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Coordinate Measurement Technique Considering the 3D-Abbe Principle

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Abstract

A new portable measuring machine for three-dimensional measurements has been developed and verified for high-accurate inspection and calibration of large parts directly on-site in production. The system can be considered as a high-precision metrological frame. It is capable to improve the accuracy of conventional coordinate measuring machines (CMMs) or can turn machine centres into qualified 3D measuring devices. In general this procedure is applicable to multi-body systems used to move a tool mounted at the last kinematic axis, for instance, CMMs, machine tools or robots. The core tasks in achieving this system are based to a large extent on the successes of: M3D3 algorithms that control the system and task specific error correction that the system offers. Tasks involved were jointly implemented by PTB.

1. Multi-lateration measuring system

The mobile measurement machine for 3D Abbe measurements (M3D3) consists of high-accurate tracking interferometers of which at least four are required (Figure 1) [1]. Each of the LaserTracer is a self-contained measuring device that is capable to automatically track a single moving retro-reflector and to measure distances continuously [2]. All LaserTracer are connected to a programmable trigger board to facilitate synchronized capturing of distance change data. A new operating software for this application was developed to enable the central control of all laser tracers, communication with a mover (CMM, robot or machine centre), handling of the data recording and evaluation of all length measurements and finally calculation of the coordinates X, Y, and Z of all retro-reflector positions.

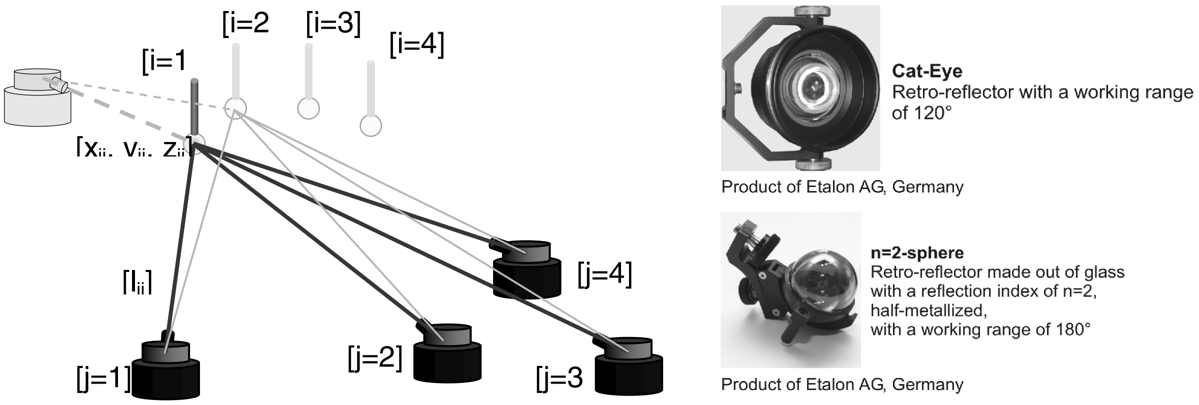

Fig. 1: Measuring principle of the M3D3 system and usable retro reflectors

Within the operating volume of the mover or CMM, four or more laser tracers are setup in non coplanar condition. At least four of them are required to determine the unknown dead path of each of the interferometers. Based on the principle of multi-lateration 3D point coordinates, x_i, y_i, z_i are calculated by using measured length changes l_{ij} according to:

$$l_{ij} + l_{0j} + w_{ij} = s_j \sqrt{\left(x_{ij} - x_{0j}\right)^2 + \left(y_{ij} - y_{0j}\right)^2 + \left(z_{ij} - z_{0j}\right)^2} \qquad \text{(eq. 1)}$$

Whereby are:

$i = 1 \ldots n$	measurement point number
$j = 1 \ldots m$	laser tracer position number $(m \geq 4)$
x_i, y_i, z_i	coordinates of measurement points (unknown)
x_{0j}, y_{0j}, z_{0j}	coordinates of laser tracer positions (unknown)
l_{0j}	unknown dead path of laser tracer j
s_j	scale factor from calibration of laser tracer j
l_{ij}	measured length change from laser tracer j to point i
w_{ij}	residual between measured and fitted distance to measurement point

A set of m × n equations emerge to solve a set of (5×m)+(3×n) unknowns which are: n point coordinates x_i, y_i, z_i, and five unknowns $x_{0j}, y_{0j}, z_{0j}, l_{0j}, s_j$ of each laser tracer. In practice, some 20 to 100 or even more points are measured resulting in an over-determined system of equations (eq. 1). The discrepancies among between the elements of the equation can be minimized by means of a least squares adjustment. There are various ways to find a solution of the over-determined system of equations (eq. 1). The approach used here minimises the weighted sum of squared discrepancies as described in eq. 2.

$$w^T P w \Rightarrow \min \qquad \text{(eq. 2)}$$

where *P* denotes a matrix of weights that describes the uncertainty of the interferometric length measurements. This depends on factors like ambient environmental conditions, opto-geometric errors of the cat's-eye retro-reflector, roundness deviation of the reference ball inside the laser tracers. One simple uncertainty model assumes that the accuracy of the measured distances considers both the length dependent and independent effects. Then *P* becomes a diagonal matrix with elements $p = (a^2 + b^2(l_{ij} + l_{0j})^2)^{-1}$, whereby *a* and *b* are constants.

The non-linear minimisation problem (eq. 2) is solved by approximating it by a sequence of linear ones. The linearization of the non-linear equation (eq. 1) yields one equation (eq. 3) for each length measurement.

$$w_{ij} = w_{ij}\big|_0 + \frac{\delta l_{ij}}{\delta x_i}\bigg|_0 dx_i + \frac{\delta l_{ij}}{\delta y_i}\bigg|_0 dy_i + \frac{\delta l_{ij}}{\delta z_i}\bigg|_0 dz_i - \frac{\delta l_{ij}}{\delta x_{0j}}\bigg|_0 dx_{0j} - \frac{\delta l_{ij}}{\delta y_{0j}}\bigg|_0 dy_{0j} + \frac{\delta l_{ij}}{\delta z_{0j}}\bigg|_0 dz_{0j} + \frac{\delta l_{ij}}{\delta l_{0j}}\bigg|_0 dl_{0j} + \frac{\delta l_{ij}}{\delta s_j}\bigg|_0 ds_j \qquad \text{(eq. 3)}$$

In equation (eq. 3), dx_i, dy_i, dz_i denote additions to the approximated point coordinates, $dx_{0j}, dy_{0j}, dz_{0j}$ represent additions to the position coordinates of the laser tracers, while dl_{0j} and ds_j denote additions to their dead path and scale factor. The differential quotients $\big|_0$ and $w_{ij}\big|_0$ are calculated from approximated values of the unknowns, for instance point coordinates provided by the mover (CMM, robot or machine centre).

In equation (eq. 2) the scale factor s_j considers that the wavelength of each laser tracer differs within the calibration uncertainty, which is taken into account by (eq. 4)

$$w_{sj} = 1 + ds_j$$

(eq. 4)

and the associated weight $p = (u_s)^{-2}$, u_s denotes the standard uncertainty of the laser wavelength calibration.

In order to simplify the notation, (eq.3) and (eq. 4) can be written in matrix form

$$w = J\beta - w_0 \qquad \text{(eq. 5)}$$

where *J* denotes the Jacobian matrix with the differential quotients, β the corrections to the approximate point coordinates and the five unknown parameters of each laser tracer. Using (eq. 5) the target function (eq. 2) becomes:

$$(J\beta - w_0)^T P(J\beta - w_0) \Rightarrow \min \qquad \text{(eq. 6)}$$

Then, the solution of this minimisation problem is given by

$$\beta = (J^T PJ)^{-1} J^T P w_0 \qquad \text{(eq. 7)}$$

if the matrix $(J^T PJ)$ is non-singular. However, in a purely net of length measurements (eq. 7) is always singular, because no coordinates are chosen to define the origin and orientation of the coordinate system. A way to solve this problem is to fix three coordinates, but a large number of different solutions are possible. A more general solution is given if the calculated point coordinates meet the six constraints (eq. 8):

$$\sum_{i=1}^{n} dx_i = 0 \quad ; \quad \sum_{i=1}^{n} dy_i = 0 \quad ; \quad \sum_{i=1}^{n} dz_i = 0 \quad ;$$

$$\sum_{i=1}^{n} (y_i dx_i - x_i dy_i) = 0 \quad ; \quad \sum_{i=1}^{n} (x_i dz_i - z_i dx_i) = 0 \quad ; \quad \sum_{i=1}^{n} (z_i dy_i - y_i dz_i) = 0 \quad ; \qquad \text{(eq. 8)}$$

or in matrix notation

$$G^T \beta = 0 \qquad \text{(eq. 9)}$$

Then the solution is given by

$$\begin{bmatrix} \beta \\ k \end{bmatrix} = \begin{bmatrix} J^T PJ & G \\ G^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} J^T P w_0 \\ 0 \end{bmatrix} \qquad \text{(eq. 10)}$$

By the constraints (eq. 8) the geometry of the net of length measurements is not deformed. The geometric meaning of (eq. 8) is that the calculated point coordinates are optimally fitted to the approximated point coordinates ($\sum (dx_i\, dx_i + dy_i\, dy_i + dz_i\, dz_i) = \min$).

Since the minimization problem is non linear, the calculation has to be iterated until the weighted sum of squared discrepancies $w^T P w$ has reached its minimum. Finally, the *n* point coordinates x_i, y_i, z_i, and five parameters $x_{0j}, y_{0j}, z_{0j}, l_{0j}, s_j$ of each LaserTracer can be received according to (eq. 10). The sub-matrix in the top left hand corner of the regular inverse in (eq. 10) is the singular variance-covariance matrix that describes the accuracy of the calculated point coordinates, the estimated position of the laser tracers and the estimated length of the dead paths.

2. Task-specific correction

One main challenge in the application of the M3D3 system for measuring large structures is how to ensure the visibility of the retro-reflector for all tracking interferometers simultaneously (Figure 2and Figure 3). Because the visibility for most of the real-world parts cannot be guaranteed, a most practicable approach is to split the measuring task into separate steps:

- Register the spatial location of the points on the surface of the workpiece using the mover (optical or tactile CMM, machine tool, robot)
- Replay all probing points exactly and measure them with the M3D3 and a retro-reflector mounted to the mover.
- Evaluate the local error vector at each measured point from the difference between the indicated mover position and the position measured with the M3D3 system
- Use the local error vector to correct the original measurement point by point (Figure 4).

This is admissible if the local error vectors can be considered as equal in a close proximity to each measurement point. This is the case if the short periodic errors of the mover are negligibly small. This means, their period should be greater than the positioning accuracy of the CMM or mover.

This novel procedure is worthwhile and applicable for CMMs with significant and repeatable systematic deviations. The first two steps mentioned above can be performed in a reversed order, for instance, if a task-specific correction table is needed prior to the actual measurement. A practical case would be the calibration of parts which are considered identical, or if an error compensation is applied to a CNC program of a machine centre prior to the machining of the parts.

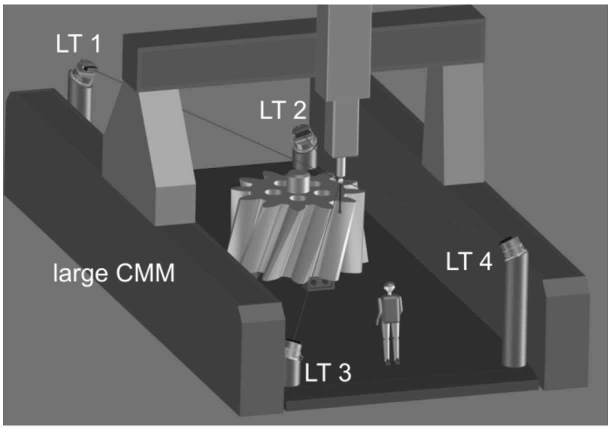

Fig.2: Tactile measurement of a large gear on bridge type CMM and positions of 4 LaserTracers (LT 1 to LT 4)

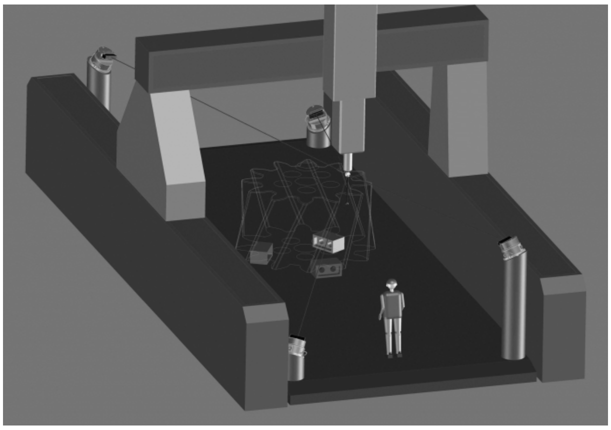

Fig. 3: Measuring the task specific error correction (workpiece removed)

The prime advantage of this procedure in comparison with the parametric error compensation approach is that no specific geometric model of the mover is required for the error correction. In addition, this procedure covers also the hysteresis effects as long as they are reproducible and the travel path along the axes is identical when measuring the real part and when determining the errors by means of the M3D3.

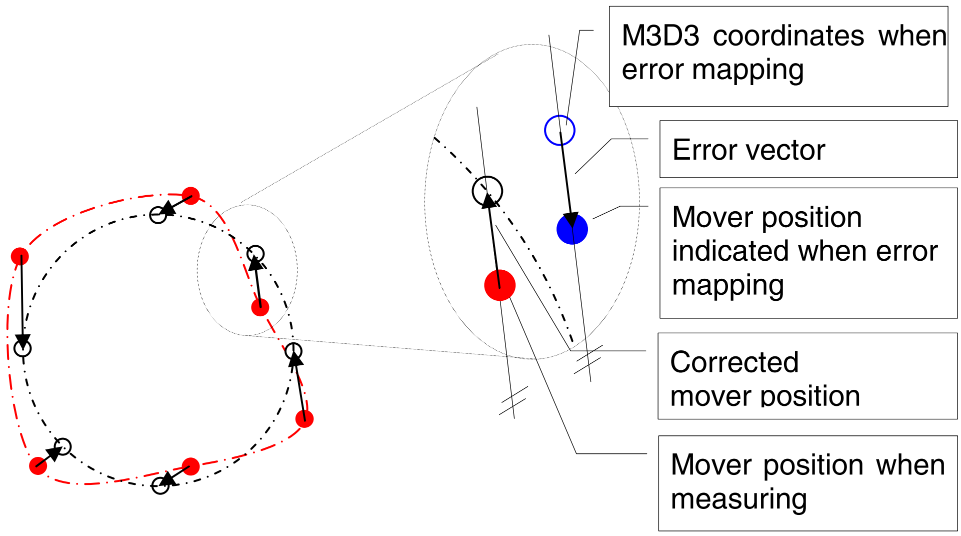

Fig. 4: Principle of task-specific error correction

3. Validation

3.1 Comparison to calibrated artefact

In order to verify the procedure for the task-specific error correction, measurements were carried out on calibrated artefacts. For these tests, the numerical error compensation of the CMM was deactivated, so that the CMM simply acted as a mover, thereby producing significant amount of errors which were however highly reproducible. The measurements performed served as verification and accuracy proof of the procedure of the task-specific error correction described above. The first experiments were conducted using a ball bar made of Zerodur® and 3 ceramic spheres, incorporating three distances in space (Figure 5). The measurement procedure consists of the following main steps:

- Measurement of the object with tactile probing
- Export all probing points from the measuring program
- Replace the probe pin with the retro-reflector
- Use probing points to position the retro-reflector when measuring with the M3D3 system
- Calculate the retro-reflector position by multi-lateration
- Determine the local error vectors by comparison
- Add the local error vector to the original measured probing points
- Re-import the corrected probing points into the measuring program
- Evaluate the corrected probing points and determine the geometric parameters of the object

Fig. 5: Measurement of a calibrated ball bar as reference

The measurement results obtained on the CMM with de-activated numerical error compensation and those of the M3D3 multi-lateration measuring system were compared to the calibration values of the ball bar. Table 1 delineates a comparative overview of the deviations and shows a considerable improvement of the sphere centres spacing when measured with the M3D3. The deviations between the M3D3 results and the calibrated lengths of the ball bar are close to the calibration uncertainty of the ball bar.

Table 1: Calibrated lengths of ball bar and measured deviations

Calibrated distances (in mm)	Calibration uncertainty (*k*=2) (in mm)	Uncorrected CMM length deviations (in mm)	M3D3 length deviation (in mm)
165.6941	0.0004	-0.0049	0.0003
399.9913	0.0004	-0.0090	0.0009
565.6715	0.0005	-0.0139	0.0012

3.2 Comparison to 1D laser interferometer

The measurement described above is a test describing in general the working procedure of M3D3 multi-lateration system. However, the observed deviations are in most cases in the range of the uncertainty of the used references (high-precision CMM, calibrated artefact). Therefore another setup was realized, applying an additional LaserTracer as reference interferometer along inclined straight measurement lines. In the procedure applied here, all five LaserTracers were locked in on one retro-reflector and a spatial grid of reflector positions similar to the setup above was measured to determine the positions of all five tracers.

One of the five LaserTracer was chosen as reference interferometer. Then, straight lines through the centre of this tracer were defined as reference lines using the known position of this tracer as fix point. As a result the selected LaserTracer is able to follow a retroreflector without tracking when moving along these lines (Figure 6), i.e. this dedicated LaserTracer acts as a non-tracking interferometer (static) and measures distance changes directly, free of Abbe errors. After adding reflector positions on those lines, the measurement was again conducted using all five tracers. However, all reflector positions were calculated by multi-lateration using only four LaserTracer forming the M3D3 measuring system. Finally distances between reflector positions along the reference lines were calculated using the measurement results of the M3D3 system. Then, these distances were directly compared with the readings of the reference interferometer. The advantage of this setup is that it uses a reference system with interferometric length accuracy which requires no manual alignment.

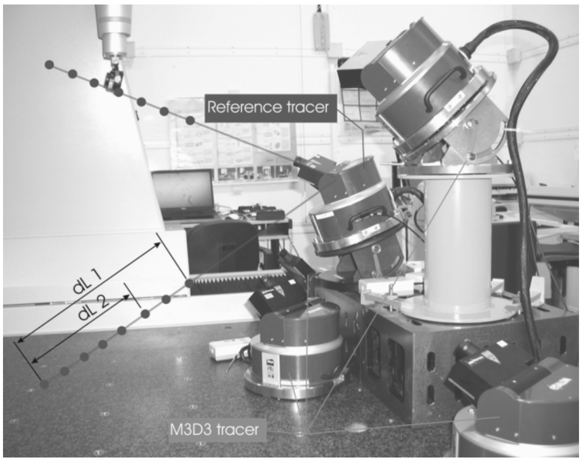

Fig. 6: Additional LaserTracer measuring on straight lines

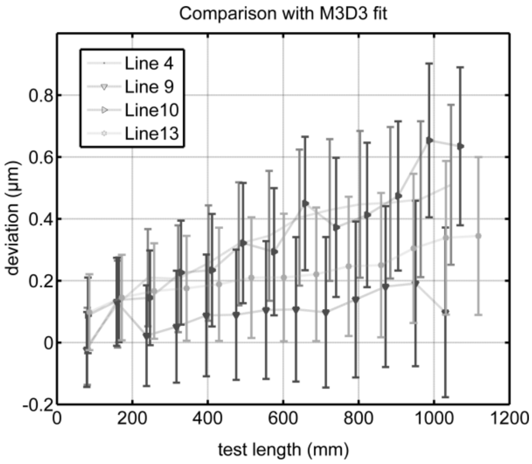

Fig. 7: Typical length deviations on inclined measurement lines

Typical length deviations between the distances measured by the M3D3 system and the reference interferometer together with the estimated length measurement uncertainties are shown in the Figure 7. The standard uncertainties (k=1) were calculated by means of uncertainty propagation using the accuracy of the measured point coordinates which are obtained from the sub-matrix in the top left hand corner of the matrix inverse in (eq. 10). For each LaserTracer a standard length measuring uncertainty of $u = \sqrt{0.1\,\mu m + (1*10^{-7})^2 (l_{ij} + l_{0j})^2}$ was assumed. The uncertainties are in the range of 0.1 µm to 0.2 µm for distances on the selected lines.

3.3 Calibration of large involute gear artefact

The measurement capabilities of M3D3 system were verified on the shop floor. The primary focus here is to demonstrate: Measurement accuracy, and testing of task-specific error correction method specifically for complex parts e.g. large involute gear artifact (Figure 8 and Figure 9). The experiments have been performed on a large, high-end CMM, which guarantees a good reproducibility of the geometric errors. The thermal conditions during the measurement have been at shop floor level, but very well controlled (stable, low gradients).

The main challenge with error mapping of complex artifact is that several probe tip offsets are usually involved in the tactile measurement while the multi-lateration operates on one single target. The requirement in this case is to mount the retro-reflector as close as possible in the position of the used probe tip. To fulfill this requirement, the measurement is split into subtasks related to all involved probe tip offsets.

For each subtask the list of probing points related to each probe tip is extracted and supplemented with additional reflector positions to ensure the correct qualification of the M3D3 system. The principle of the dataflow is shown in Figure 10. M3D3 system's software supports import and export of points for the evaluation software QUINDOS and Calypso.

Figure 11 shows the differences of the gear parameters obtained in the first on-site calibration using the M3D3 system and the PTB reference values obtained under laboratory conditions. The error bars show the expanded uncertainties (k=2) associated with the gear parameters of the on-site calibration. These uncertainties have been determined by the Virtual CMM technique [3].

Fig. 8: Large involute gear artefact

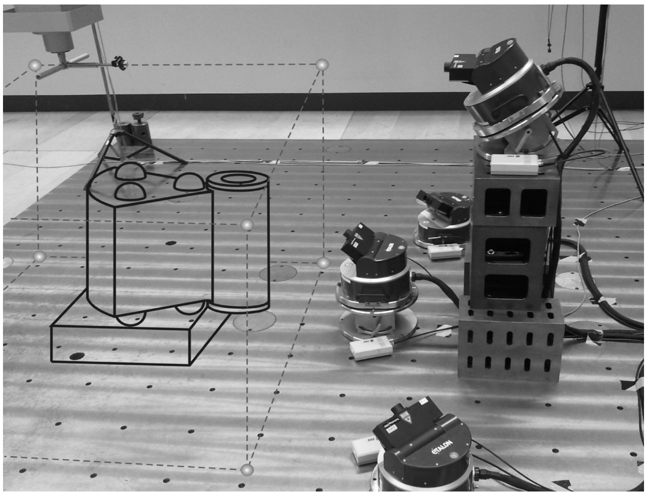

Fig. 9: M3D3 shop floor demostration on involute gear standard

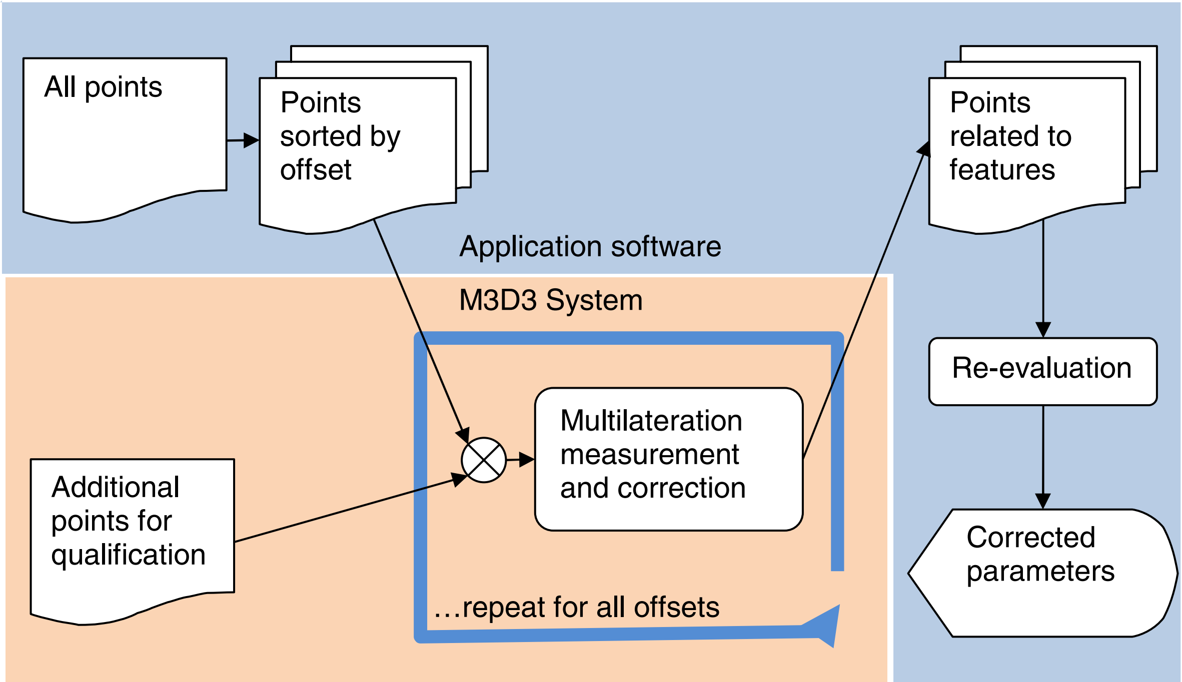

Fig. 10: Basic dataflow for the offline correction when different tool offsets are involved

The good conformance with the reference values can be interpreted as a first validation of the suggested calibration method under shop floor conditions.

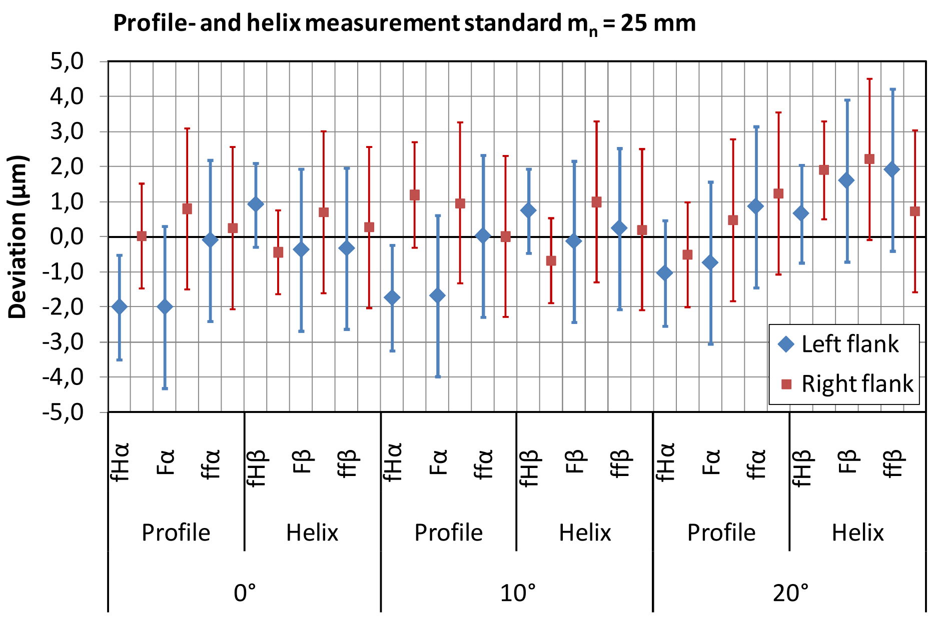

Fig. 11: Deviations of On-Site calibrated gear parameters from PTB reference values

Conclusions

The M3D3 system realizes the Abbe principle in 3D. Therefore, large objects up to several meters can be calibrated with higher accuracy. As the system consists of portable tracking laserinterferometer (LaserTracer) it offers the possibility to perform the calibration on customer's site.

Acknowledgement

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