# EVALUATION OF CYLINDRICITY AND SPHERICITY FROM FORM DATA USING SUPPORT VECTOR REGRESSION

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***Abstract:*** *Manufactured components are subjected to measurement in order to verify their conformance to specifications. From the measurement data, approximate functions are established to assess errors on geometry of the components. Misalignment in setting of the components for cylindricity and sphericity measurements leads to certain degree of distortion in the measured data. The size-suppression inherent in these form measurements leads to data representing deviations in microns. Therefore, limacon cylinders and limacoids are respectively used as approximating functions for the evaluation of cylindricity and sphericity errors. In this research, Support Vector Regression has been used to arrive at these functions for minimum zone evaluation.*
***Key words*****:** *Minimum Zone, Form error, Support Vector Regression, Cylindricity, Sphericity*

## 1. INTRODUCTION

A number of tolerances are specified on engineering components during design stage and the major emphasis is on size and geometry of components. Geometrical specifications include form tolerances like straightness, flatness, circularity, etc. Cylindricity and sphericity tolerances are also specified for certain components to satisfy functional requirements. After manufacture, components are verified for conformances to these tolerances by a measurement system. Co-ordinate Measuring Machines (CMMs) can be used to capture the geometrical aspects of the surface from discrete measurement points. Form testers yield data representing deviations from a measurement datum. Cylindricity measurement is done using a roundness measuring instrument with additional straight datum. For measuring sphericity of rolling elements in bearings, the ball profiles are obtained in two or three equatorial planes at 90˚ to each other using a roundness measuring instrument. Ideally, sphericity measurement requires a special arrangement in which latitudinal angle can be controlled (Samuel and Shunmugam, 2002). After obtaining measurement data, data has to be processed suitably to verify the conformance of the component to the specified tolerances. Computer Aided Inspection systems assist the manufacturers in measurement and processing of measurement data.

In general, form measuring equipments/setups are used with the features well-aligned with a measurement datum. Since the size is suppressed and only deviations from the measurement datum are measured, such data is referred to as form data. Measurement data obtained from roundness measuring equipments would also contain profile distortion due to eccentricity in measurement setup. Thus, the assessment features used should compensate for these factors. Researchers have shown that a limacon cylinder (Venkaiah and Shunmugam, 2007) and a limacoid (Samuel and Shunmugam, 2003) can be used as reference features to evaluate cylindricity and sphericity errors respectively when measurement is done using form measuring equipments. To process the measurement data for cylindricity and sphericity, a number of algorithms are available. Murthy et al analyzed the spherical surfaces and devised methods for its measurement and evaluation (Murthy et al, 1979). An approach based on Monte Carlo, simplex and spiral search was also put forward to evaluate form errors (Murthy et al, 1980). Least Squares approach was applied to arrive at geometric errors (Shunmugam, 1986). Though the LS method is based on sound mathematical principles, the error values produced are not always a minimum. A median technique was also proposed (Shunmugam, 1986). Using discrete Chebyshev approximations, Dhanish and Shunmugam have arrived at minimum zone values for cylindricity and sphericity (Dhanish and Shunmugam, 1991). Kanada proposed an evaluation based on statistical methods (Kanada, 1995). Computational geometric techniques have also been proved applicable in form evaluation. Samuel and Shunmugam proposed the evaluation of sphericity from form data using computational geometric techniques (Samuel and Shunmugam, 2002). An approach to evaluate cylindricity was also put forward (Venkaiah and Shunmugam, 2006).

Of late, Support Vector Regression has been found applicable in this area. Prakasvudhisarn et al (Prakasvudhisarn et al, 2003) proposed an approach based on ν-SVR to evaluate straightness and flatness errors based on data obtained from a coordinate measuring machine. SVR has a firm grounding on statistical learning theory (Vapnik, 1995) and essentially employs structural risk minimization. Unlike LS which is based on normal distribution assumption, SVR is not based on any assumptions. Modelling of nonlinear functions is simplified by the use of Kernel functions. Also, the parameters depend only on a subset of data points termed support vectors thereby reducing the search efforts. Thus, in this paper, ν-SVR has been applied to evaluate cylindricity and sphericity errors from form data.

This paper discusses the concepts involved in evaluating cylindricity and sphericity using support vector regression. A brief overview of SVR is given first. Following this, method for evaluation of cylindricity is put forward. Evaluation of sphericity is dealt subsequently. Data is taken from literature and is mentioned in the appendix. Data used in this work represent the deviations from a measurement datum in microns. The results are tabulated and discussed.

## 2. OVERVIEW OF SUPPORT VECTOR REGRESSION

Given *"Q"* sets of points of the form $\{X_k, d_k\}_{k=1}^{Q}, X_k \in \mathbb{R}^n, d_k \in \mathbb{R}^n$, it is required to find an appropriate approximant that models the functional dependence of *d* on *X*. Support Vector Machines for regression approximate function of the form

$$f(X,W) = \sum_{i=1}^{Q} w_i \phi_i(X) + w_0 = W^T \Phi(X) + w_0 \qquad (1)$$

where $\Phi(X) = (\phi_1(X), ...., \phi_m(X))^T$ is the high dimensional feature vector of input points ($x$), $W$ represents the weight vector, which, depending on the dimension of fit required, would be $w_1$, $w_2$, and so on, and $w_0$ represents the bias. A common formulation is the minimization of $||W||^2$.

From the parameters obtained, it is necessary to estimate a zone enclosed within two hyperplanes such that all data points lie within these hyperplanes and the distance of separation (zone width) is minimal. There may be a few outliers for a given approximate function. To compensate for these outliers, two sets of non-negative slack variables $\xi_i, \xi_i^{'}$, $i = 1, ...., Q$ are introduced for each data point which will help quantify the total error over the training set. In this ν-SVR (Scholkopf et al, 1998 and 1999), "2ε" (zone width) is also determined along with the other parameters where ν represents fraction of errors in the data set. Size of ε is traded off against model complexity and slack variables via a regularization constant *C* and ν.

## 3. EVALUATION OF CYLINDRICITY

Measurement data for evaluating cylindricity is taken using roundness measuring equipment with an additional straight datum. Data from three sections on the cylinder can be obtained in the form of $(r_i, \theta_i, z_i)$ where $z$ is the height of the cylinder. These values are in the form of polar coordinates where $r_i$ represents deviations from a datum.

The size of the cylinder would be suppressed and misalignment between the axes of machine table and

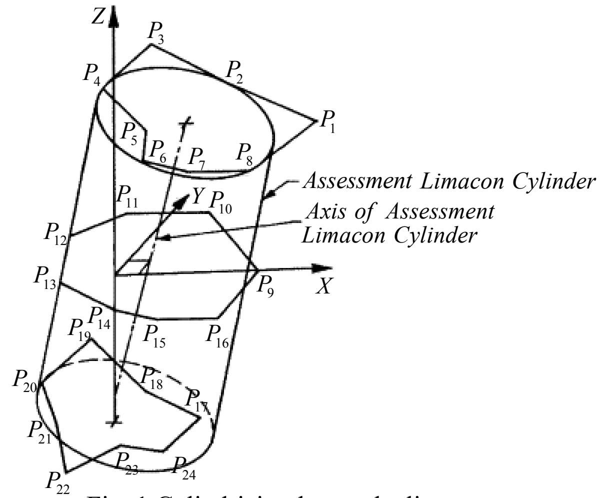


Fig. 1 Cylindricity data and a limacon cylinder

component would introduce distortion in the circularity profile obtained at each section. Therefore, a limacon-cylinder is used as assessment feature to evaluate cylindricity errors from form data. The deviations of a measured point with reference to the limacon-cylinder is represented as

$$e_i = r_i - \left(R_0 + x_0 \cos\theta_i + y_0 \sin\theta_i + l_0 z_i \cos\theta_i + m_0 z_i \sin\theta_i\right) \quad (2)$$

where $R_0$ is the estimated radius, $(x_0, y_0)$ are the estimated trace coordinates of axis of the assessment limacon-cylinder, $l_0$ and $m_0$ are slope values for the axis. By convention, deviation of a measured point lying outside the assessment limacon cylinder is taken as positive and a point inside is considered to have negative deviation.

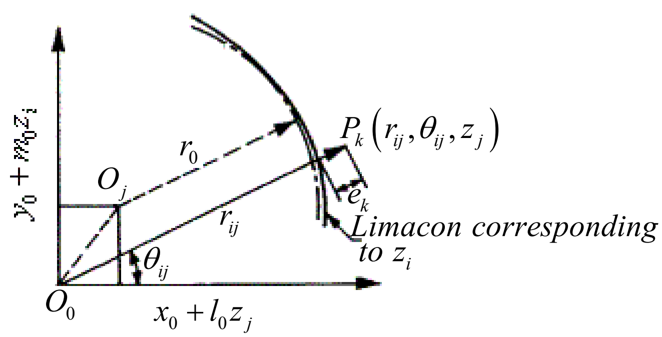


Fig. 2 Deviations at a point

Deviations are shown in Fig. 2.

Based on these characteristics, ν-SVR for cylindricity can be formulated. To comply with minimum zone requirements, it is required to estimate two concentric limacon-cylinders in the form of Eqn. 2 such that all the data points are enclosed within them and the distance of separation is minimal. This distance would represent the cylindricity error

$$\Delta = |e_{\max}| + |e_{\min}| = 2\varepsilon \tag{3}$$

where $2\varepsilon$ is the distance between the hyperplanes. If W is taken to represent the trace coordinates $x_0$, $y_0$ and the slope values $l_0$, $m_0$, primal for $\nu$-SVR is formulated as

$$Minimize: L_P(W,\Phi,\Xi,\Xi',\varepsilon) = \frac{1}{2}\left\| x_0 \quad y_0 \quad l_0 \quad m_0 \right\|^2 + C \cdot \left( \nu\varepsilon + \frac{1}{Q}\sum_{i=1}^{Q} \xi_i + \xi_i' \right) \tag{4}$$

Subject to

$$\left( \begin{bmatrix} x_0 & y_0 & l_0 & m_0 \end{bmatrix} \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \\ z_i\cos\theta_i \\ z_i\sin\theta_i \end{bmatrix} + w_0 \right) - r_i \le \varepsilon + \xi_i$$

$$r_i - \left( \begin{bmatrix} x_0 & y_0 & l_0 & m_0 \end{bmatrix} \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \\ z_i\cos\theta_i \\ z_i\sin\theta_i \end{bmatrix} + w_0 \right) \le \varepsilon + \xi_i'$$

$$\xi_i \ge 0, \xi_i' \ge 0, \varepsilon \ge 0$$

ν is kept at 0 initially and iterated in steps by a predefined step length. Lagrange multipliers $\lambda$, $\lambda'$ and so on are taken for the constraints and primal variables and the Wolfe dual (Scholkopf et al, 1998) is obtained. From Eqn. 4, if xi = $[\cos\theta_i \sin\theta_i \; z_i\cos\theta_i \; z_i\sin\theta_i]^T$, the dual is obtained as

$$Maximize:$$

$$W(\Lambda,\Lambda') = \sum_{i=1}^{Q}\left(\lambda_i' - \lambda_i\right) r_i - \frac{1}{2}\sum_{i=1}^{Q}\sum_{j=1}^{Q}\left(\lambda_i' - \lambda_i\right)\left(\lambda_j' - \lambda_j\right) k(x_i, x_j) \tag{5}$$

Subject to

$$\sum_{i=1}^{Q}(\lambda_i - \lambda_i') = 0, \quad 0 \le \lambda_i \le \frac{C}{Q}, \quad 0 \le \lambda_i' \le \frac{C}{Q},$$

$$\sum_{i=1}^{Q}(\lambda_i + \lambda_i') \le C \cdot \nu$$

where $k$ is a kernel of the form $k(x,y) = \phi(x)\cdot\phi(y)$ (Satish Kumar, 2004) satisfying Mercers theorem, and $\Lambda, \Lambda'$ is the vector containing $\lambda, \lambda'$ respectively. On solving the dual, the values of $x_0$, $y_0$, $l_0$ and $m_0$ can be obtained as

$$\hat{W} = \sum_{i=1}^{Q}(\hat{\lambda}_i - \hat{\lambda}_i') \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \\ z_i\cos\theta_i \\ z_i\sin\theta_i \end{bmatrix} = \sum_{k=1}^{n_s}(\hat{\lambda}_k - \hat{\lambda}_k') \begin{bmatrix} \cos\theta_k \\ \sin\theta_k \\ z_k\cos\theta_k \\ z_k\sin\theta_k \end{bmatrix} \tag{6}$$

Using Karush Kuhn Tucker (KKT) conditions, only those points that satisfy the conditions $0 \le \lambda_i \le C/Q$; $0 \le \lambda_i' \le C/Q$ will have ξ or ξ' as 0 (Smola and Scholkopf, 2003). These points are support vectors and lie on the enclosing hyperplanes. Since this reduces the constraints of Eqn. 4 to equalities, $w_0$ and ε can be obtained. Using these values, $e_i$ is calculated for all points based on Eqn. 2. If all $e_i$'s are found to be lesser than the obtained value of $\varepsilon$ (i.e. if all points lie within the estimated zone), the value of ν is incremented with the step length and the procedure is repeated until optimum is reached. The results for data set given in Table A1 are documented in Table 1 and discussed in Sec 5.

## 4. EVALUATION OF SPHERICITY

Data for sphericity evaluation $(r_i,\theta_i,\beta_i)$ can be obtained from roundness measuring equipments using special arrangement in which pickup can be positioned at desired latitudinal angles $(\beta_i)$. Such measurement would give only the deviation from a datum and the size would be suppressed. Even if a truly spherical surface is measured, the measurement data would be distorted due to offset between the centres of component and the measuring instrument. The deviations of a measured point from the established feature are given by

$$e_i = r_i - \left(R_0 + x_0\cos\beta_i\cos\theta_i + y_0\cos\beta_i\sin\theta_i + z_0\sin\beta_i\right) \tag{7}$$

where $(r_i,\theta_i,\beta_i)$ is the spherical coordinates of the measured point (Fig. 3), $(x_0, y_0, z_0)$ is the centre and $R_0$ is the radius of the sphere from which the limacoid is obtained.

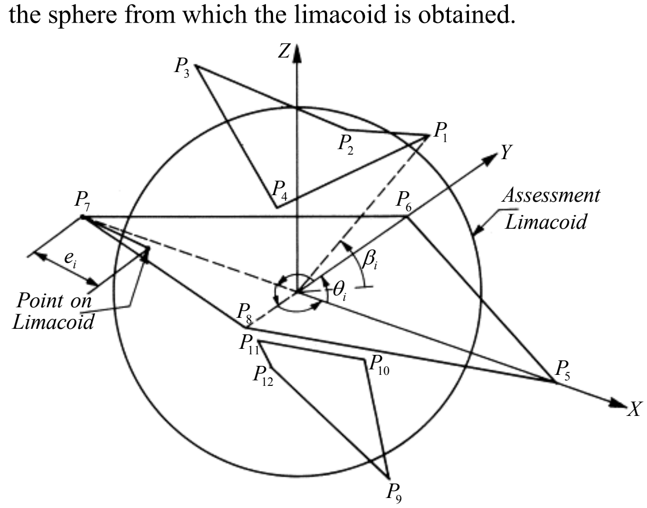


Fig. 3 Sphericity data and a limacoid ($r_i$ in microns, $\theta_i$ and β in degrees)

To evaluate sphericity, primal and dual of ν-SVR mentioned in Eqn. 4 and Eqn. 5 are modified. W is taken to represent the centre $(x_0, y_0, z_0)$. The input $x_i$ is given as $(\cos\beta_i\cos\theta_i, \cos\beta_i\sin\theta_i, \sin\beta_i)$. Similar to cylindricity evaluation, fraction of errors is set at 0 initially. The inputs are given and the quadratic program is solved to get values of the centre. Based on KKT, support vectors are identified and radius ($r_0$) and $\varepsilon$ are estimated. Similar to cylindricity, it is checked whether all $e_i$'s (from Eqn 7) comply with the $\varepsilon$ obtained. The procedure is repeated by iterating ν with the predefined step length. The data set taken is given in Table A2. The results are documented in Table 2 and are discussed further in Sec 5.

## 5. RESULTS AND DISCUSSIONS

The code for the devised algorithm was written in MATLAB. ν was initially varied in steps of 0.025 and as the converging point was obtained, the step length was made finer to 0.001. This reduced the computational time. C was fixed as 200 for all cases.

The results for sphericity evaluation are given in Table 2. The results obtained are compared with published results (Samuel and Shunmugam, 2002) and are found to coincide. The algorithms in the present work are dependent on the support vector points and hence parameters obtained for minimum zone solution are unique.

Table 1. Results for cylindricity evaluation

| | | |
|---|---|---|
| Trace coordinates of axis of the assessment limacon-cylinder | **$x_0$ (μm)** | 0.53553 |
| | **$y_0$ (μm)** | 0.58578 |
| slope values | **$l_0$ (μm)** | 0.82842 |
| | **$m_0$ (μm)** | -0.12132 |
| radius of the assessment limacon-cylinder | **$r_0$ (μm)** | 2.67157 |
| Fraction of Errors | **ν** | 0.015 |
| Regularization Constant | **C** | 200 |
| Cylindricity Error | **Zone (μm)** | 1.92893 |

Results for cylindricity evaluation are mentioned in Table 1. The data set taken (Table A1) is a benchmark data set taken from literature which validates the algorithm. It can be seen that the value of form error obtained, 1.9289 μm agrees with the published value (Venkaiah and Shunmugam, 2007)

The proposed algorithms involve quadratic programming and speed of the algorithm depends on the order of complexity. Solving a QP is equivalent to solving a linear program with further constraints and hence can be assumed to have polynomial time complexity. In comparison with the search methods or related optimization algorithms, the proposed algorithm is faster.

Table 2. Results for sphericity evaluation

| | | |
|---|---|---|
| Estimated coordinates of center of the assessment feature (limacoid) | **$x_0$ (μm)** | 1.41421 |
| | **$y_0$ (μm)** | 0.24264 |
| | **$z_0$ (μm)** | 0.58578 |
| Radius of assessment lfeature | **$r_0$ (μm)** | 3 |
| Fraction of Errors | **ν** | 0.04 |
| Regularization Constant | **C** | 200 |
| Sphericity Error | **Zone (μm)** | 2.82842 |

## 6. CONCLUSIONS

This paper proposes a novel approach to evaluate cylindricity and sphericity errors based on Support Vector Regression. SVR offers many advantages. One of the main advantages of SVR is the presence of a global minimum. In comparison with published values, it can be seen that the algorithm produces minimum zone results as proposed by the standards. Also, unlike other optimization algorithms, only the support vector points are processed. This reduces the time and complexity when compared to the other optimization based algorithms.

Since the measurement data is taken from form measuring instruments, limacon cylinders and limacoids are used for evaluating cylindricity and sphericity respectively. Data sets used are benchmarked data sets available in literature and the parameters obtained are found to be coinciding with the published values. The present algorithm always gives the minimum error zone for a data set. Practitioners can easily implement them in their computer aided form measuring instruments for the evaluation of cylindricity and sphericity.

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**APPENDIX: DATA SETS** (Shunmugam, 1986)

**Table A1. Data for evaluating Cylindricity**

| **i** | **$r_i$ (μm)** | **$\theta_i$ (deg)** | **$z_i$** |
|---|---|---|---|
| 1 | 5 | 0 | 1 |
| 2 | 3 | 45 | 1 |
| 3 | 4 | 90 | 1 |
| 4 | 3 | 135 | 1 |
| 5 | 1 | 180 | 1 |
| 6 | 2 | 225 | 1 |
| 7 | 2 | 270 | 1 |
| 8 | 3 | 315 | 1 |
| 9 | 4 | 0 | 0 |
| 10 | 4 | 45 | 0 |
| 11 | 3 | 90 | 0 |
| 12 | 3 | 135 | 0 |
| 13 | 3 | 180 | 0 |
| 14 | 2 | 225 | 0 |
| 15 | 2 | 270 | 0 |
| 16 | 3 | 315 | 0 |
| 17 | 3 | 0 | -1 |
| 18 | 2 | 45 | -1 |
| 19 | 4 | 90 | -1 |
| 20 | 3 | 135 | -1 |
| 21 | 2 | 180 | -1 |
| 22 | 3 | 225 | -1 |
| 23 | 1 | 270 | -1 |
| 24 | 2 | 315 | -1 |

**Table A2. Data for evaluating Sphericity**

| **i** | **$r_i$ (μm)** | **$\theta_i$ (deg)** | **$\beta_i$ (deg)** |
|---|---|---|---|
| 1 | 5 | 0 | 90 |
| 2 | 5 | 0 | 45 |
| 3 | 3 | 45 | 45 |
| 4 | 4 | 90 | 45 |
| 5 | 3 | 135 | 45 |
| 6 | 1 | 180 | 45 |
| 7 | 2 | 225 | 45 |
| 8 | 2 | 270 | 45 |
| 9 | 3 | 315 | 45 |
| 10 | 4 | 0 | 0 |
| 11 | 4 | 45 | 0 |
| 12 | 3 | 90 | 0 |
| 13 | 3 | 135 | 0 |
| 14 | 3 | 180 | 0 |
| 15 | 2 | 225 | 0 |
| 16 | 2 | 270 | 0 |
| 17 | 3 | 315 | 0 |
| 18 | 3 | 0 | -45 |
| 19 | 2 | 45 | -45 |
| 20 | 4 | 90 | -45 |
| 21 | 3 | 135 | -45 |
| 22 | 2 | 180 | -45 |
| 23 | 3 | 225 | -45 |
| 24 | 1 | 270 | -45 |
| 25 | 2 | 315 | -45 |
| 26 | 3 | 0 | -90 |