

# Projection Moiré Profilometry using Liquid Crystal Digital Gratings

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## Abstract:

A projection moiré profilometry using a pair of liquid crystal digital gratings (LCDG) is applied to determine a step height using two liquid crystal digital gratings (LCDGs). Moiré contours are filtered over to remove the images of the original grating from the moiré counter when a pairs of grating images are traveled to the same direction in-phase electrically. The new idea of this report is proposed to overcome the  $2\pi$  ambiguity of moiré contour by changing the fringe interval of moiré contours which are adjusted the period of LCDGs. The measured accuracy has been achieved to  $\pm 0.7 \text{ m m}$  in the range of 96 mm of steep height by projecting two different of grating periods as 6.7 and 13.3 time of original grating.

## 1. Introduction

In this report, we propose the projection moiré profilometry by means a phase-shifting method using a pair of liquid crystal digital gratings (LCDG) to measure a steep height or separated areas. Moiré contours are filtered over to remove the images of the original grating from the moiré counter when a pairs of grating images are traveled to the same direction in-phase electrically. The new idea of this report is proposed that we overcome the  $2\pi$  ambiguity of moiré contour by dual grating period method by changing the fringe interval of moiré contours which are adjusted the period of LCDGs.

Recently, there are requirements for three-dimensional profile with measuring speed and high spatial resolution. One of the useful methods is stereoscopic analysis. A light-sectioning method and a grating-projection method are one of the popular three-dimensional profilometry. A shadow moiré topography is smart to remove the image of the original grating from the moiré counters by moving the grating[1]. We have succeeded to measure a step height shape by means the frequency modulation method for shadow moiré type of topography by rotating the grating [2]. A projection type of moiré topography requires only a small system which is made of a projector and a camera with gratings. However there are problems of measurement time and accuracy because of the effect of original gratings. We have proposed a structured LC digital grating, especially for the grating projection method[3].

We proposed the three-dimensional shape measurement by a phase-shifting grating projection method by the structured LC digital grating. In this paper, we propose a projection moiré profilometry by phase-shifting using a pair of liquid crystal digital gratings (LCDG) to measure steep height areas and separated areas.

## 2. Theory and experimental setup

### Projection Moiré Profilometry

Figure 1 shows an experimental setup a projection moiré profilometry by means the phase-shifting using LCDGs. A pair of liquid-crystal digital gratings (LCDGs) and imaging lenses are set parallel to the optical axis. One LCDG works as a projection grating and the other is reference grating. A deformed grating image that is illuminated by light source to s sample is overlapped to the reference grating and is produced moiré contours.

The intensity  $I_x$  of the moiré contour at the object with height  $h$  from the reference plane on the CCD camera is written as <sup>1)</sup>,

$$I_x = \frac{I_0}{4} \left[ 1 + \gamma \cos \frac{2\pi}{p} x + \gamma \cos \frac{2\pi}{p} \left( x + \frac{ah_N}{m(L+h_N)} \right) + \gamma^2 \cos \frac{2\pi}{p} \left( 2x + \frac{ah_N}{m(L+h_N)} \right) + \gamma^2 \cos \frac{2\pi}{p} \left( \frac{ah_N}{m(L+h_N)} \right) \right], \quad (1)$$

where, the maximum of intensity  $I_0$ , the visibility of moiré contour, the distance between imaging lenses  $a$ , the pitch of LCDG, the distance between the imaging lens and reference plane  $L$ , the distance between the imaging lens and LCDG  $b$ , and the magnitude of the imaging lens at the reference plane  $m (= L/b)$ , respectively.

The height  $h_N$  is expressed by the geometrical relation by the fringe order  $N$  as,

$$h_N = \frac{LmNp}{a - mNp} \quad (2)$$

In Eq. (1), the first term is the bias, and the second to fourth terms are the fundamental and second harmonic frequencies of the original gratings, the so-called noise terms, and the last

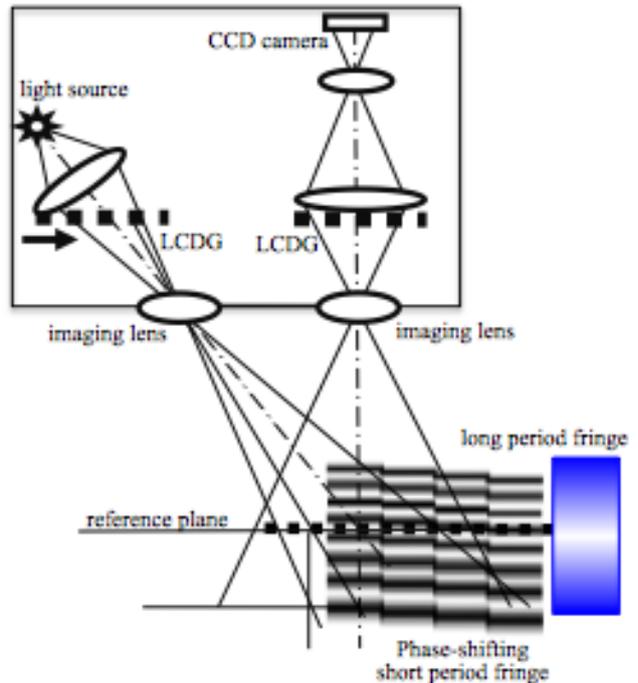


Fig. 1: Optical configuration of projection moiré profilometry 1

term is the moiré contours. We remove the second to fourth terms in Eq. (1) and extract the moiré contours. When the images of the grating patterns of the LCDGs travel in the x-direction, synchronized electrically with velocity  $V$  and integrating the exposure time  $t$  of the CCD camera, we can remove the noise grating images of the components from the second to fourth terms in Eq. (1).

### 3. Step height and separated area measurement by the dual grating-period method

The fringe order  $N$  is expressed using the phase of moiré contours by the phase-shifting method. It is easy to apply phase shifting by changing the phase of the LCDG. However, there is ambiguity, such as the integer of the fringe order. It can be determined the fringe order using the dual grating-period method and the precise phase by the short- and long-grating periods because the sensitivity of moiré contours depends upon the grating period of moiré contours. Figure 2 shows the dual grating-period method, which consists of short- and long-periods of grating. The long period of moiré contour covers the dynamic range of the measurement in one fringe, or a few periods in the case of an easy unwrap area.

First, the fringe order  $N$  of the short period of moiré contour is calculated, and the phase  $\phi_s$  of the short period of moiré contours can be easily unwrapped using the fringe number as follows:

$$\phi_s = 2\pi N + \phi_o, \quad (3)$$

where indicates the phase of the short period of moiré contours, which is analyzed by the phase-shifting method.

The fringe order  $N$  of the short period of moiré contours can be determined as

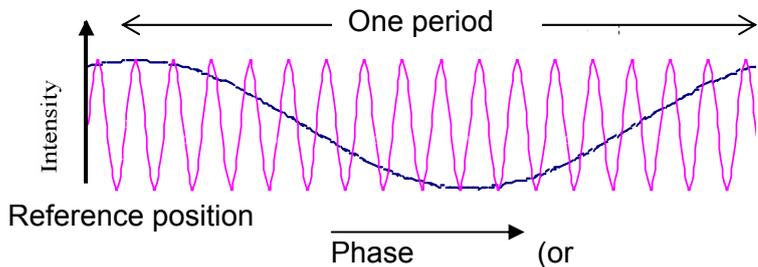


Fig. 2: Proposed technique using short- and long-grating periods

$$N = \frac{1}{2} \left[ \frac{\phi_L}{\pi} \cdot \frac{p_L}{p_s} \right] + \frac{1}{4} \left( 1 - \frac{\phi_o}{|\phi_o|} \right), \quad (4)$$

where the phase of the long period of moiré contours  $\phi_L$ , the period pitch of grating  $p_L$ , the phase data of the short-grating period  $\phi_o$ , and Gaussian bracket [ ].

To compensate for the error  $\varepsilon$  of the wrapped phase within  $\pm \pi p_S / 2 p_L$  resulting from the phase error between the phase of the long-grating period and the phase to change the phase of the short-grating period  $\phi_S$  to  $\phi_{Ls} (= \phi_S p_S / p_L)$ , we compare the phase difference between the phases of short- and long-grating periods as follows.

$$\varepsilon = \phi_{Ls} - \phi_L \quad (5)$$

The determination parameter  $k$  is shown as,

$$k = \frac{\pi}{2} \cdot \frac{p_S}{p_L} \quad (6)$$

If  $\varepsilon$  is larger than  $k$ , we need to correct the wrapped phase. We assume the next equation with correction as  $j = 1$  and without correction as  $j = 0$ . Finally, we obtain the corrected phase as follows.

$$\phi_S' = \left[ \frac{\phi_L p_L}{\pi p_S} - \frac{1}{2} \frac{\varepsilon}{|\varepsilon|} j \right] \pi + \frac{1}{2} \left( 1 - \frac{\phi_o}{|\phi_o|} \right) \pi + \phi_o \quad (7)$$

Here,  $\frac{\varepsilon}{|\varepsilon|}$  means the discriminant term, which gives the sign of  $\varepsilon$ .

Figure 3 shows a simulated result in a case where random error at the long-grating period of the moiré contours is added. The simulation parameters used were: pitch 6.08 mm per line for the long-grating period, and 0.456 mm per line for the short-grating period. The simulation is used as a maximum phase error with 7% for the long-grating period and a maximum phase error with 5% for the short-grating period. As the error is much larger than the phase value of the short-period grating period, the unwrapped error is observed. However, there are no unwrapped errors using determination parameter  $k$ .

Our proposed method is a robust enough measurement to determine the fringe order  $N$  as a correction of the unwrap error.

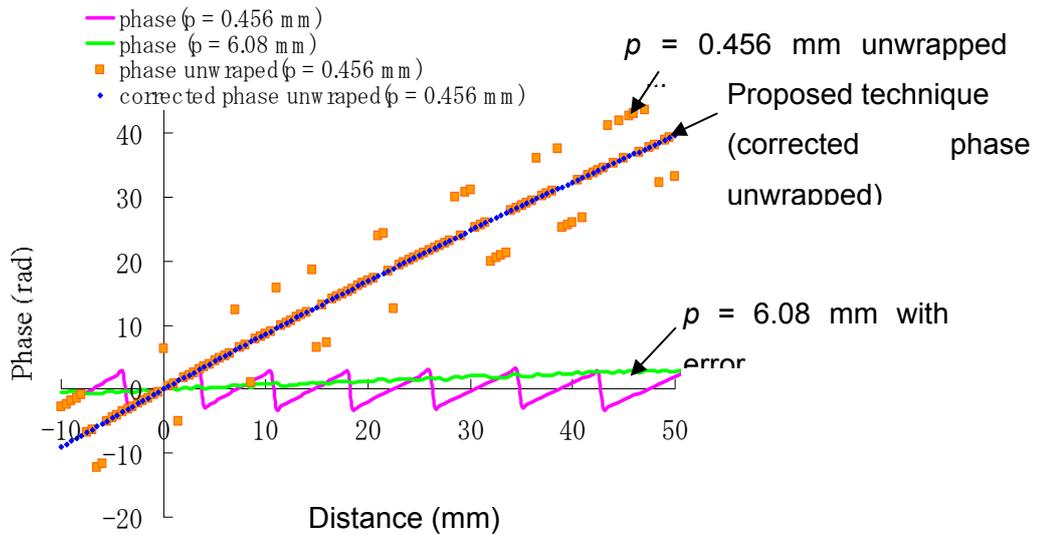


Fig. 3: Simulation of error reduction by proposed technique

#### 4. Experimental results of step height measurement

Figure 4 shows the measured results of a stair model with three moiré contours for different heights. We calculated wrapped moiré contours with a high-grating period by using the low-grating period of the moiré data. The low-grating period has a pitch of 3.04 mm. The high-grating period has a pitch of 0.456 mm and a pitch of 0.228 mm. In the figure, 3.04 shows the individual result of a grating pitch of 3.04 mm, and 0.456 shows the result using a low-grating pitch of 3.04 mm and a high-grating pitch of 0.456 mm, 0.228 shows the result using a low-grating pitch of 3.04 mm and a high-grating pitch of 0.228 mm. We compare each height in Table 1. From measurement 2 in this table, we estimate that the maximum height error is less than 0.6 mm.

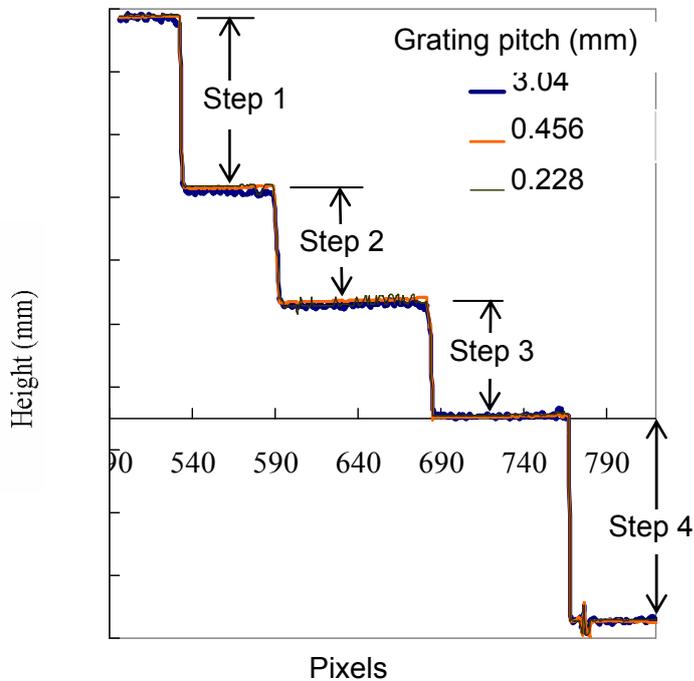


Fig. 4 Experimental result of step height

## 5. Conclusion

We have proposed phase-shifting projection moiré topography using a pair of liquid crystal digital gratings (LCDGs) to measure step height or separated areas. Our novel idea is that we overcome the  $2\pi$  ambiguity of moiré contours and determined the fringe number by changing the fringe interval of moiré contours, which are adjusted to the LCDG period in this case by 6.7 and 13.3 times the pitch. The proposed method is to determine the fringe number of moiré contours by dual grating period method and the precise phase by long and short grating periods because the sensitivity of moiré contours depends on the grating period of moiré contours.

Finally, we have checked that the measured accuracy has been achieved to  $\pm 0.7\text{mm}$  in the range of 96 mm of step height by projecting two different of grating periods as 6.7 and 13.3 time of original grating.

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