MIMO NON-LINEAR SENSORS CALIBRATION BASED ON GENETIC ALGORITHMS

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Abstract:

The increasing integration between electronics and mechanical engineering brings to the industrial market very hi-tech sensors, often non-linear, capable of more than a single input and single output. A problem more and more relevant for sensors like these is calibration. Classic linear calibration procedures, when applied to this extremely engineered sensors, lead to poor accuracy and are generally not satisfactory.

The case study is the calibration of a bi-dimensional laser based position sensor, in particular a positive sensitive detector, that is an optical position transducer based on series of photodiodes commonly used as multidimensional sensor. To perform the calibration a micrometric positioning table was used to test the whole photodiode active area in both directions. The sensor studied showed a very linear behaviour in the central region of the working range, and a limited nonlinearity closer to the range limits and was to be used to verify robot movement capabilities; to reduce uncertainty associated with nonlinearities, a set of non-standard, non-linear, calibrations were performed, pointing out residual values in order to compare different algorithms. In a previous work, authors have already tested a linear model against an algorithm based on radial basis functions (RBF) and Nelder-Mead simplex method. Object of this paper is the definition of a procedure based on RBF and genetic algorithms for multi-dimensional interpolation of data cloud and a comparison between this updated procedure results and the ones of the previous studied algorithms.

The reference model for calibration was a black box with two inputs, X and Y position of the laser spot, and two outputs, voltages Vx and Vy, while the calibration procedure was split in two separate layers, one for each output depending on both inputs. Given N data points in a M-dimensional environment and N values that represent the non linearity residual, purpose of the algorithm is to approximate a data cloud with a real function, that is represented as a sum of a polynomial (linear) part and L radial basis functions, each associated with a different center (node) and weighted by an appropriate coefficient, that the procedure also allow to assess.

When no starting guess for nodes are given in input, nodes coordinates are the output of a non-linear optimizer based on a genetic algorithm, whose goal is to locally minimize the objective function. The algorithm stops itself whenever it reaches a certain tolerance level, a user-specified number of nodes or when the previous iteration has a better value of the objective function. This study has been performed for various RBF classes, and shows an increased accuracy, thus a better metrological behaviour, with respect to the standard linear (planar) calibration model traditionally used.

Keywords: calibration, radial basis function, multi input, uncertainty, genetic algorithm.

1. INTRODUCTION

Positive Sensitive Detectors (PSDs) are optical position transducer based on series of photodiodes commonly used in mechatronics as multidimensional sensor. The case study is the calibration of a bi-dimensional laser based position sensor SC-10D from UDT Sensors, which shows a very linear behaviour in the central region of the working range, and a limited nonlinearity closer to the range limits.

To reduce uncertainty associated with those nonlinearities, as the sensor was to be used to verify robot movement capabilities, a set of non-standard, non-linear, calibrations were performed.

The reference model for calibration was a black box with two inputs, X and Y position of the laser spot, and two outputs, voltages Vx and Vy, while the calibration procedure was split in two separate layers, one for each output depending on both inputs, as is shown in figure 1:

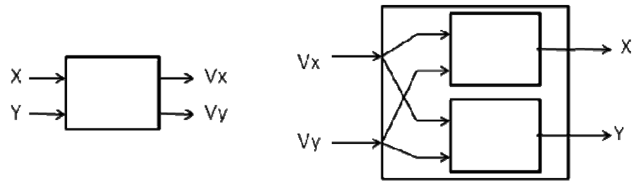

Figure 1. Layered reference model used for calibration

To perform the calibration a micrometric positioning table was used testing the whole photodiode active area in both directions following the setup displayed in figure 2. Results presented here are related to a calibration test performed using a set of 81 points, 41 used for numerical model set-up and 40 for model validation. Different tests with more points were performed with compatible results.

A starting linear model for the transducer behaviour was created based on the points acquired, and residual values (i.e. deviation from linearity) were pointed out, in order to be tested against different algorithms. Considering one direction at a time, the position has been expressed as the composition of a linear part and a residual one , an example of the which can be seen in figure 3, along with the following expression (valid for the X direction):

$$X(V_x,V_y)=A_0+A_x\cdot V_x+A_y\cdot V_y+\varepsilon(V_x,V_y)$$
$$\varepsilon(V_x,V_y)=\sum_{i=1}^{L}\alpha_i\cdot\varphi_i(V_x,V_y) \tag{1}$$

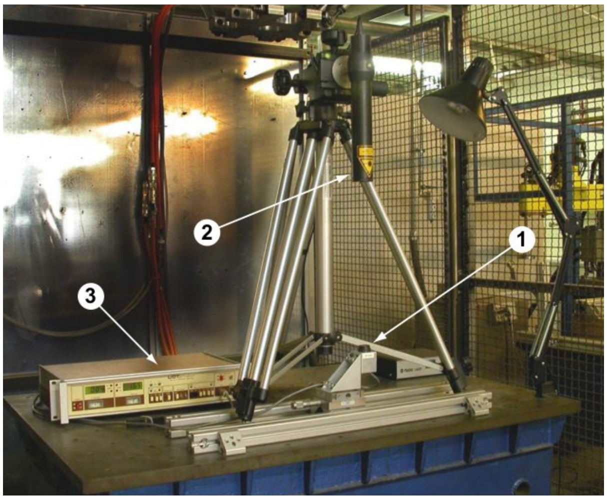

Figure 2 – Experimental setup: sensor (1), laser (2), conditioning device (3)

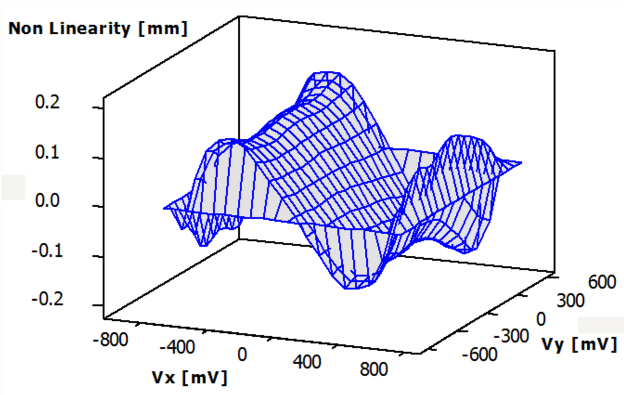

Figure 3 – Non linearity behaviour for the X direction

2. METHODS AND MODELS

For the description of the sensor non-linearity an algorithm using radial basis functions (RBF) multi-dimensional interpolation of data clouds has been used: the rationale for doing this is that radial basis functions give a "local" response instead of the traditional polynomial function whose response are "global" in the whole input domain.

Given the vector *x* representing the input of the calibration diagram (*Vx* and *Vy* in the laser photodiode presented) and the scalar y the output (measure estimation) of the calibration data, a radial basis function (RBF) is a real-valued function whose value depends only on the distance from the origin, so that

$$\varphi(x) = \varphi(\|x\|) \tag{2}$$

or alternatively on the distance from some other point c, called a center, so that

$$\varphi(x, c) = \varphi(\|x - c\|) \tag{3}$$

Any function φ that satisfies this property is a radial function. The norm is usually Euclidean distance, although other distance functions are also possible.

In this case a gaussian radial function has been choose where the distance *r* is given by

$$r^2 = \left(\frac{x_1 - c_1}{\sigma_1}\right)^2 + \left(\frac{x_2 - c_2}{\sigma_2}\right)^2 \tag{4}$$

for every center (node) c_i and

$$\varphi(x, c_i) = e^{-r^2} \tag{5}$$

Given N data points $(x_1, x_2, .., x_N)$ in a M-dimensional environment (each point is identified by a vector), and N values $(y_1, y_2, .., y_N)$ (the non linearity residual), purpose of the algorithm is to approximate this data cloud with the real function:

$$y(x) = \varepsilon(x) = \sum_{i=1}^{L} \alpha_i \cdot \varphi_i(x, c_i) \tag{5}$$

where the approximating function $y(x)$ is represented as a sum of L radial basis functions, each associated with a different center (node) c_i and weighted by an appropriate coefficient α_i.

The weights α_i can be estimated using the methods of Linear Least Squares, because the approximating function is linear in the weights.

Position of the center c_i and the parameters σ_i are estimated thanks to a genetic algorithm, which will be explained in next paragraph, and whose goal is to locally minimize the objective function:

$$\sigma_0^2 = \frac{\sum_{k=1}^{N} (y_k - y(\bar{x}_k))^2}{N - L} \tag{6}$$

An outlook of this two-folded, iterative process can be found in figure 4.

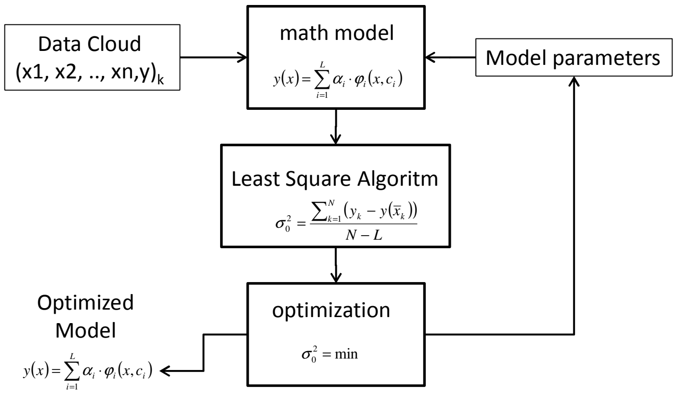

Figure 4 – Twofold algorithm used for calibration.

3. SIMULATION

The optimization procedure is based on a custom genetic algorithm (GA) [10,11], with the aforementioned target function σ_0 to be minimized.

For each generation a fixed number of 100 individuals is generated, with every individual characterized by a sequence of 4L parameters, given L the number of RBFs.

The generation is mainly mono-parental, with every individual gene sequence based on a single individual of the previous generation.

Every gene is generated using a Normal distribution centered on the parent value, with an ex-ante assigned variance and upper and lower limits [12].

Give FS the sensor full scale, for center position c_i coordinates variance was imposed to be $(FS/50)^2$, while limits were set equal to 2FS. The remaining σ_i parameters were set with a variance of $(FS/100)^2$ and limited between 0.1 FS and 2 FS.

After generation, to avoid problems caused by the mono-parental simplification, a random sample of 5% of the individuals is chosen, and a random parameter is chosen and switched from an individual to another.

The selection mechanism for determining which individuals for each generation are allowed to reproduce is based on a selective pressure degree system, assigning a probability of child generation inversely proportional to fitness of the model, represented by the target function value, following the model:

$$p_c = \left(\frac{1}{\sigma^2_{0_i}}\right) / \sum_j \left(\frac{1}{\sigma^2_{0_j}}\right) \qquad (7)$$

Where p_c is the probability of each individual of the next generation to be a child of the i^{th} individual of the current.

To preserve the best solution of each generation, the best performer is both involved in new individuals generation, and cloned without regenerating its genes.

As can be noticed in figure 5, after 200 generations, the best performer gives the same result for each generation, indicating a stable result with a value below 0.005 mm, therefore, for each test a limit of 250 generation was imposed.

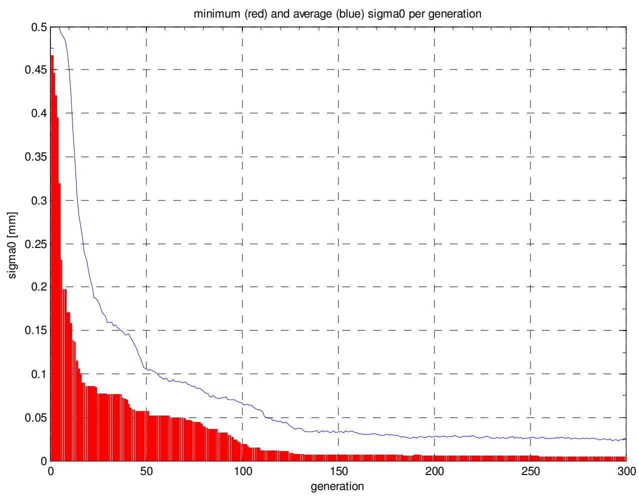

Figure 5. Maximum (red) and average (blue) fitness of model for each generation.

4. RESULTS

As stated before, for the calibration of this sensor have been tested with a Gaussian model tuned using genetic algorithm. The algorithm was applied to a set of 41 experimental data couples used as reference data to be fitted, and it has been tested on a different set of 40 points, obtaining the results displayed in the following table and graphs.

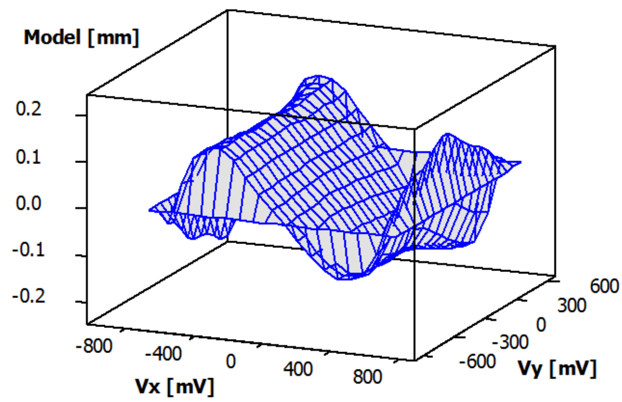

Figure 6 – Linear model residual for the X direction

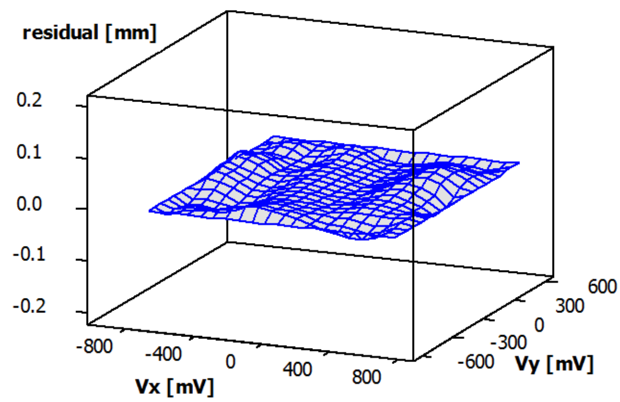

Figure 7 – RBF proposed model residual

	Non linearity $\varepsilon(V_x, V_y)$	Residual on calibration data	Residual on control points
	[mm]	[mm]	[mm]
Max	0.178	0.030	0.031
Min	-0.203	-0.033	-0.049
Standard dev.	0.114	0.014	0.016

Table 1 – Non linearity (residual of linear) and proposed model residuals

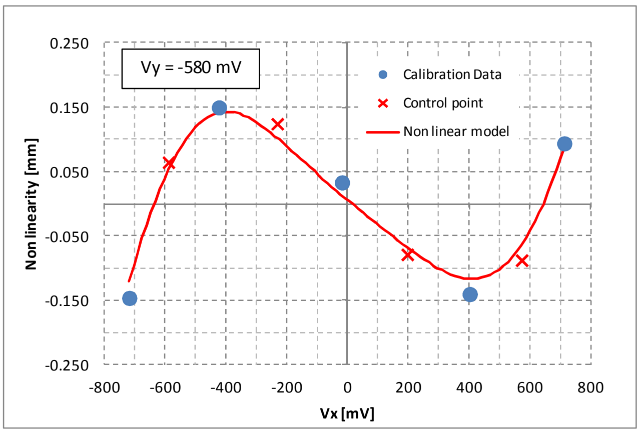

Figure 8 –Non-linearity: calibration data, control points and model results

Figure 8 shows a "section" of the non-linearity surface (i.e. the residual of a linear best fit) with the indication of calibration data points (circles) and control points (crosses). Same results are available also in different Vy tension level, as shown in figure 9.

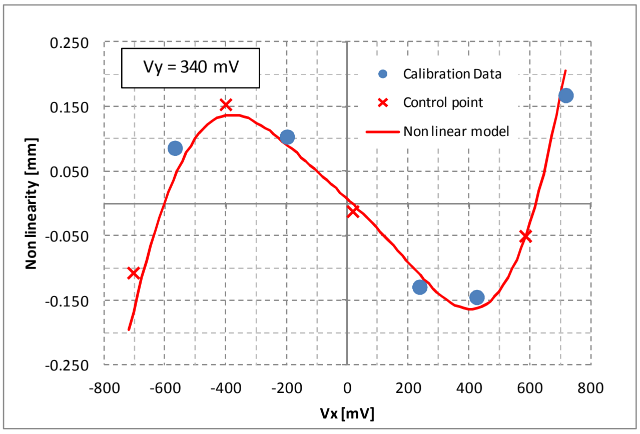

Figure 9 –Non-linearity: calibration data, control points and model results

5. DISCUSSION

Overall uncertainty evaluation is considered to be given as a composition as two contributors: uncertainty related to model characterization and residual variance unexplained by the model itself. It is common practice to neglect the first part, as is by definition smaller than the second given a reasonable number of samples; however, by taking into account both, uncertainty of predictions made using the calibrated model can be expressed as:

$$u^2(y) = \sigma^2_{y(x)} + \sigma^2_0 \tag{8}$$

where

$$\sigma^2_{y(x)} = [\varphi(x)]^T \cdot C_{\alpha\alpha} \cdot [\varphi(x)] \tag{9}$$

and

$$C_{\alpha\alpha} = \sigma^2_0 \cdot \left[[\varphi(x_k)] \cdot [\varphi(x_k)]^T \right] \tag{10}$$

Since in the case study uncertainty given by characterization of the model was negligible with respect to residual variance, and being the latter a major player in the algorithm itself, the simplifying following assumption was made:

$$u(y) = \sigma_0 \tag{11}$$

Furthermore, this has been used to validate the results, as the further sets tested showed a result compatible with the prediction given this standard uncertainty.

6. CONCLUSIONS

A method to describe a non-linear behaviour of transducer using various kind of RBFs has been described. The same algorithm has been tested in the case study of a two-dimensional contactless position sensor – a photodiode – capable of measuring the two coordinates of a laser-light point hitting his square surface.

The use of a non linear model increased accuracy, thus a better metrological behaviour, with respect to the standard linear (planar) calibration model traditionally used.

Because of strongly non-linear optimization problems, a genetic algorithm for model parameters has been proposed. The results show how both, model and method, can reduce uncertainty in the case of multi-input non linear sensors calibration.

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