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MEASUREMENT AND EVALUATION OF THREE INTERSECTING AXES OF PRECISION EQUIPMENT

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Abstract: *In many precision machines and equipment, there are 2 or 3 axes intersecting at a point as per the drawing theoretically. One example is gyro spin axes construction. The other examples are N/C Machine tools with spindle and table and space equipment. In all of the above examples the axes are meeting at a common point. Presently each axis is measured separately for its straightness or perpendicularity with some surface. Though the axes can be measured separately, one cannot say to what extent they are meeting at a common point. And there are no standards to specify this error. As there are no methods so far to consider all the three axes simultaneously, satisfying condition, that they meet at a common point, a measurement and evaluation technique has been attempted in this paper. As the axis measurement data can be obtained on a CMM, different evaluation methods are explained in this paper. The algorithm takes into account the condition of passing through a point. This technique helps in managing the knowledge about each axis and the knowledge about all three axes. This knowledge and method of evaluation helps in managing such situations. The article also suggests different ways of specifying the accuracy in combination. A method to evaluate the axes when all the three axes are mutually perpendicular and later when any three axes are closely approaching is established by the author and now the method is established for any 3 axes meeting at a point*
Key words: *GD & T, Evaluation of Intersecting axes*

1. INTRODUCTION

Measurement of straightness, flatness etc related to one dimension or two dimensions is well established. These are dependent on the availability of a suitable measuring instrument and an evaluation technique based on available 2D geometric tolerancing standard and evaluation technique. In any precision equipment or machine, there two or three mutually perpendicular axes, corresponding to the slides and/or spindle. The present practice is to measure and evaluate each axis for its linearity/straightness, because of the limitations of a measuring instrument and because of the non-availability of a suitable evaluation standard. Presently the trend is complete elimination of 2D drawings and migration to 3D with dimensioning and tolerancing in 3D drawings. In this regard a new ASME Y14.41-2003, STANDARD FOR CAD in to the digital domain is already available (Digital product definition data practices and X dimensioning and tolerancing Y14.41 & Y14.5M). In this context there is a need for suitable evaluation techniques for application in 2D and 3D tolerancing. A method for evaluation of two mutually perpendicular axes has been reported by the author (Murthy 1994) , which evaluates an artefact with number of holes along two perpendicular rows for its hole positioning accuracy and alignment of the holes. A method for evaluation of three mutually perpendicular axes has been reported by the author (Murthy 2005), which evaluates the three mutually perpendicular axes of precision equipments. A method for evaluation three closely approaching axes is reported by the author (Murthy 2007).

In this paper considering the need for evaluating the three coordinate axes of any precision machine, which are not mutually perpendicular like in bevel gear boxes, a method for evaluation of the axes supposed to passing through a point, is attempted as there are no methods or standards for such application.

Matlab has been used for developing the algorithm and for simulation and verification of the proposed method.

2. PROBLEM DEFINITION

Fig.1 shows a Cartesian coordinate system with three mutually perpendicular axes and another set of three axes representing the measurements corresponding to the three axes of a machine, equipment like helicopter gear box/gyroscopic etc. The problem is to locate the likely intersection point (a, b, c) of the three measured axes and the orientation of each of these axes.

3. ANALYSIS

Fig.2 shows coordinated on all the three axes and their designation (axis numbering). The procedure for obtaining the likely intersection point of the three axes is determined as follows.

a) Theoretically a plane passes through any two intersecting lines (axes). In Fig.2 plane 1 contains axes 1 and 2. Similarly plane 2 contains axes 2 and 3. Also plane 3 contains axes 3 and 1.

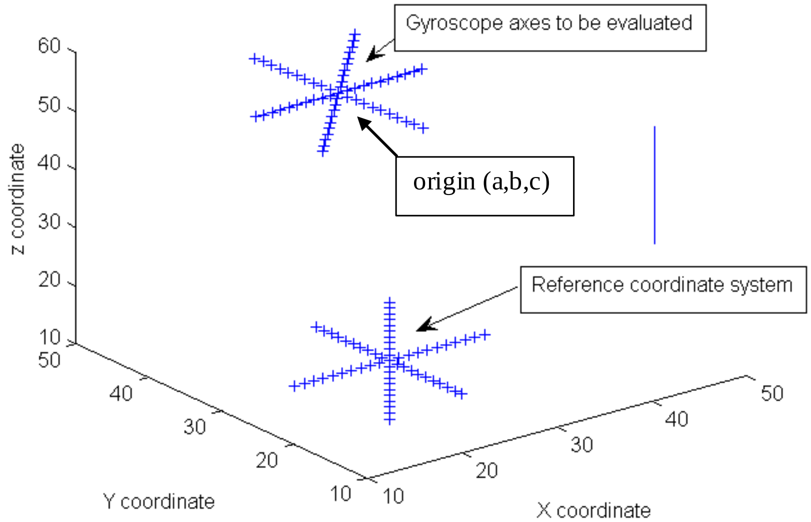

Fig. 1 Reference coordinate system and the other 3 axes to be evaluated

b) Plane 1 of the form $A_1x + B_1y + C_1z - 1 = 0$ is fitted for the coordinated data corresponding to axes 1 and 2.
c) Plane 2 of the form $A_2x + B_2y + C_2z - 1 = 0$ is fitted for the coordinated data corresponding to axes 2 and 3.
d) Plane 3 of the form $A_3x + B_3y + C_3z - 1 = 0$ is fitted for the coordinated data corresponding to axes 3 and 1.

The best fit plane (Plane 1) is fitted by minimizing the squares of the deviations (E) from the property of the plane (Murthy, 1985, 1987) namely

$$E = \Sigma(A_1x + B_1y + C_1z - 1)^2 \qquad (1)$$

Differentiating partially with respect to A_1, B_1, C_1, the equations to solve for A_1, B_1, C_1, are given by:

$$\begin{bmatrix} \Sigma(x_1^2 + x_2^2) & \Sigma (x_1y_1 + x_2y_2) & \Sigma (x_1z_1 + x_2z_2) \\ \Sigma (x_1y_1 + x_2y_2) & \Sigma(y_1^2 + y_2^2) & \Sigma (y_1z_1 + y_2z_2) \\ \Sigma (x_1z_1 + x_2z_2) & \Sigma (y_1z_1 + y_2z_2) & \Sigma(z_1^2 + z_2^2) \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} = \begin{bmatrix} \Sigma (x_1+x_2) \\ \Sigma (y_1+y_2) \\ \Sigma (z_1+z_2) \end{bmatrix} \qquad (2)$$

where suffix 1 , 2 refer to the coordinate data on the axis 1 and 2 respectively. (And x_1 is the short form of representation for x_{1i} for i=1 to n1, x_{2j} for j=1 to n2, x_{3k} for k=1 to n3 etc. where n1, n2, n3 represent the number of data on axes 1, 2 and 3.)

Knowing A_1, B_1, C_1 etc from equations (2) to (4)
The equations of the planes 1, 2 and 3 are given by:

$A_1x + B_1y + C_1z - 1$
(5)

$A_2x + B_2y + C_2z - 1$
(6)

$A_3x + B_3y + C_3z - 1$
(7)

e) Since any three planes (non parallel) intersect at a point the point of intersection is found by solving the equation of the three planes.

This point of intersection of the planes is also the point of intersection of the three axes, which is the required likely intersection point.

Similarly the other two planes are determined by:

$$\begin{bmatrix} \Sigma(x_2^2 + x_3^2) & \Sigma (x_2y_2 + x_3y_3) & \Sigma (x_2z_2 + x_3z_3) \\ \Sigma (x_2y_2 + x_3y_3) & \Sigma(y_2^2 + y_3^2) & \Sigma (y_2z_2 + y_3z_3) \\ \Sigma (x_2z_2 + x_3z_3) & \Sigma (y_2z_2 + y_3z_3) & \Sigma(z_2^2 + z_3^2) \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \\ C_2 \end{bmatrix} = \begin{bmatrix} \Sigma (x_2+x_3) \\ \Sigma (y_2+y_3) \\ \Sigma (z_2+z_3) \end{bmatrix} \qquad (3)$$

and

$$\begin{bmatrix} \Sigma(x_3^2 + x_1^2) & \Sigma (x_3y_3 + x_1y_1) & \Sigma (x_3z_3 + x_1z_1) \\ \Sigma (x_3y_3 + x_1y_1) & \Sigma(y_3^2 + y_1^2) & \Sigma (y_3z_3 + y_1z_1) \\ \Sigma (x_3z_3 + x_1z_1) & \Sigma (y_3z_3 + y_1z_1) & \Sigma(z_3^2 + z_1^2) \end{bmatrix} \begin{bmatrix} A_3 \\ B_3 \\ C_3 \end{bmatrix} = \begin{bmatrix} \Sigma (x_3+x_1) \\ \Sigma (y_3+y_1) \\ \Sigma (z_3+z_1) \end{bmatrix} \qquad (4)$$

By solving planes 1 and 2 (equations 5 and 6) the direction ratios of the axis 2 shown in Fig.2 are obtained. They are given by

$L_2 = B_1C_2 - B_2C_1$
$M_2 = C_1A_2 - C_2A_1$
$N_2 = A_1B_2 - A_2B_1$

And the direction cosines are given by

$l_2 = L_2/\sqrt{(L_2^2 + M_2^2 + N_1^2)}$

$m_2 = M_2/\sqrt{(L_2^2 + M_2^2 + N_2^2)}$
$n_2 = N_2/\sqrt{(L_2^2 + M_2^2 + N_2^2)}$
(8)

Similarly the direction cosines of other two axes are given by

$L_3 = B_2C_3 - B_3C_2$
$M_3 = C_2A_3 - C_3A_2$
$N_3 = A_2B_3 - A_3B_2$

And the direction cosines are given by

$l_3 = L_3/\sqrt{(L_3^2 + M_3^2 + N_3^2)}$
$m_3 = M_3/\sqrt{(L_3^2 + M_3^2 + N_3^2)}$
$n_3 = N_3/\sqrt{(L_3^2 + M_3^2 + N_3^2)}$
(9)

$L_1 = B_3C_1 - B_1C_3$
$M_1 = C_3A_1 - C_1A_3$
$N_1 = A_3B_1 - A_1B_3$

And the direction cosines are given by

$l_1 = L_1/\sqrt{(L_1^2 + M_1^2 + N_1^2)}$
$m_1 = M_1/\sqrt{(L_1^2 + M_1^2 + N_1^2)}$ (10)
$n_1 = N_1/\sqrt{(L_1^2 + M_1^2 + N_1^2)}$

The likely point of intersection of the axes (a, b, c) is obtained by solving the equations (5) to (7) and is given by

$$\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (11)$$

4. APPLICATION

The above method has been applied for different data and the results are satisfactory. However it is recommended that the fitted line is checked visually as shown in Fig.3 for its goodness of fit. Fig. 3 shows the data given in Table 1 and the fitted planes (and then the three axes). Table 1 also indicates the results namely the likely point of intersection and direction cosines of the axes.

5. ERROR

The distance between the theoretical intersection point and the computed intersection point is one way of specifying the error. In the above example the theoretical centre (a, b, c) is

(5.645 6.048 37.598)

and the computed centre is

(5.605 5.945 36.548)

and the deviation/error is 1.0559.

or in terms of relative displacement (Δx, Δy, Δz) it is

(0.0396 0.1026 1.0501)

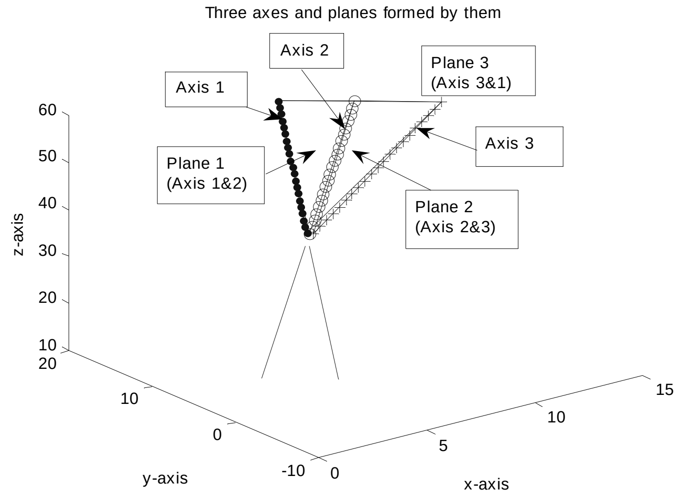

Fig.2 Representation of axes and planes formed by pair of axes

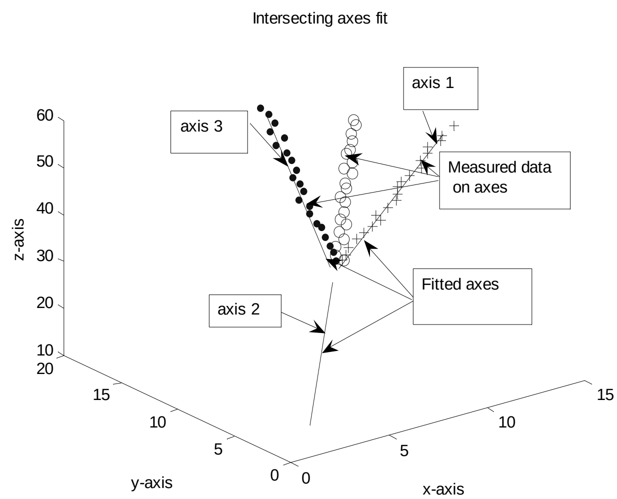

Fig.3 Representation of fitted axes and planes

6. RESULTS AND CONCLUSIONS

In this paper a method to find the probable point of intersection of three axes of precision equipment is proposed, as there are no standards or methods for the same. A method for specifying the error is also given. In the future work methods to maintain the orientation of the axes as required and to pass through the specified point is being worked out to arrive at the overall error specification

7. ACKNOWLEDGEMENTS

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Table 1. Axes data for all the three axes (1, 2, 3)

S. No	Axis 1			Axis 2			Axis 3		
	x	y	z	x	y	z	x	y	z
1	5.606	5.946	36.549	6.333	6.375	38.533	6.104	6.704	38.420
2	6.455	6.302	39.481	6.080	6.629	39.467	6.377	7.407	39.240
3	6.743	6.580	40.421	6.441	7.269	40.400	6.464	7.788	40.061
4	7.276	6.829	41.362	6.861	7.228	41.334	6.601	8.470	40.882
5	7.685	6.825	42.303	6.987	7.842	42.268	6.852	9.235	41.703
6	8.043	6.786	43.244	7.369	7.912	43.202	6.670	9.391	42.524
7	8.421	6.668	44.185	7.255	8.236	44.136	6.923	10.399	43.344
8	8.293	6.827	45.126	7.546	8.395	45.069	7.113	10.737	44.165
9	8.927	6.898	46.067	7.915	8.983	46.003	6.969	11.450	44.986
10	9.385	6.945	47.007	7.860	9.270	46.937	7.404	11.844	45.807
11	9.546	7.091	47.948	8.305	9.512	47.871	7.552	12.334	46.627
12	9.731	7.488	48.889	8.360	9.755	48.805	7.467	12.881	47.448
13	9.905	7.473	49.830	8.915	10.095	49.739	7.888	13.253	48.269
14	10.295	7.354	50.771	8.718	10.433	50.672	8.131	14.092	49.090
15	10.908	7.392	51.712	9.109	10.373	51.606	8.233	14.678	49.911
16	11.065	7.760	52.653	9.168	11.037	52.540	8.060	15.375	50.731
17	11.516	7.883	53.593	9.296	10.933	53.474	8.597	15.619	51.552
18	11.572	8.030	54.534	9.663	11.390	54.408	8.306	16.289	52.373
19	12.168	7.846	55.475	9.847	11.774	55.341	8.798	16.810	53.194
20	12.199	7.861	56.416	10.249	12.056	56.275	8.927	17.518	54.014
21	12.985	8.112	57.357	10.174	12.199	57.209	8.762	17.959	54.835
	Results of best fit axis and intersection point					**axis**	**l**	**m**	**n**
						1	**0.3235**	**0.0987**	**0.9411**
	a	**b**	**c**			**2**	**-0.2189**	**-0.2798**	**-0.9348**
	5.6059	**5.9457**	**36.5487**			**3**	**0.1510**	**0.5459**	**0.8241**

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