

A FUZZY LOGIC BASED SYSTEM ANALYSIS APPROACH FOR DESIGN OF EXPERIMENTS

Weckenmann A. and Brenner P. -F.

Chair Quality Management and Manufacturing Metrology, University Erlangen-Nuremberg,
Naegelsbachstrasse 25, D-91052 Erlangen, Germany
E-mail: peter-frederik.brenner@qfm.uni-erlangen.de

Abstract: Design of Experiments (DoE) is mainly used during process planning, manufacturing and assembly. When the task of a DoE project is to analyse and optimize complex products or processes, the phase system analysis will become a crucial step in the DoE approach. The paper combines the use of fuzzy logic and multi criteria decision making methods with DoE and points out an approach for system analysis of products and processes. The system analysis approach can be integrated into software for DoE.

Key words: Quality Management, Design of Experiments, System Analysis, Fuzzy Logic.

1. INTRODUCTION

The purpose of "Design of Experiments" (DoE) is to obtain information about a product or a process usually in order to achieve a continuous optimization of parameters with reference to one or more output variables.

DoE is mainly used during process planning, manufacturing and assembly. In these phases of the product life cycle, a model of the existing processes is usually used to explain why Design of Experiments should be applied (Montgomery, 1991).

So-called assignable causes (inputs) and extraneous variables (stochastic inputs) affect the response variables (outputs) of a technical system. The cause-and-effect relationships within the process or product are partially or completely unknown.

In DoE, inputs are understood to be assignable causes that affect the response variable. The response variables are outputs that describe the observed result of experiments. An unintentional and uncontrollable influencing variable is termed an extraneous variable. The influencing variables used for the experiments are also called factors (Figure 1).

The procedure in Design of Experiments can be divided into five phases: system analysis, experimental strategy, realization of experiments, evaluation and validation.

System analysis of products and processes is a crucial step in the framework of DoE. The objective of the phase system analysis is to discover process or product variables, called factors, which influence the product specifications or the process yield output.

The identification of the factors is done with the help of systematic quality management tools like Ishikawa-Diagrams, Brainstorming and Metaplan-

Technique. Furthermore, relevant factors with a supposed impact on the product quality or the process output have to be chosen and factor levels have to be pointed out (Pfeifer, 2002).

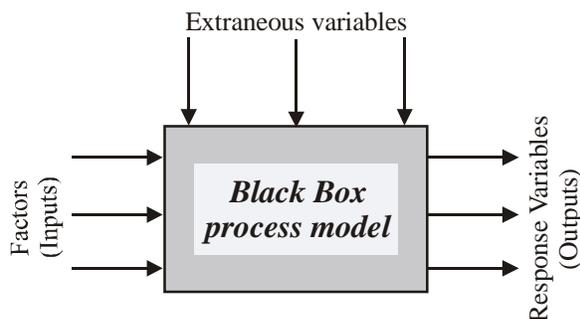


Fig. 1. Black box process model in DoE.

When DoE is used to optimize a process or a product, an interdisciplinary team is usually formed out of all those involved in the process or product. It is also necessary that not only the technology experts but also the methodology experts (DoE experts) are present in the team.

Software for Design of Experiments mostly features the set up, the analysis and the evaluation of experiments but not the system analysis step. It offers tools to create experimental designs, generates the experimental designs into spreadsheets for data entry and supplies the user with mathematical evaluation methods. System analysis tools for structuring products and processes and for gathering relevant information are mostly not integrated in Software-Packages for DoE (Weckenmann & Brenner 2006).

When the task of a DoE project is to analyse and optimize complex products or processes with numerous response variables and factors, the initial phase system analysis will become a crucial step in the DoE approach.

2. FUZZY LOGIC

Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth, the truth values between "completely true" and "completely false".

Fuzzy logic is a type of logic that recognizes more than simply true and false values. Fuzzy logic is a form of knowledge representation suitable for concepts that cannot be defined precisely, but which depend upon their context. Human judgements including preferences are often vague and cannot be estimated with an exact numerical value. A more realistic approach may be to use linguistic assessments instead of numerical values.

A linguistic variable is a variable which values are words or sentences in a natural or artificial language (Zimmermann, 1991).

A linguistic variable takes on linguistic terms. Fuzzy linguistic terms often consist of two parts: a fuzzy predicate (primary term): e. g. expensive, old, rare, good, etc. and a fuzzy modifier: e. g. very, likely, almost impossible, extreme unlikely, etc.

A fuzzy modifier makes a composite linguistic term out of the primary term (Klir & Yuan, 1995). Each linguistic term can be associated with a fuzzy set, which has a defined membership function.

3. MULTI CRITERIA DECISION MAKING

System analysis in DoE is a group decision-making process under multiple criteria that have to be taken into consideration. In general with the use of so-called Multi Criteria Decision Making (MCDM) methods a finite number of given alternatives can be screened, prioritized, selected and ranked.

The Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) is one of the classical MCDM methods (Chen, 2000). It implies that the selected alternative has the shortest distance from a so-called Positive Ideal Solution (PIS) and the farthest from a Negative Ideal Solution (NIS). The number of decision makers, different evaluation criteria and the degree of uncertainty has to be taken into consideration for solving this task.

In conventional MCDM methods, the ratings and the weights of the different criteria are known precisely. In (Chen et al., 2006) a MCDM approach is presented which is extended with fuzzy logic models in order to describe uncertainty in the multi criteria decision making process.

4. APPLICATION OF FUZZY MULTI CRITERIA DECISION MAKING FOR SYSTEM ANALYSIS

In the methodological approach of DoE subjective and vague or imprecisely defined qualitative information that has to be handled during system analysis. In order to make

the system analysis more consistent and effective, a system analysis approach based on fuzzy logic can be applied.

The purpose of the approach is to evaluate the importance of different factors with respect to different criteria in order to be able to select the most important factors, which have to be integrated in experiments. For this the concept of linguistic variables is applied.

Each linguistic variable can be assigned to one or more qualitative linguistic values which are connected to a quantitative value through the mechanism of fuzzy membership functions. The weights of different criteria and the ratings of qualitative criteria are considered as linguistic variables. The linguistic variables are expressed in positive trapezoidal fuzzy numbers.

Decision makers use linguistic variables to evaluate the importance of the criteria and the ratings of factors with respect to a criterion.

The first step deals with converting the fuzzy decision data crisp scores. The fuzzy data may be linguistic terms or fuzzy numbers. If the fuzzy data takes linguistic terms as its values, they are transformed into fuzzy numbers first. By using fuzzy ranking methods, the fuzzy numbers of each attribute are assigned crisp scores. The result of the first step is a decision matrix which contains all crisp real numbers.

In the second step, an appropriate MCDM method has to be applied to determine the rank order of alternatives.

With the help of a MCDM method the importance of factors can be determined as a relevant assessment status during the system analysis phase.

In the special application of Design of Experiments given factors are selected and grouped with respect to their importance as a result of the assessment by the team members.

5. CONCEPT OF THE FUZZY LOGIC SYSTEM ANALYSIS APPROACH

The system analysis approach described in the paper combines fuzzy logic and multi criteria decision making method. As an output of the approach the most important factors that should be included in experiments can be chosen. This is the MCDM part of the approach.

For the evaluation process of the factors linguistic variables are used in order to determine the importance of the factors. This is the fuzzy logic part of the approach.

The approach consists of the following steps:

- Identifying response variables

At the beginning of the system analysis phase in the framework of Design of Experiments response variables (outputs) of the technical system have to be identified by the DoE team.

- Definition of a response variable

In the next step an output has to be defined which is the response variable of the experiments. Before the evaluation of the response variables different criteria have to be defined which are used for evaluation. The following

criteria can be used in order to evaluate the importance of the response variables: importance for the customer, importance for the company, importance for the experiments and problems.

An interdisciplinary DoE team consists of a number of K decision makers. In the following the members of the DoE team are called decision makers D_k ($k = 1, 2, \dots, K$).

- Identifying factors

In the next step the decision makers have to run a systematic survey in order to identify factors of the system which may have an impact on the chosen response variable.

Therefore the decision makers can use brainstorming as an idea generating session.

The output of the brainstorming is a set of possible factors of the analysed technical system that have to be taken into consideration called F_i ($i = 1, 2, \dots, m$). This step is the beginning of the multi criteria decision making process.

- Determination of a fuzzy scale

Generally in a fuzzy multi criteria decision making study decision makers evaluate alternatives and criteria then they choose the relevant scale in order to translate linguistic variables.

In this approach the fuzzy scale is determined at first. Decision makers in the DoE team determine the fuzzy scale by sharing their attitude and thoughts. An example of a fuzzy scale for a linguistic variable is shown in Figure 2.

There is a need to find a general fuzzy scale for all of the decision makers. Because in some approaches one decision maker uses the term “very important” in order to say same thing for that the other decision maker uses “very high”.

That misunderstanding results in a deviation in other phases of the evaluation process. So it is important to determine the fuzzy scale and use it during the decision process as a “common language”. Also determining the fuzzy scale will help decision makers to evaluate the importance of the factors.

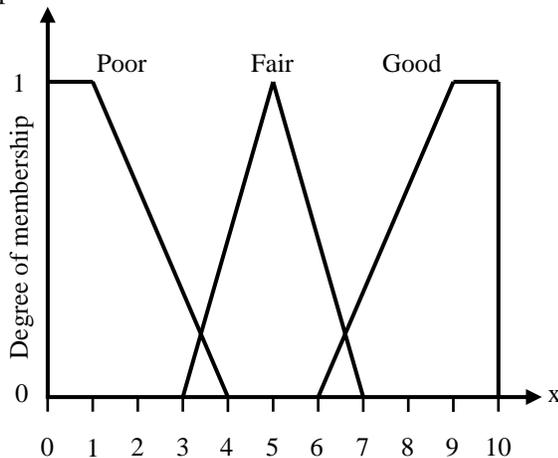


Fig. 2. Linguistic terms of a linguistic variable for evaluation.

- Defining criteria

To evaluate factors of the system different evaluation criteria have to be determined. These criteria must be connected and related with response variables of the technical system.

For a system analysis study in Design of Experiments the following criteria may be important: accuracy and repeatability of a given factor as well as effect of a given factor on the response variable and repeatability of setting the different levels of a given factor.

Pareto principle says that most of the results in any situation are determined by a small number of causes. In system analysis for DoE it means that a response variable is influenced by a small number of factors. The small number of factors must be determined.

The output of this activity is a set of criteria called C_j ($j = 1, 2, \dots, n$).

- Weighting criteria

In order to weight the importance of criteria for the decision making process decision makers define and choose linguistic variables. To assess the importance of a specific criterion linguistic variables such as “poor”, “fair” or “good” are used.

Decision makers in the team determine the importance of weights. Linguistic variables, which are chosen by decision makers, are translated into trapezoidal fuzzy numbers. For translation the predefined fuzzy scale is used.

A linguistic variable with its linguistic terms is translated into a trapezoidal number that can be defined as:

$$\bar{x} = (x_1, x_2, x_3, x_4) \quad (1)$$

For example the linguistic term “poor” of the linguistic variable as shown in Figure 2 can be represented as fuzzy number (0, 1, 1, 4) and the linguistic term “fair” can be represented as fuzzy number (3, 5, 5, 7).

- Evaluating factors

The decision makers in the DoE team choose a linguistic variable in order to determine the importance between factors and the response variable. The terms of the linguistic variable are for example “very high”, “medium high” or “low”.

Linguistic variables, which are chosen by decision makers, are translated into trapezoidal fuzzy numbers. For translation a fuzzy scale is used.

With respect to a decision maker there is a trapezoidal fuzzy number for each relationship between factors and criteria. Because of more than one decision maker in the DoE team the range of the fuzzy rating of each decision maker has to be taken into consideration. Therefore an aggregation method has to be used.

According to (Chen 2006) the following aggregation method can be applied.

The aggregated fuzzy rankings of a factor with respect to a criterion are calculated as:

$$\bar{e}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}) \quad (2)$$

With

$$a_{ij} = \min_k \{a_{ijk}\}$$

$$b_{ij} = \frac{1}{K} \sum_{k=1}^K b_{ijk}$$

The aggregated fuzzy weights of a criterion are calculated as:

$$\bar{w}_j = (w_{1j}, w_{2j}, w_{3j}, w_{4j}) \quad (3)$$

With

$$w_{1j} = \min_k \{w_{1jk}\}$$

$$w_{2j} = \frac{1}{K} \sum_{k=1}^K w_{2jk}$$

$$w_{3j} = \frac{1}{K} \sum_{k=1}^K w_{3jk}$$

$$w_{4j} = \max_k \{w_{4jk}\}$$

A linear scale transformation is used to transform the various criteria scales into comparable scales. The set of criteria can be separated into benefit criteria (the larger the rating, the greater the preference) and cost criteria (the smaller the rating, the greater the preference). Therefore, a normalized fuzzy-decision matrix can be constructed as:

$$\bar{r}_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*} \right) \quad (4)$$

$$\bar{r}_{ij} = \left(\frac{a_j^-}{d_{ij}^-}, \frac{a_j^-}{c_{ij}^-}, \frac{a_j^-}{b_{ij}^-}, \frac{a_j^-}{a_{ij}^-} \right) \quad (5)$$

With $d_j^* = \max_i d_{ij}$ for benefit criteria

$a_j^- = \min_i a_{ij}$ for cost criteria

The normalization ensures the property in which the elements are standardized (normalized) trapezoidal fuzzy numbers.

To take into account the different importance of the criteria defined before a weighted fuzzy matrix is constructed:

$$\bar{v}_{ij} = \bar{r}_{ij} \cdot \bar{w}_j \quad (6)$$

with the multiplication of any two trapezoidal fuzzy numbers that are an approximate trapezoidal fuzzy number:

$$\bar{x} \cdot \bar{y} \approx (x_1 \cdot y_1, x_2 \cdot y_2, x_3 \cdot y_3, x_4 \cdot y_4) \quad (7)$$

The elements v_{ij} of the decision matrix can be approximated by normalized positive trapezoidal fuzzy

$$c_{ij} = \frac{1}{K} \sum_{k=1}^K c_{ijk}$$

$$d_{ij} = \max_k \{d_{ijk}\}$$

numbers. For approximation of the elements a so-called fuzzy positive ideal solution (A^*) and the so-called fuzzy negative-ideal solution (A^-) can be generated by the following formulae:

$$A^* = (v_1^*, v_2^*, \dots, v_n^*) \quad (8)$$

$$A^- = (v_1^-, v_2^-, \dots, v_n^-) \quad (9)$$

With $v_j^* = \max_i \{v_{ij4}\}$

And $v_j^- = \min_i \{v_{ij1}\}$

$i = 1, 2, \dots, m ; j = 1, 2, \dots, m.$

For each factor F_i the distance from the negative and positive ideal solution is calculated:

$$F_i^* = \sum_{j=1}^n d_{ij}(v_{ij}, v_j^*) \quad (10)$$

$$F_i^- = \sum_{j=1}^n d_{ij}(v_{ij}, v_j^-) \quad (11)$$

with the distance between two fuzzy numbers x and y :

$$d(\bar{x}, \bar{y}) = \sqrt{\frac{1}{4}(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2 + (x_4 - y_4)^2} \quad (12)$$

A so-called closeness coefficient determines the ranking order for each factor.

$$R_i = \frac{F_i^-}{F_i^- + F_i^*} \quad (13)$$

The closeness coefficients are the basis for the decision making with the selection of factors.

- Selection of factors

According to the descending order of the closeness coefficients decision makers can determine the ranking order of all factors and select the most important ones among the collected factors. By comparing the closeness coefficients of each factor the most important factors which should be integrated in the experiments can be identified by the DoE team in a group decision-making-process. Factors are selected and grouped by their relevant importance. This is the result of the system analysis

approach. With the identified factors an appropriate experimental strategy can be defined in the following phase in the framework of DoE. The following phases of DoE will directly use the factors and response variables. The basis for experimental investigations is set up in the pre experimental planning phases. The selection of the factors to be investigated experimentally is of major importance.

7. CONCLUSION

The paper combines the use of fuzzy logic and multi criteria decision making methods with Design of Experiments and gives an approach for system analysis of products and processes. The fuzzy scale for the translation of linguistic variables into quantitative fuzzy numbers during the assessment process can be influenced by the team, so that the method is adoptable to the users' needs. The system analysis approach can be integrated into software for DoE.

8. ACKNOWLEDGEMENTS

The work is supported by the German Research Foundation (Deutsche Forschungsgemeinschaft e.V.) under DFG-Project Number WE 918/30-1.

9. REFERENCES

- Chen C.-T., Lin C.-T. and Huang S. F. (2006). A fuzzy approach for supplier evaluation and selection in supply chain management. *Int. J. Production Economics*, 102 (2006) pp. 289-301
- Chen, C. T. (2000). Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems* 114, pp. 1-9
- Klir G. J. and Yuan B. (1995). *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. Prentice-Hall Inc., USA.
- Montgomery D. C (1991). *Design and Analysis of Experiments*, Wiley, New York
- Pfeifer T. (2002). *Quality Management: Strategies, Methods, Techniques*, Hanser, Munich
- Weckenmann A. and Brenner P.-F., *A Process Modeling Approach for Design of Experiments Software*, In: Dolgui A., Moriel G. and Pereira C. E. (Ed.): *Proceedings of the INCOM 2006 - Volume I: Information Systems, Control and Interoperability*. (12th IFAC Symposium on Information Control Problems in Manufacturing (INCOM 2006) Saint-Etienne, France 17.-19. May 2006). Elsevier, Oxford, pp. 799-804.
- Zimmermann H. (1991). *Fuzzy Set Theory and its Applications*, Kluwer Academic Publishers, Lond