DEFINITION OF A MEASURABILITY THRESHOLD OF GEOMETRIC TOLERANCES IN RELATION TO MEASUREMENT UNCERTAINTY AND DIMENSIONAL PARAMETERS

Matteo Lancini[1], Ileana Bodini[1], Simone Pasinetti[1], David Vetturi[1]

1 University of Brescia, Italy, matteo.lancini@ing.unibs.it

Abstract:

In order to avoid final product malfunction and to allow for assembly integration, geometric specifications and dimensional tolerances are commonly used in mechanical design. However the feasibility of geometric specification measurement and verification is often neglected and the influence of measurement uncertainty in geometric tolerances evaluation underestimated. The authors propose updated results of a mathematical and numerical model, based on Monte Carlo simulations, developed in order to define a measurability threshold of geometric tolerances in relation to measurement uncertainty and geometric parameters, such as feature dimensions, meant to help the designer to define measurable geometric specifications.

Starting from EN ISO 14253-2:2011 and EN ISO 14253-3:2011 standards, a perpendicularity tolerance between a cylindrical feature and a planar one has been simulated. A mathematical model has been defined for each feature, in order to assess both misalignment and its uncertainty when starting from the estimate of geometric entities obtained from point coordinates measured by a Coordinate Measurement Machine (CMM).

Monte Carlo Analysis of these simulation underlined how geometric parameters, such as dimensions of the features involved, can act as magnifiers for measurement uncertainty when verifying a geometric specification: there could be cases where this magnification effect could lead to non-measurability of misalignment and non-verifiability of the geometrical specification requested.

Keywords: measurability threshold, geometric tolerances, measurement uncertainty, dimensional parameters

1. INTRODUCTION

Coordinate measuring machines (CMM) play a key role in quality control and acceptance, especially when used to check geometrical product specification (GPS) [1]. While improvement efforts usually focus on CMM single-point accuracy, GPS verification uncertainty is also related to geometrical properties of the feature being measured, which could lead to a measurability threshold higher than the specification requested.

To demonstrate this, the case of orthogonality between a cylinder and a plane has been investigated. This case has been chosen due to its mechanical significance, especially in precision engineering, where cylindrical feature could be used as reference [2-3].

2. METHODS AND MODELS

2.1 CMM simulation

To simulate uncertainty related to CMM measurement a set of points was chosen on each feature surface, with the wider possible spatial distribution, in order to minimize resulting orientation uncertainty, but with a minimum number, 8 for feature, to simulate the common case in precision industry, where small dimensions make using more points unfeasible.

For each point coordinates an equal and uncorrelated uncertainty has been supposed, with a uniform distribution. Though this is not the most realistic assumption [4], in the absence of detailed information on the uncertainty characteristics of the CMM used, this has been considerate a suitable solution.

2.2 Plane model

The model takes root in the estimation of plane center, dimension, and normal versor using a least square method (LSM) applied to points coordinates simulated, minimizing the square sum of Euclidean distances between points measured and the feature surface.

The reference plane π is defined using a point C_0 belonging to the plane itself and its normal versor n , so that the definition of π could be function of six values:

$$\pi = f_{plane}\left(C_0, \vec{n}_{plane}\right) \quad (1)$$

LSM approach evaluates those parameters, as displayed by Figure 2, and minimizing the quantity

$$\sum_i \left[\overrightarrow{P_i C_0} \cdot \vec{n}_{plane}\right]^2 = g_{plane}\left(C_0, \vec{n}_{plane}\right) = \min \quad (2)$$

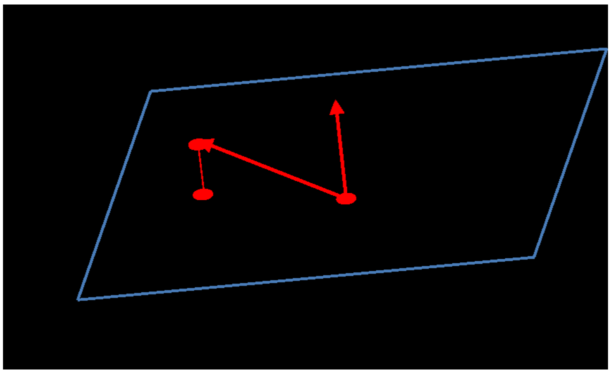

Figure 2. Plane definition

2.3 Cylinder model

With the same approach, the cylinder model is defined using its base center C0, its axis versor v and its radius R, leading to a function of 7 parameters:

$$\gamma = f_{cil}\left(C_0, \overrightarrow{v_{cil}}, R\right) \qquad (3)$$

Such values are obtained by minimizing the quantity

$$\sum_i \left[\overline{P_i H_i} - R\right]^2 = g_{cil}\left(C_0, \overrightarrow{v_{cil}}, R\right) = \min \qquad (4)$$

where Pi are points measured on the cylinder, H\i are the projection on cylinder axis of the measured points, and the minimization procedure is a non-linear least square problem numerically resolved.

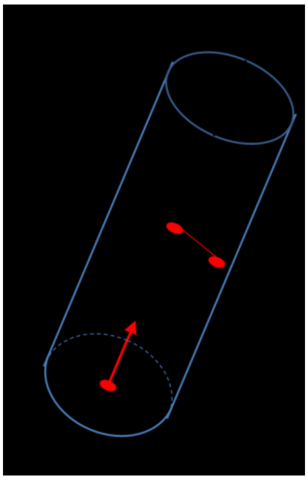

Figure 3. Cylinder definition

2.4 Misalignment definition

Following GPS common procedure, misalignment is not expressed as a solid angle, but as a maximum acceptable distance between actual surface and designed one. Following this principle, misalignment could be expressed as an angle between plane normal versor and cylinder axis versor, time the cylinder height, leading to a definition of misalignment as

$$t = H \cdot \sin(\alpha) \qquad (5)$$

where H is the cylinder height (see Figure 4).

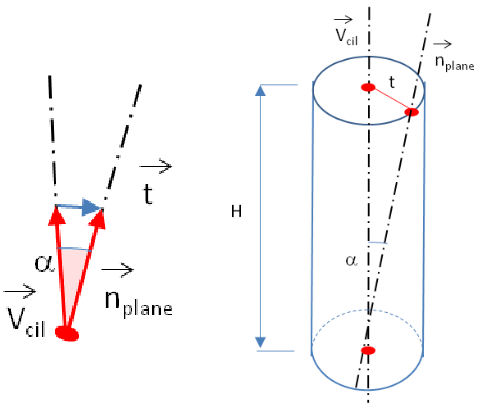

Figure 4. misalignment t and versors angle α

3. SIMULATIONS

For each geometrical configuration a set of different Montecarlo simulations (MCM), with 10000 runs each, were undertaken to assess misalignment uncertainty.

Geometrical and dimensional parameters varied for this investigation were the plane dimensions, cylinder radius, between 20 and 250 mm, and cylinder height, between 10 and 120 mm.

To keep result interpretation as simple as possible Lx and Ly dimensions were always simulated with the same value.

To evaluate influence of CMM accuracy on misalignment measurability a second set of simulations was also obtained using different values of half-width a(0.5 1 and 2 µm) for the uniform distribution used to simulate CMM coordinate uncertainty.

4. RESULTS

For each set of parameters used in the simulations, both the misalignment t and the angle α, resulting from simulations, follow a non-negative, asymmetric distribution, an example of the which could be seen in figure 5.

Since the simulated misalignment is known to be null and both the definition of angle α and misalignment t provide a nonnegative value, the 95th percentile of the resulting MCM runs is assumed as an indicator of the extended uncertainty of misalignment with a 95% confidence level [5].

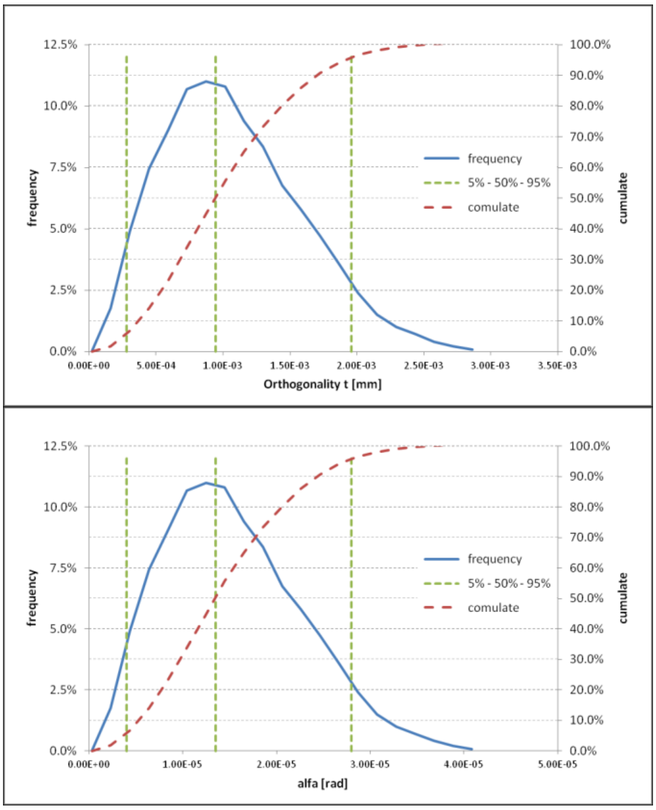

Figure 5 Orthogonality t (upper) and angle α (lower), result of 10000 MCM simulations for the case R = 8 mm, H = 70 mm, Lx = Ly = 30 mm, a=1µm.

Using a fixed accuracy estimate of 1µm, dependency of orthogonality uncertainty t on principal dimension (Lx=Ly and H) was sought, as shown in Figure 6.

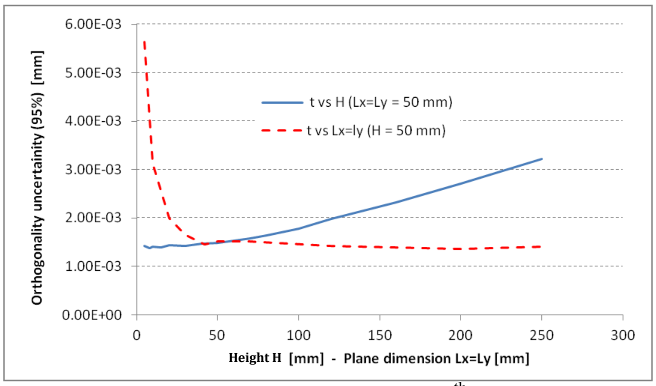

Figure 6. Orthogonality uncertainty (95th percentile based estimate) vs dimensions

Figure 6 shows how misalignment uncertainty t depends on both plane dimensions (continuous line) and on cylinder height (dotted line): with small values of Lx plane size play a bigger role in the uncertainty budget, on the other hand, when plane dimension is bigger than cylinder radius, misalignment uncertainty depends on cylinder height following a linear direct proportionality.

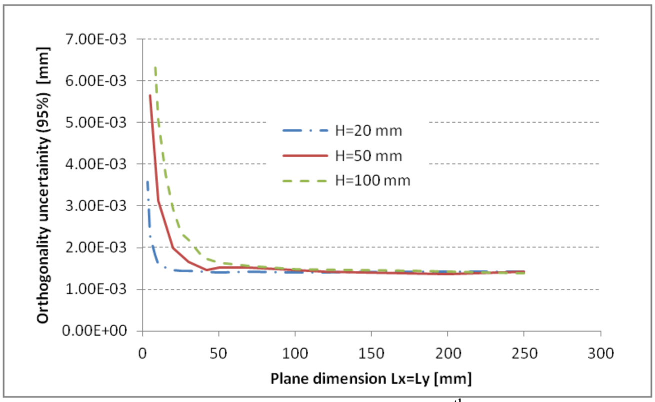

Figure 7. Orthogonality uncertainty (95th percentile based estimate) vs dimensions

A plot of the results at different cylinder heights, which can be seen in figure 7, shows how plane dimensions contribution to orthogonality uncertainty is higher where the size is smaller, with a vertical asymptote in zero, and flattens in an horizontal asymptote with increasing plane size.

This suggests that the plane size allows other contributors (such as cylinder geometry or CMM accuracy) to be relevant only over a certain value, below the which misalignment uncertainty is limited by its dependence on plane size, and over the which size is not relevant.

To further investigate cylinder dimensions effects, results of simulations, shown in Figure 8, where two parameters influence on uncertainty was explored: 95% orthogonality uncertainty is displayed in function of height H and for different, fixed, dimensions of the plane.

As can be noticed orthogonality uncertainty is heavily dependent on cylinder height: a high value, while improving accuracy in the cylinder orientation measurement, could in fact amplify uncertainty associated with plane orientation.

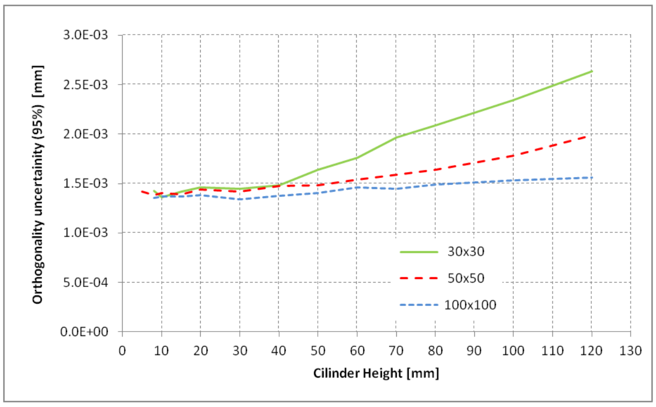

Figure 8. Orthogonality uncertainty in different geometrical configurations (R = 8 mm, Lx=Ly on chart).

In the cylinder case, a limit is found below the which orthogonality uncertainty does not depend on the height of the feature, while, for higher values, a linear relationship can be noticed.

Plane dimension have also a concurrent effect on orthogonality measurement: as can be noticed by Figure 8, only if these dimensions are small enough cylinder height has some effect on uncertainty, otherwise another contributor is predominant and leads to a lower limit of measurability.

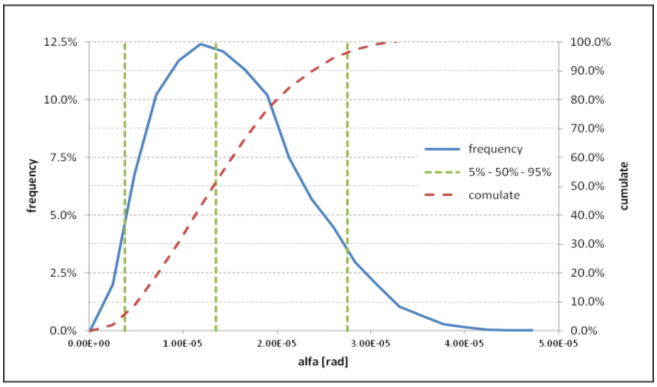

Figure 9. Angle α, result of 10000 MCM simulations for the case R = 20 mm, H = 70 mm, Lx = Ly = 30 mm

Radius of the cylinder displayed no significant influence on uncertainty on the explored range, as can be notice on Figure 9, where α obtained using a radius R = 20 mm has a distribution compatible, in terms of 5th,50th and 95th percentile, with the one depicted in Figure 5, obtained using R = 8 mm therefore its influence has been no further explored.

Extending the radius range was also considered of no interest due to the fact that smaller values would have made the assumption of being able to measure 8 points on its surface unrealistic, while higher values would have given a different problem, with a cylinder bigger than the plane under consideration.

5. DISCUSSION

To describe in more general terms how plane-cylinder orthogonality uncertainty depends on dimensions involved, a geometrical normalization has been proposed, in the reference case of Lx = Ly, defining the ratio between H, cylinder height, and Lx, plane dimension.

$$\lambda = H / Lx \quad (6)$$

A second set of MCM runs has been used to investigate orthogonal uncertainty dependence on CMM accuracy, using values of 0.5, 1 and 2 µm. These results were integrated in the previous and all orthogonality uncertainty results were expresses in terms relative to CMM accuracy, to stress the magnifications effect found.

Misalignment uncertainty has been estimated thanks to the 95th percentile of t population, divided by the half-width Δ of CMM coordinates distribution, obtaining:

$$\delta = t_{95\%} / \Delta \quad (7)$$

Results of this normalization can be observed in figures 10 and 11 where δ is plotted as function of λ, making evident how is the ratio between principal dimensions (H and Lx) to define orthogonality uncertainty, thus measurability, while radius has a negligible contribution.

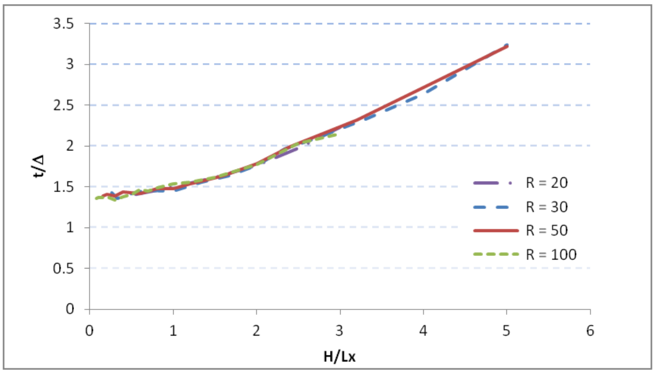

Figure 10 Normalized uncertainty vs normalized dimension (with fixed values of CMM accuracy a=1 µm)

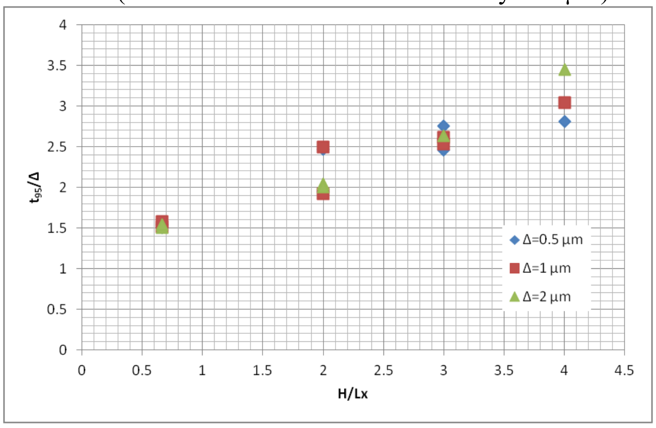

Figure 11 Normalized uncertainty vs normalized dimension (with different values of R in range 10-30 mm)

6. CONCLUSIONS

Often quantitative expression of geometric product specifications is managed by the mechanical designer, which is mainly concerned about functional aspects of the project, while quality control experts focus mainly on CMM accuracy to assess measurability.

In precision mechanics this leads to very narrow acceptance intervals to guarantee functional performances of the product, however, measurability cannot be always achieved due to the very geometrical properties of the item.

This work underlines how geometrical parameters, such as dimensions of the feature involved, can act as magnifiers for measurement uncertainty when verifying a geometric specification.

The case presented, in particular, could be used as a guide to understand how ratio between feature dimensions could pose limits to measurability of misalignment, and provide a template to develop suitable models to describe key features in the design phase.

Such an approach could avoid cases where the described uncertainty magnification effect could take uncertainty in the measurement at a value equal or higher than the measurement estimate itself, leading to non-measurability of misalignment and non-verifiability of the geometrical specification requested.

It is belief of the author that mechanical designers should be aware of such a limit, on its dependence on geometrical features, and take it into account when drafting specifications.

REFERENCES

[1] EN ISO 14253:2011 "Geometrical Product Specifications (GPS) – Inspection by measurement of workpieces and measuring equipment", 2011.

[2] M. Lancini, I. Bodini, D. Vetturi, "Enhancing micropositioning accuracy of a six axis hexapod throught uncertainty evaluation", *Annals of DAAAM 2009*, ISSN 1726-9679 pp. 529-530, Vienna, Austria, 2009.

[3] D. Vetturi, M. Lancini, I. Bodini "How geometrical tolerances affect the measurement of reciprocal alignment of two different assemblies: a case study" *Proc. ESDA 2012,* ISBN 9780791838778, Istanbul, Turkey, 2010.

[4] A. Forbes "Surface fitting taking into account uncertainty structure in coordinate data", *Meas. Sci. Technol.* vol 17 p 553-558, 2006.

[5] H. Coleman, W. Steele, "Experimentation, Validation and Uncertainty Analysis for Engineers", Wiley 2009