SIMPLIFIED ALGORITHM OF COORDINATE MEASUREMENTS UNCERTAINTY EVALUATION

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Abstract:

Long-lasting works on the ISO 15530 series can be a proof for complexity of uncertainty evaluation of coordinate measurements for different characteristics defined according to the rules of geometrical product specification (GPS) [1]. There are two most acknowledged and commonly accepted methods. First is described in the document ISO 15530-3:2011 [2] which provides in detail the procedure of uncertainty evaluation with the use of calibrated workpiece or measurement standard. Second procedure is based on the use of computer simulation that is known from many publications and mentioned in ISO/TS 15530-4:2008 [3].

The author is one of the elaborator of the methodology and the software (EMU-CMMUncertainty™) which makes possible evaluation of the measurement uncertainty for bridge and horizontal arm CMMs. Some details concerning the software operation have been published in [4-8]. The software is developed with assumptions some of which are similar as in the method using the computer simulation. The most important assumption is that the uncertainty of measurement for particular characteristic (size, dimension, form, orientation, location and run-out) is calculated with the formula describing the characteristic as a function of **differences of coordinates of characteristic points** of the workpiece (the generality condition).

For easier understanding of the methodology a simplified version was developed which includes in the uncertainty budget just the MPE_E instead of the geometrical errors and probing head errors of the CMM. Of course, this assumption gives overestimated uncertainty values but enables the sensitivity analysis. The paper includes some interesting cases of the sensitivity analysis.

Keywords: uncertainty, coordinate measurements, CMM, geometrical product specification

1. INTRODUCTION

Determination of measurement uncertainty in general, and especially uncertainty of the coordinate measurement, is not an easy task. One of the purposes of determining the uncertainty of measurement is to raise awareness of the person performing the measurements: which factors have the greatest contribution in the uncertainty, or in other words, what elements of the measurement strategy require particular attention. This information is only accessible through the uncertainty budget.

Coordinate measurements are indirect measurements Directly, coordinates of points are measured, then the software calculates the required geometrical characteristics: size as well as form, orientation, location and run-out deviations. The relationship between the coordinates of the probing points and the values of the measured characteristics is not overt – it's in the form of algorithms. Furthermore, the calculation is performed in two steps, and it also complicates formulating the measurement model. At first, from the coordinates of probing points belonging to one feature so called geometric elements (planes, cylinders, etc.) are constructed. In the second step from geometric elements required characteristics are calculated (eg, parallelism or position deviation).

Jakubiec pointed out the possibility of analytical evaluation of the uncertainty of coordinate measurement in [4-6]. Jakubiec and Płowucha have developed the methodology and software for the evaluation of uncertainty that follows the GUM guidelines [7, 8]. Some elements of the methodology can also be used for approximate evaluation of uncertainty and are discussed in this publication. Appropriate simplified method of calculation for estimating the coordinate measurement is based on two fundamental assumptions:

- there is a small minimum number of characteristic points of the workpiece sufficient to determine the specific characteristics of the workpiece (dimensional and geometric deviations) and at the same time the measurement uncertainty,
- the only easily accessible information on the accuracy of the measuring machine is the formula for the maximum permissible error of indication for length measurements.

2. GENERAL MEASUREMENT MODEL

All the necessary formulas expressing individual characteristics as a function of coordinate differences of sampling points can be derived based on the formulas for distance of two points, the distance of the point and the line [5] and the distance of the point and the plane [8].

The distance l of a point S from the straight line p defined by a point P and unit vector u parallel to the straight line is calculated as

$$l(S,p) = |(P-S)\times u| \quad (1)$$

The distance l of a point S from the plane p defined by a point P and unit normal vector u is calculated as

$$l(S,p) = |(P-S)\cdot u| \quad (2)$$

It turns out that each measured geometrical characteristic l can be expressed as a function of coordinate differences r_i, $i = 1...N$, of a few pairs of characteristic points of the workpiece

$$l = f(r_1, r_2, \dots, r_N) \tag{3}$$

Using this function, one can (assuming no correlation) calculate combined measurement uncertainty with the well-known formula

$$u_c(l) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial l}{\partial r_i} \right)^2 u^2(r_i)} \tag{4}$$

The corresponding partial derivatives can be calculated analytically (which is quite time-consuming) or numerically. To estimate the combined uncertainty of the coordinate measurement result $u_c(l)$ in addition to the partial derivatives appearing in the formula (4) also the uncertainties $u(r_i)$ of measurement of the particular differences r_i are needed. The presented methodology assumes that the expanded uncertainty of measurement U coordinate differences pair of points can be estimated from the formula for the maximum permissible error of indication in the measurement of the length *MPEE*, which usually takes the form of

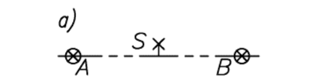
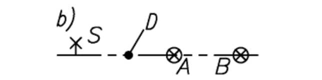
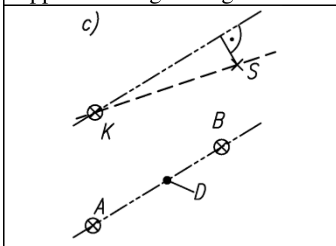
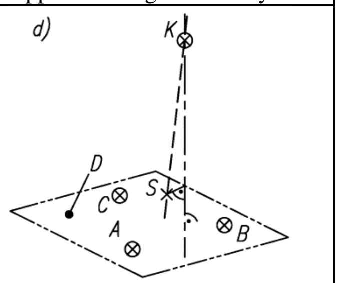
$$MPE_E = A + BL \quad or \quad MPE_E = A + L/K \tag{5}$$

where L is the measured length and in the described approach also the appropriate difference of coordinates.

3. MEASUREMENT MODELS BASED ON THE POINT-STRAIGHT LINE DISTANCE

Models based on the distance of the point from the straight line require 3-5 characteristic points. They are summarized in Table 1. More information about the use of these models can be found in [5].

Table 1. The measurement models based on the point-straight line distance

 The straight line determined by points *A* and *B*; point S lies within the segment *AB* Application e.g.: straightness	 The straight line determined by points *A* and *B*; point S lies outside the segment *AB* Application e.g.: coaxiality
 The straight line includes the point *K* and is parallel to the segment *AB* Application e.g.: parallelism of axes - cylindrical tolerance zone	 The straight line includes the point *K* and is perpendicular to the plane *ABC* Application e.g.: perpendicularity of axis - cylindrical tolerance zone

For the first two models (Tab. 1a, b) the formula for the distance of point S from the line AB is

$$l = \sqrt{(y_{AS} \cdot z_{AB} - z_{AS} \cdot y_{AB})^2 + (z_{AS} \cdot x_{AB} - x_{AS} \cdot z_{AB})^2 + (x_{AS} \cdot y_{AB} - y_{AS} \cdot x_{AB})^2} \Big/ \sqrt{x_{AB}^2 + y_{AB}^2 + z_{AB}^2} \tag{6}$$

The meaning of the distance l is as follows:

- for the straightness the distance l is the value of the deviation,
- for the coaxiality the distance l equals the half of the deviation value.

Example. Evaluation of the uncertainty of coaxiality measurements for two holes (Fig. 1).

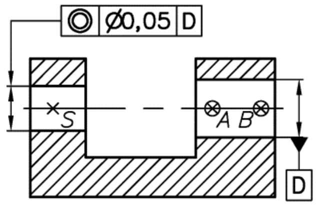


Fig. 1 The drawing for the uncertainty budged of coaxiality measurement; coordinates of characteristic points: *A*(50, 50, 50), *B*(60, 50, 50), *S*(100, 50, 50)

It was assumed that the axis of the measured holes is parallel to the x axis, the datum hole is probed in two sections 10 mm apart, and the toleranced hole is 50 mm away from the datum hole (the characteristic points on Fig. 1 are given in the CMM coordinate system). The measurement is performed on a CMM with the MPE_E in the form of

$$MPE_E = 2 + L/300 \tag{7}$$

The measured characteristic (coaxiality) is a function of six differences of coordinates

$$l = f(x_{AB}, y_{AB}, z_{AB}, x_{AS}, y_{AS}, z_{AS}) \tag{8}$$

The calculated partial derivatives' values (sensitivity coefficients) are summarized in the 4th column of the table 2. Due to large number of data it is more efficient to calculate the derivatives numerically. The table includes also differences of coordinates of points A and B as well as A and S, expanded uncertainties and standard uncertainties of these differences and products of the standard uncertainties and the sensitivity coefficients.

Table 2. Uncertainty budget for coaxiality measurement

Differences of coordinates, mm	Expanded uncertainty U_i, µm	Standard uncertainty u_i, µm	Sensitivity coefficient $\partial l/\partial r_i$	$\frac{\partial l}{\partial r_i} u_i$, µm
$x_{AB} = 10$	2,03	1,015	0	0
$y_{AB} = 0$	2	1	5	5
$z_{AB} = 0$	2	1	5	5
$x_{AS} = 50$	2,17	1,085	0	0
$y_{AS} = 0$	2	1	1	1
$z_{AS} = 0$	2	1	1	1
			$u_c =$	14,42

The standard combined uncertainty calculated from the formula (4) u_c = 14,42 µm.

The presented model enables analysis of the influence of the measurement strategy on the measurement uncertainty. The influence of the distance between the extreme probing sections of the datum feature (datum length) is shown on Fig. 2, and the influence of the distance of the toleranced feature from datum on Fig. 3.

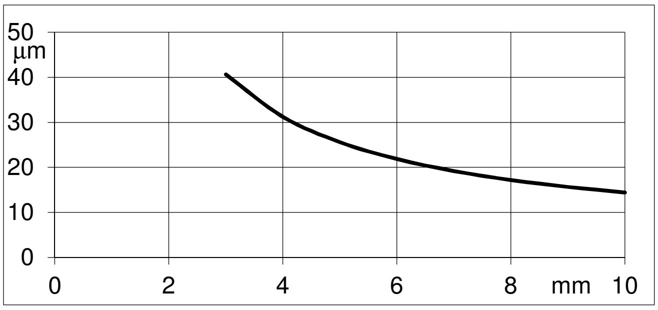


Fig. 2 The influence of the distance between the probing sections of the datum feature on the standard uncertainty of coaxiality measurement (for the distance of the toleranced feature from datum *BS* = 40 mm); ordinate – distance *AB*, abscissa – standard uncertainty *u*.

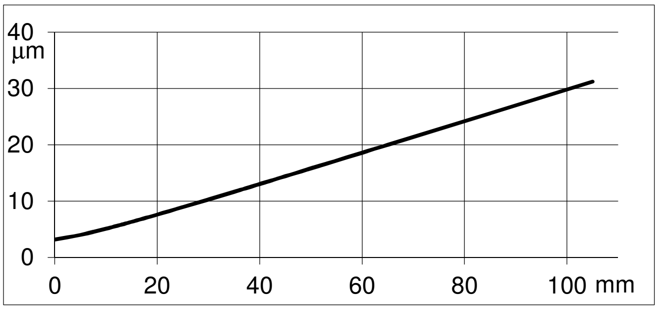


Fig. 3 The influence of the distance of the toleranced feature from the datum on the feature on the standard uncertainty of coaxiality measurement (for the distance between the extreme probing sections of the datum *AB* = 10 mm); ordinate – distance *AS*, abscissa – standard uncertainty *u*.

4. MEASUREMENT MODELS BASED ON THE POINT-PLANE DISTANCE

The models based on the point-plane distance require the coordinates of 4-7 characteristic points [8].

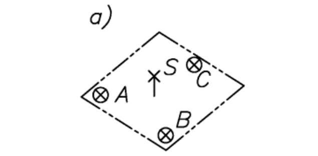
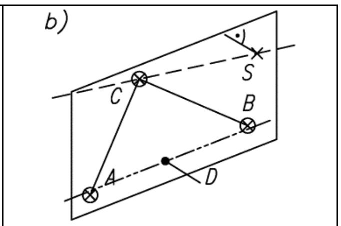
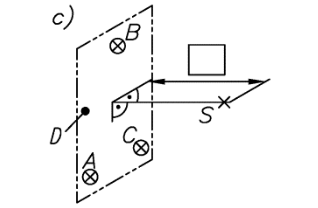
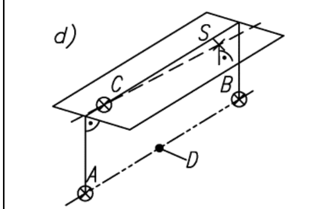
For the models *a*, *b* and *c* from the Tab. 3 the formula for the distance *l* of the point *S* from the plane *ABC*

$$l = \left| \frac{a x_{SA} + b y_{SA} + c z_{SA}}{m} \right| \tag{9}$$

where

$$\begin{aligned} a &= y_{BA} z_{CA} - z_{BA} y_{CA} \\ b &= x_{CA} z_{BA} - x_{BA} z_{CA} \\ c &= x_{BA} y_{CA} - x_{CA} y_{BA} \\ m &= \sqrt{a^2 + b^2 + c^2} \end{aligned} \tag{10}$$

Table 3. The measurement models of geometrical deviations based on the point-plane distance

 The plane is determined by the points *A*, *B* and *C*; point *S* nominally lies on the plane *ABC* within the points *A*, *B*, *C* Application e.g.: flatness	 The plane is determined by the points *A*, *B* and *C*; point *S* nominally lies on the plane *ABC* outside the points *A*, *B*, *C* Application e.g.: parallelism of axes in plane normal to the common plane
 The plane is determined by the points *A*, *B* and *C*, point *S* nominally lies outside of the plane *ABC* Application e.g.: position of a point, axis or plane	 The plane is perpendicular to the plane determined by the points *A*, *B* and *C*, parallel to the straight line *AB* and includes point *C* Application e.g.: parallelism of axes in the common plane

The meaning of the distance *l* is as follows

- for the flatness and the parallelism of axes, the distance *l* is the value of the deviation,
- for the position, the distance *l* equals the half of the deviation value.

Example. Evaluation of the measurement uncertainty of parallelism of axes in the direction normal to the common plane (Fig. 4).

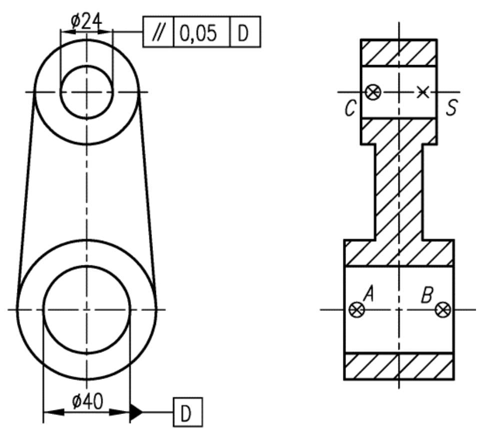


Fig. 4. The drawing for the uncertainty budged of measurement of parallelism of axes; coordinates of characteristic points: *A*(50, 50, 50), *B*(80, 50, 50), *C*(55, 50, 200), *S*(75, 50, 200)

It was assumed that the axes of the measured workpiece holes are parallel to the *x* axis of the CMM, the datum hole is probed in two sections 30 mm apart, and the toleranced hole is probed in two sections 20 mm apart. The distance between the holes is 150 mm (the characteristic points on Fig. 4 are given in the CMM coordinate system). The measurement is performed on the same CMM as in previous example.

The measured characteristic (parallelism) is a function of nine differences of coordinates

$$l = f\left(x_{BA}, y_{BA}, z_{BA}, x_{CA}, y_{CA}, z_{CA}, x_{SA}, y_{SA}, z_{SA}\right) \quad (11)$$

The calculated (numerically) partial derivatives' values (sensitivity coefficients) are summarized in 4th column of the table 4. The table includes also differences of coordinates of points *B* and *A*, *C* and *A* as well as *S* and *A*, expanded uncertainties and standard uncertainties of these differences and products of the standard uncertainties and the sensitivity coefficients.

Table 4. Uncertainty budget for parallelism measurement

Differences of coordinates, mm	Expanded uncertainty U_i, µm	Standard uncertainty u_i, µm	Sensitivity coefficient $\partial l/\partial r_i$	$\frac{\partial l}{\partial r_i} u_i$, µm
x_{BA} –30	2,1	1,05	0	0
y_{BA} = 0	2	1	0,67	0,67
z_{BA} = 0	2	1	0	0
x_{CA} = –5	2,02	1,01	0	0
y_{CA} = –150	2	1	1	1
z_{CA} = 0	2,5	1,25	0	0
x_{SA} = –25	2,08	1,04	1	1,04
y_{SA} = 0	2	1	0	0
z_{SA} = –150	2,5	1,25	0	0
			u_c =	1,57 µm

The standard combined uncertainty calculated from the formula (4) u_c = 1,57 µm.

CONCLUSIONS

The paper shows that it is possible to evaluate the measurement uncertainty basing on the models that follow the principle of the coordinate measurement, namely, that it is an indirect measurement.

It is demonstrated on examples that each geometrical characteristic, and especially each geometrical deviation, can be expressed as a function of the coordinates (more precisely: coordinates' differences) of some characteristic points of the workpiece. In some cases, the points are measured directly but more often, they are measured in the indirect manner. Some of the characteristic points are the points belonging to the surface of the workpiece, but the points belonging to the derived features can be used too. Most often, these are axis points but may also be point of the symmetry plane or a sphere centre point.

The uncertainty budget of an indirect measurement includes the sensitivity coefficients expressing the weight factor for particular standard uncertainty components influencing the expanded uncertainty.

The sensitivity coefficients are the values of partial derivatives. The presented examples, and especially the example based on the coaxiality measurement, show that the weight factors can differ significantly, what cannot be shown when using other techniques of uncertainty evaluation.

The presented methodology is very simplified version of the originally developed method of uncertainty evaluation for coordinate measurements [7]. The simplifications were done for two reasons. First, author wanted to bring attention to the theoretical fundamentals of the developed methodology. The author believes that, apart from others also important but secondary assumptions and models, it was possible to justify the substantial correctness of the new methodology for estimating uncertainty of coordinate measurement.

Moreover, the presented methodology has practical significance. The uncertainty evaluation of coordinates' differences basing on the MPE_E is considerable simplification comparing to the methodology used in the EMU-CMMUncertainty™ but it enables interested parties self-assessment of the uncertainty.

The presented methodology has substantial didactic qualities – it shows that the coordinate measurement is an indirect measurement and therefore the uncertainty significantly depends on the measurement strategy [9].

The proposed methodology is illustrated by two simple examples, where in the measurement model (and therefore in the uncertainty budget) only a few elements occur. Similar calculations can be done for any characteristic, especially for those based on the models given in [7].

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