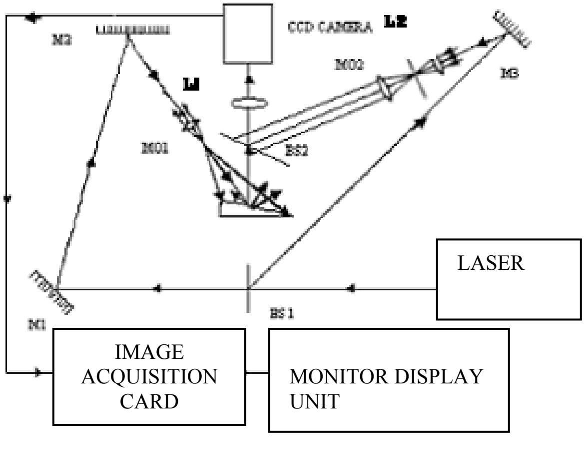
Therefore, in the regions where the speckle patterns are correlated would appear dark. The condition $\Delta = (2n+1)\pi$ corresponds to a bright fringe and denotes all those regions where both the speckle patterns before and after deformation are uncorrelated.

The angle of incidence and observation for our experimental setup is very small. Using small-angle approximation for the configuration, the phase Δ is given by: $\Delta = \frac{4\pi}{\lambda}w_0$ (6)

Where, w_0 is the displacement along z direction (i.e. out-of-plane displacement) and λ is the wavelength of laser light used to illuminate the object.

3. EXPERIMENTAL SETUP

Schematic of the DSPI setup for recording the fringe patterns of the static deformations of the object is shown in Fig.2.



M1, M2, M3 : MIRRORS
BS1, BS2 : BEAM SPLITTERS
MO1, MO2 : MICROSCOPIC OBJECTIVES
L1, L2 : LENSES

Fig.2: *Experimental setup*

3.1 Main equipments used

a) Laser

Type: He–Ne laser

Power: 10 mW

Wavelength: 632 nm

b) Camera

SDC-313 PD CCD camera

DC = 12V, 2.5W

c) Object

Aluminium plate of size 100mm x 50mm x 5mm

4. EXPERIMENTATION DETAIL

An aluminium plate of size 100mmx50mmx5mm (length, width and thickness) was used as a cantilever beam with length along x-axis and the load of 25 gm was applied at the free end in the z direction.

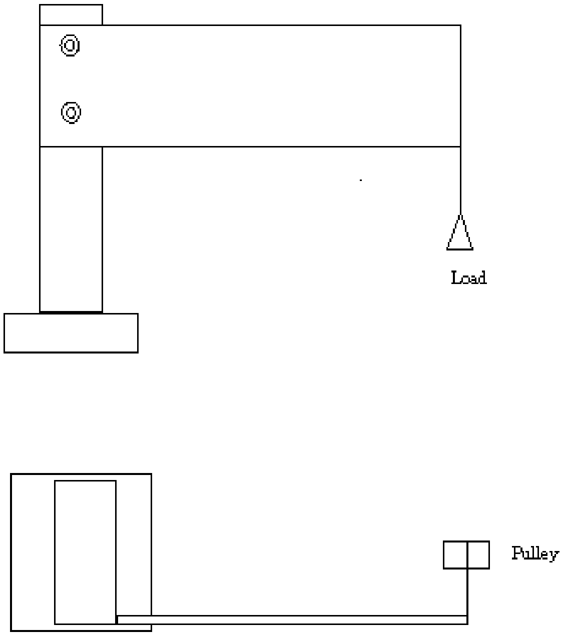


Fig.3: *Sketch of the loading arrangement*

The deflection occurred along the direction of load applied. A beam of laser was allowed to fall on the beam splitter BS1. The beam got splitted into two beams, one of which falls on mirror M1 and another on M3. The object beam from mirror M1 falls on M2. After reflecting from M2, it passed through microscopic objective MO1 and illuminates the object. The second beam which is a reference beam after reflection from M3 passed through MO2 and BS2. The objective beam combined with the reference beam to form a speckle interferogram is converted into a video signal by CCD camera.

The video analog output from CCD camera is fed to the PC-based image-processing system. LabVIEW 7.0 based program in graphical programming language was developed to acquire, process and display the image. The first image was grabbed before deformation, which served as reference image, and the second image was grabbed after deformation. The subtracted image was displayed on computer screen. The speckle interferogram for the deflection of the cantilever beam obtained after subtraction is shown in Fig.4. This DSPI fringe pattern has inherent speckle noise. Presence of undesirable bright speckles in the area of dark fringes and in the other dark region and similarly the dark speckles in the bright region make inaccuracy in measurement from speckle interferogram. In order to remove the speckle noise from the fringe pattern filtering was done using MATLAB 7.0.

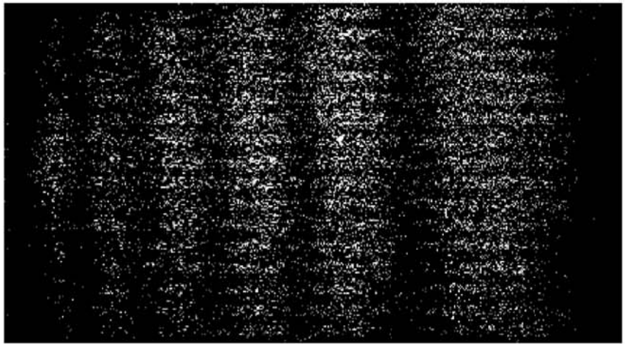


Fig.4: *Original speckle interferogram for the deflection of the cantilever beam*

4.1 Filtering

To filter the specklegram, a large number of experiments were conducted and finalized implementation of a filtering scheme, which efficiently removes the noise. The filtering scheme constitutes of maximum filter, median filter, wiener filter and again maximum and median filter of different matrix size followed by histogram equalization. Final specklegram derived out from histogram equalization is given in fig.5.

Fig.5: *Final speckle interferogram for the deflection of the cantilever beam*

5. RESULTS AND DISCUSSION

5.1 Measurement

The final fringe pattern shown in Fig.5 was used as a reference and the distance between different fringes was measured manually using scale, which is shown in Fig.6. The fringes are counted from the fixed end towards the free end.

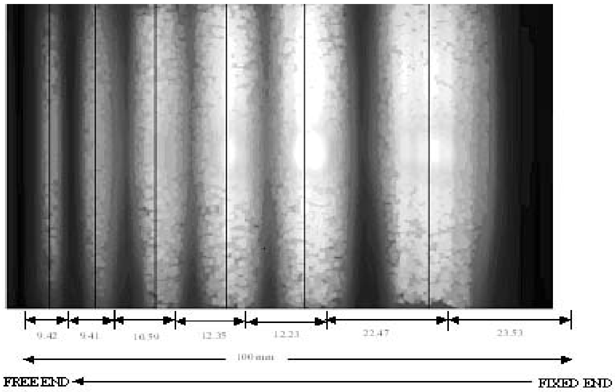


Fig.6: *Measurement of the distance between the fringes*

5.2 Calculations

5.2.1 Deflection

After counting the number of fringes, the deflection of the cantilever beam at the place of fringes was calculated using the out-of-plane static deformation relationship between the deflection (w_0) and fringe order (n) as given in eq. 1.

Deflection at a distance of 90.58mm from the fixed end was calculated from the number of fringes which comes to be 1.896 μm. Deflection at free end (at a distance 100mm from the fixed end) of the cantilever beam was also measured by Mitutoyo dial gauge which comes to be 2μm which corresponds to a deflection of 1.718 μm at a distance 90.58mm from the fixed end. The error in result is due to poor least count of the dial gauge.

The results obtained after calculating deflection at different fringe numbers are shown in the Table: 1.

5.2.2 Young's modulus

After measuring the distance of the fringes from the fixed end (x) and calculating the deflection of the cantilever beam at different fringes (w_0), the Young's modulus was calculated using the relationship given below.

$$w_0 = \frac{F}{2EI}\left(lx^2 - \frac{x^3}{3}\right) \tag{7}$$

Where,

$$F = 0.245\,N$$

$$l = 0.1\,m$$

$$I = \frac{bd^3}{12}$$

Where,

$$I = 520.83 * 10^{-12}\,m^4$$

$$b = 0.05\,m$$

$$d = 0.005\,m$$

The results obtained after the calculation of Young's modulus are shown in the tabulated form in Table: 1.

Table 1: *Calculation of Young's modulus*

S.No.	**Fringe No.(n)**	**Deflection (w_0) μm**	**Distance (x) m**	**Young's modulus (E) GPa**
1	1	0.316	0.02353	37.970
2	2	0.632	0.04600	66.670
3	3	0.948	0.05823	69.458
4	4	1.264	0.07058	70.880
5	5	1.580	0.08117	71.580
6	6	1.896	0.09058	71.048

After calculating the Young's modulus of the Aluminium plate for different fringes, the average value was calculated for n=2 to 6 and found to be 69.92GPa. In this case the Young's modulus value for n=1 was not included. This was done due to the fact that the first fringe forms near the fixed end and due to tightening of the bolts, the stress concentration is developed and this fringe is not formed at its actual location. The Young's modulus was also calculated using UTM and the value obtained was 71.31Gpa. The standard deviation of Young's modulus obtained by out-of-plane displacement was 0.784 and by UTM was 1.627.

6. CONCLUSION AND FUTURE SCOPE

6.1 Conclusion

Experimental investigations reveal that DSPI can be effectively used to monitor/measure static deformations and mechanical properties of components. Accuracy of measurement is increased by reducing speckle noise in the DSPI fringes. The result for Young's modulus of elasticity of aluminium using out-of-plane displacement method was calculated as 69.92GPa. It is within 0.1% of the standard value of 70GPa. The value of Young's modulus obtained by the UTM was 71.31Gpa.The standard deviation in value of ‘E’ using DSPI method comes to be 0.784 while it is1.627 in case of UTM. Hence, data of DSPI are more consistent.

The variation in result of measurement of deflection at the distance of 90.58mm from the fixed end, obtained from the filtered speckle interferogram and measurements obtained from the dial gauge was found to be 0.178μm. This error in result was due to poor least count of the dial gauge.

6.2 Future scope

Digital speckle pattern interferometry (DSPI) is a promising field having a variety of applications in the measurement of displacement/deformation, vibrations analysis, contouring, non-destructive testing, etc. It is a powerful ‘non-contact’ technique used to measure accurately static and dynamic displacements and deformations of surfaces down to sub micron scale.

Information such as modulus of elasticity, Poisson’s ratio, flexural strength, loading direction, shear direction, number of mode, mode shape, etc. can be precisely extracted from the speckle interferogram. This real-time full-field deformation measurement technique has large number of applications in industry for design, testing and quality control of the components.

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