# DEFORMATION COMPENSATION OF A STABILIZED RESONATOR UNDER HIGH PRESSURE CHANGE BASED ON A MODIFIED POUND-DREVER-HALL METHOD

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**Abstract**:

Recently, the resolution of length measurements based on optical interferometry reached sub-nanometer order. However, the accuracy of the optical interferometry is limited by the knowledge of absolute air refractive index. In this work, we have proposed a method to measure the absolute air refractive index by measuring the free-spectral-range and the resonance frequency of a Fabry-Perot cavity, whose inside is filled by vacuum and by air. During the air refractive index measurement, the geometrical length of the Fabry-Perot cavity must be constant in vacuum and in air. In this paper, a compensation of the Fabry-Perot cavity length deformation owing to a large pressure difference between air and vacuum is proposed.

**Keywords**: Air refractive index, Fabry-Perot cavity, Free spectral range, Resonance frequency, Pound-Drever-Hall method

## 1. INTRODUCTION

According to the ITRS's report in 2012 [1], the resolution of the semiconductor lithography will reach nanometer order or less in the near future. Measurement range and tolerance are expected to be one meter order and sub-nanometer order, respectively [2]. Therefore, metrology methods with a relative precision of $10^{-9}$ (= nanometer/meter) or less must be required for the ultra-precision engineering society.

Optical interferometry is widely used in the nanotechnology and the ultra-precision engineering because of main advantages of nondestructive probing of test objects and high resolution. In the optical interferometry, an absolute distance can be precisely detected by the relation of $L = N \times \lambda_o / 2n_{air}$, where $L$, $N$, $\lambda_o$ and $n_{air}$ are a geometrical length, a fringe number, the light wavelength and the air refractive index. This equation shows that the air refractive index directly influences the measurement precision and accuracy. Therefore, the measurement of the absolute air refractive index is very important in high-precision optical metrology.

Some researchers proposed to solve the air refractive index errors, which includes the fluctuation and the absolute value of air refractive index. The fluctuation of air refractive index, which reduces the precision of length measurement, was compensated with an uncertainty of $10^{-9}$ order by a compensation method using two-color interferometry [3], or the air refractive index stabilization method based on Fabry-Perot (FP) cavity [4]. The absolute air refractive index, which limits the accuracy of the length measurement, is estimated by theoretical methods as the Edlen's and the Ciddor's calculations [5],[6] that based on the information of environmental parameters. In addition, the absolute air refractive index can be measured by an experimentally method using a Zeeman type laser interferometer and a vacuum chamber [7]. However, the accuracy of these methods was limited at $10^{-8}$ order, and cannot meet the recent requirement of the ultra-precision engineering society. Therefore, the measurement of the absolute air refractive index with an accuracy of $10^{-9}$ order or less becomes a challenge in the optical metrology.

In this paper, a measurement method of the absolute air refractive index, which bases on measuring the free-spectral-range (FSR) [8] and the resonance frequency ($f_{res}$) measurement of the FP cavity in vacuum and in air, is proposed. Theoretically, if the FP cavity length is constant in both vacuum and in air, the absolute air-refractive index can be accurately determined. However, the measurement accuracy is limited by the deformation of FP cavity that is caused by the pressure change from air to vacuum (or from vacuum to air). In order to achieve the target measurement accuracy of less than $10^{-9}$, the pressure deformation must be compensated.

## 2. PRINCIPLE OF ABSOLUTE AIR REFRACTIVE INDEX MEASUREMENT

### 2.1. Free spectral range and resonance frequency measurement based on phase modulation method
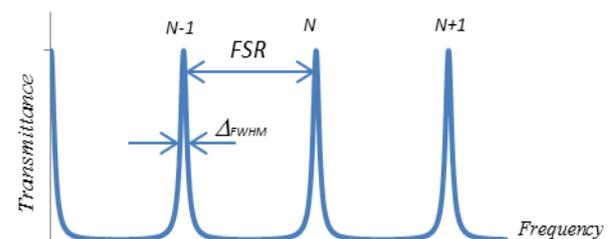


Figure 1. Transmittance of a Fabry-Perot cavity. FSR: free spectral range, $\Delta_{FWHM}$: full width at half maximum

Figure 1 shows the transmitted light intensity of a FP cavity on the frequency domain. The FP cavity allows laser light transmit through only when the light frequency is approaching a resonance frequency. The FSR frequency ($f_{FSR}$) of the FP cavity is defined as the difference between two continuous resonance frequencies ($f_{N-1}$ and $f_N$) as following equation

$$f_{FSR} = f_N - f_{N-1} \tag{1}$$

The FSR of the FP cavity is related to its geometrical length ($L$) and the environmental refractive index ($n$) as

$$f_{FSR} = \frac{c}{2nL} \qquad (2),$$

and the resonance frequency ($f_{res}$) can be calculated as

$$f_{res} = N f_{FSR} = N \frac{c}{2nL} \qquad (3),$$

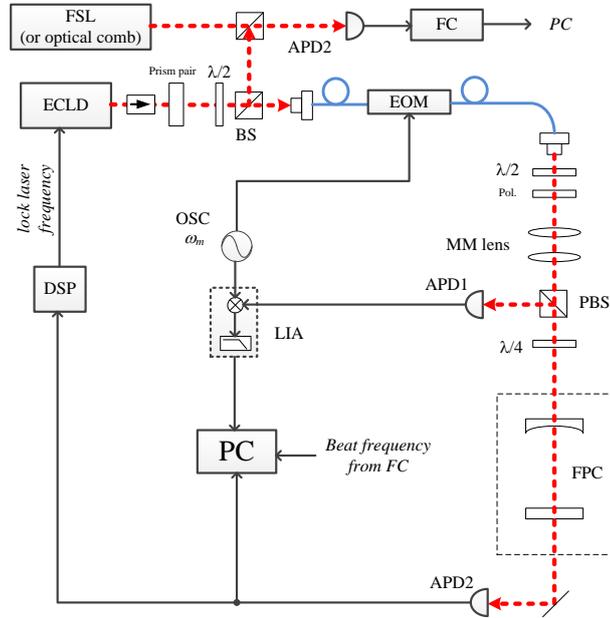where $c$ is the light speed in vacuum, $N$ is the fringe number.



Figure 2. Experimental setup for the FSR and the resonance frequency measurement. FSL: frequency stabilized laser, ECLD: external cavity laser diode, BS: beam splitter, MM Lens: mode matching lens, EOM: electro-optic modulator, APD: avalanche photo detector, OSC: oscillator, LIA: lock-in amplifier, FC: frequency counter.

Figure 2 shows the experimental setup for the measurement of the FSR and the resonance frequency of the FP cavity. In figure 2, a laser beam that is emitted from an external cavity laser diode (ELCD) is modulated by an electro-optic modulator (EOM) with a modulation angular frequency ($\omega_m$), which almost equals the FSR frequency. The system bases on the Pound-Drever-Hall (PDH) method [9]. The reflected beam from the FP cavity is detected by an avalanche photodetector (APD), after that the signal is demodulated by a lock-in amplifier (LIA) with a demodulation angular frequency of $\omega_m$. If the modulation angular frequency ($\omega_m$) equals the FSR angular frequency, the LIA error signal crosses the null point, and the FSR frequency can be determined by a null method. The uncertainty of the FSR measurement depends on the slope inclination of the LIA signal around the null point, the signal to noise ratio, and the stability of the optical path inside the FP cavity. The measurement uncertainty reached $10^{-8}$ order at the current time [8].

In the system, we also can measure the resonance frequency using the PDH frequency locking method and the beat frequency detection. In this case, the modulation frequency is much smaller than the FSR frequency. After being locked to resonance point, the laser beams from the ECLD and from a frequency stabilized laser (FSL) are sent to photodetector and the beat frequency can be detected by a frequency counter. Consequently, the resonance frequency is calculated from the FSL frequency ($f_{FSL}$) and the beat frequency ($f_{beat}$) as $f_{res} = f_{FSL} \pm f_{beat}$. The uncertainty of this frequency measurement can be $10^{-11}$ order or less by using an iodine-stabilized laser or a stabilized optical frequency comb as the frequency reference.

### 2.2. Length stabilizated resonator

The system of the absolute air refractive index measurement, which includes a length stabilization system with two deformation sensing triangular resonators, is shown in figure 3. The length stabilization system is employed to compensate the FP cavity deformation caused by the pressure change, which can result an error of $10^{-7}$ order to the air refractive index measurement.

Two triangular resonators (M1-M3-M4 and M2-M5-M6) are used for sensing the displacements of the FP cavity mirrors (at two positions P1 and P2) that are caused by the material deformation under the pressure change. An iodine-stabilized laser (FSL) with the stabilization of $10^{-11}$ order is employed and is used as the light source and the reference frequency for this system. The emitted laser beam from the FSL is modulated with an electro-optic modulator (EOM2) and is divided into two beams by a beam splitter (BS), and then is fed into two resonators. Two reflected beams from the resonators are detected by two avalanche photo-detectors (APD2 and APD3) and the signals are demodulated in two lock-in amplifiers (LIA2 and LIA3). The LIA error signals are used to lock the resonators (1 and 2) to the laser frequency by the PDH method [9]. Two piezo-electric actuators (PZT1 and PZT2) are used to control and lock the two tracking positions (P1 and P2) and stabilize the FP cavity length. In order to ensure the stabilization of the refractive index, the triangular resonator's environment is filled by thermo and pressure stabilized helium gas.

Since the triangular resonators are locked to a resonance position with the laser frequency ($f_{FSL}$), the round lengths ($L$) is equal to an integer number ($N$) of the wavelength ($c / nf_{FSL}$) as following equations

$$L_1 = N_1 \frac{c}{n_{He} f_{FSL}} \qquad (4),$$

and

$$L_2 = N_2 \frac{c}{n_{He} f_{FSL}} \qquad (5),$$

where $n_{He}$ is the environmental refractive index (helium).

Another optical system for measurement the FSR and the resonance frequency of the FP cavity is combined in figure 3. The FSR measurement theory was presented in the section 2.1. An ECLD with a frequency tuning range of larger than 1 [THz] and a high laser power is used as the light source for the FSR measurement method. The FSR frequency of the FP cavity is detected by the null point of the LIA1's error signal [8]. To measure the resonance frequency of the FP cavity, the ECLD is locked to the $N^{\text{th}}$

Figure 3. System schematic of absolute air refractive index measurement with length stabilization using two deformation sensing triangular resonators. FSL: frequency stabilized laser, ECLD: external cavity laser diode, BS: beam splitter, M: mirror, MM Lens: mode matching lens, EOM: electro-optic modulator, APD: avalanche photo detector, OSC: oscillator, LIA: lock-in amplifier, PID: proportional-integral-derivative controller, PZT: piezo electric actuator, FC: frequency counter, PC: personal computer, P1 and P2 are locked positions of the FP cavity.

order of the FP cavity by the PDH method [9] and the beat signal of the ECLD frequency ($f_{res}$) and the stabilized laser frequency ($f_{FSL}$) is detected by a frequency counter.

If the ECLD is locked to a same order of resonance frequencies when the FP cavity is being filled by vacuum and by air, the laser frequency will shift approximately 128[GHz]. A stabilized optical comb system will be employed to measure the resonance frequencies with an uncertainty order of $10^{-12}$ or less [12].

### 2.3. Absolute air refractive index measurement

Based on a precise measurement of the free-spectral-range (FSR) and the resonance frequency ($f_{res}$) of the Fabry-Perot (FP) cavity, we would like to introduce the principle of the absolute air refractive index measurement. Since the FSR and the resonance frequency are measured in vacuum and in air, Eq. (3) can be expressed as

$$f_{resA} = N_{air} f_{FSRair} = N_{air} \frac{c}{2n_{air} L_{air}} \qquad (6),$$

$$f_{resV} = N_{vac} f_{FSRvac} = N_{vac} \frac{c}{2n_{vac} L_{vac}} \qquad (7),$$

where $f_{air}$ and $f_{vac}$ are the measured FSRs, $f_{resA}$ and $f_{resV}$ are the measured resonance frequencies, $N_{air}$ and $N_{vac}$ are the fringe numbers, $L_{air}$ and $L_{vac}$ are the FP cavity's geometrical lengths, in air and in vacuum respectively. From equations (6) and (7), we can determine the air refractive index in three different ways by following equations.

*(1) Bases on the free spectral range measurement:*

$$n_{air} = \frac{f_{FSRvac}}{f_{FSRair}} \times \frac{L_{vac}}{L_{air}} \times n_{vac} \qquad (8).$$

The process includes two FSR measurements in vacuum and in air that results $f_{FSRvac}$ and $f_{FSRair}$, respectively. The FSR of the FP cavity is detected by the null point of the LIA error signal while the modulation frequency is being scanned around the FSR frequency.

*(2) Bases on the resonance frequency measurement with different fringe orders in air and in vacuum: ($N_{air} \neq N_{vac}$)*

The measurement includes four steps. First, the chamber is set in vacuum and the FSR of the FP cavity in vacuum ($f_{FSRvac}$) is measured. In the second step, the ECLD frequency is locked to the $N_{vac}^{th}$ order resonance frequency of the FP cavity by the PDH method, the resonance frequency ($f_{res}$) is detected by a beat signal ($f_{beat}$) with an iodine-stabilized laser ($f_{FSL}$). The beat signal is calculated as

$$f_{Beat} = \left| f_{FSL} - f_{res} \right| \qquad (9).$$

The fringe number is an integer and can be determined from the following equation

$$N_{vac} = INT \left( \frac{f_{resV}}{f_{FSRvac}} \right) \qquad (10),$$

where $INT(x)$ is the integer part of $x$.

In the next step, a sample air is pumped into the chamber and is stabilized, the fringe numbers change are recorded by counting the number of fringe ($N_{change}$) using a LIA output signal. Finally, after the sample air is stable, the laser

frequency will be locked to a new fringe order ($N_{air} = N_{vac} + N_{change}$) of the FP cavity then the new resonance frequency is measured by Eq. (9), the air refractive is calculated as

$$n_{air} = \frac{f_{resV}}{f_{resA}} \times \frac{N_{air}}{N_{vac}} \times \frac{L_{vac}}{L_{air}} \times n_{vac} \qquad (11).$$

In this technique, the difference between the fringe numbers in air and in vacuum ($N_{air} \neq N_{vac}$) allows us to reduce the beat frequency signal ($f_{beat}$) to be less than 1 [GHz] that can be detected by a normal avalanche photo-detector.

(3) *Bases on the resonance frequency measurement at the same fringe order in vacuum and in air:* ($N_{air} = N_{vac}$)

With the similar steps from Eq. (11) but the fringe number ($N$) is maintained by continuously locking the ECLD to the FP cavity during the air pump process (or $N_{air} = N_{vac}$), the air refractive index can be calculated as

$$n_{air} = \frac{f_{resV}}{f_{resA}} \times \frac{L_{vac}}{L_{air}} \times n_{vac} \qquad (12).$$

Because the fringe number ($N$) is maintained, the resonance frequency shift is large ($f_{resV} - f_{resA} \approx 128$ [GHz]) and is out of the APD's bandwidth, a stabilized optical comb system is employed for the resonance frequency measurement. The resonance frequency can be measured by following equation

$$f_{res} = f_{ceo} + mf_r \pm f_{beat} \qquad (13),$$

where $f_{ceo}$ is the carrier–envelope-offset frequency of the optical comb, $f_r$ is the repetition frequency, $m$ is the number of comb modes and $f_{beat}$ is the beat signal between the $m^{th}$ comb mode ($f_{ceo} + mf_r$) and the measurement frequency ($f_{res}$). The sign of the beat frequency in Eq. (13) stands for the measurement frequency is higher or lower than the $m^{th}$ comb frequency.

Figure 4 shows the schematic of a frequency measurement using an optical comb [11]. In figure 4, the laser beam from a FSL passes through an optical comb generator, which is driven by a repetition frequency ($f_r$). The measurement frequency ($f_{res}$) passes through an acousto-optic modulator (AOM) and the 1st order frequency of AOM is shifted ($f_{res} + f_{AOM}$). The beat signal of the optical comb with the 0th and the 1st orders of the AOM are detected by two APDs and measured by two frequency counters after passing through two band-pass filters. By comparing the two beat frequencies, the sign of $f_{beat}$ in Eq. (13) could be observed. The mode number ($m$) could be determined by slightly shifting the repetition frequency and measuring the beat frequency change. Because the repetition frequency of the optical comb is in several hundreds of MHz, the beat frequency is minimized and can be detected. By applying the optical comb method, the resonance frequency can be measure with an uncertainty of $10^{-12}$ order or less [12].
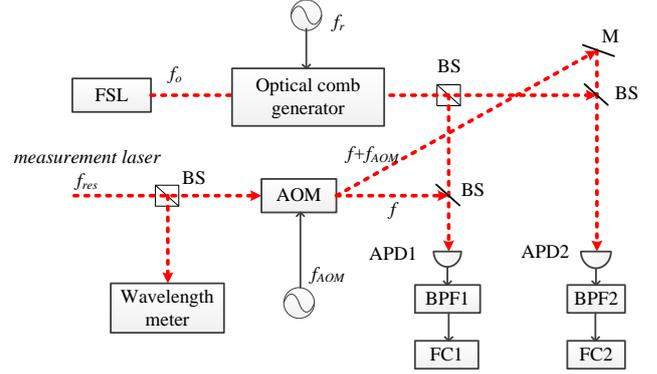


Figure 4. Resonance frequency measurement using optical comb. FSL: frequency stabilized laser, BS: beam splitter, M: mirror, AOM: acousto-optic modulator, APD: avalanche photo detector, BPF: band-pass filter, FC: frequency counter.

In above equations (8), (11) and (12), the FP cavity's length in air is approximate in vacuum ($L_{air} \approx L_{vac}$) by using the length stabilization system that was mentioned in section 2.2.

## 4. UNCERTAINTY ESTIMATION

In this section, the uncertainty of the absolute air refractive index are estimated and compared from the three equations (8), (11) and (12). From these comparisons, the best method for the absolute air refractive index measurement can be selected.

### 4.1. Uncertainty of the air refractive index measurement based on the FSR measurement

The air refractive index combined uncertainty is expressed from Eq. (8) and is described as the following equation

$$\left(U_{n_{air}}\right)^2 = \left(\frac{u(f_{FSRvac})}{f_{FSRvac}}\right)^2 + \left(\frac{u(f_{FSRair})}{f_{FSRair}}\right)^2 +$$
$$+ \left(\frac{u(L_{vac})}{L_{vac}}\right)^2 + \left(\frac{u(L_{air})}{L_{air}}\right)^2 + \left(\frac{u(n_{vac})}{n_{vac}}\right)^2 \qquad (14),$$

Where $U_{n_{air}} = \frac{u(n_{air})}{n_{air}}$.

As mentioned in section 2.1, the uncertainty of the FSR measurement is approximately

$$\frac{u(f_{FSRvac})}{f_{FSRvac}} \approx \frac{u(f_{FSRair})}{f_{FSRair}} \approx 4.1 \times 10^{-8} \qquad (15).$$

If the FPC length stabilization system reached the target uncertainty of $10^{-10}$ order, the uncertainty of the FP cavity length can be

$$\frac{u(L_{vac})}{L_{vac}} \approx \frac{u(L_{air})}{L_{air}} \approx \frac{u(L)}{L} = 10^{-10} \qquad (16).$$

The vacuum uncertainty can be neglected by an order of $10^{-12}$ since the vacuum pressure reaches 0.01 [Pa]. Additionally, the uncertainty of air refractive index is calculated by Eq. (14) as

$$U_{n_{air}} = \sqrt{2 \times \left(4.1 \times 10^{-8}\right)^2 + 2 \times \left(10^{-10}\right)^2 + \left(10^{-12}\right)^2}$$
$$\approx 5.8 \times 10^{-8} \tag{17}.$$

The uncertainty of this air refractive index measurement mostly depends on the FSR measurement and is limited at a few parts in $10^8$. In order to reach the target accuracy of the air refractive index measurement, the FSR uncertainty must be improved to $10^{-10}$ order.

### 4.2. Uncertainty of the air refractive index measurement based on the resonance frequencies detection at different resonance fringes in vacuum and in air

From Eq. (11), the air refractive index's combined uncertainty is expressed by following equation

$$\left(U_{n_{air}}\right)^2 = \left(\frac{u(n_{air})}{n_{air}}\right)^2 = \left(\frac{u(f_{resV})}{f_{resV}}\right)^2 + \left(\frac{u(f_{resA})}{f_{resA}}\right)^2 + \left(\frac{u(N_{vac})}{N_{vac}}\right)^2 +$$
$$+ \left(\frac{u(N_{air})}{N_{air}}\right)^2 + \left(\frac{u(L_{vac})}{L_{vac}}\right)^2 + \left(\frac{u(L_{air})}{L_{air}}\right)^2 + \left(\frac{u(n_{vac})}{n_{vac}}\right)^2 \tag{18}.$$

In the above equation, the uncertainty of the length stabilization system is the same as Eq. (17). The uncertainty of the resonance frequency detection depends on the uncertainty of the iodine stabilized frequency. The beat frequency uncertainty can reach $10^{-11}$ order by using a precise frequency counter. From Eq. (9),

$$[u(f_{resA})]^2 \approx [u(f_{resV})]^2 = [u(f_{res})]^2 = [u(f_{FSL})]^2 + [u(f_{beat})]^2$$
$$= (2.5 \times 10^{-11} \times f_{FSL})^2 + (10^{-11} \times f_{beat})^2 \approx 12[kHz] \tag{19}$$

The fringe number is calculated by Eq. (10) and because $N$ is an integer, the error of the fringe number is concluded to be 0.5 while $N \approx 1579801$ with the FSR of 300 [MHz] (or $L \approx 0.5$ [m]). Finally, the uncertainty of the air refractive index measurement is concluded as

$$U_{n_{air}} = \sqrt{2 \times \left(\frac{12[kHz]}{473.9[THz]}\right)^2 + 2 \times \left(10^{-10}\right)^2 + 2 \times \left(\frac{0.5}{1579801}\right)^2 + \left(10^{-12}\right)^2}$$
$$\approx 4.47 \times 10^{-7} \tag{20}.$$

The uncertainty is limited by $10^{-7}$ order, which mostly due to the error of the integral fringe number detection. In order to avoid this error, fringe number in Eq. (11) must be eliminate by locking the laser at a same resonance point.

### 4.3. Uncertainty of the air refractive index measurement based on measuring the resonance frequencies at a same fringe order

From Eq. (12), we can calculate the combined uncertainty of the air refractive index measurement as

$$\left(U_{n_{air}}\right)^2 = \left(\frac{u(n_{air})}{n_{air}}\right)^2 = \left(\frac{u(f_{resV})}{f_{resV}}\right)^2 + \left(\frac{u(f_{resA})}{f_{resA}}\right)^2 +$$
$$+ \left(\frac{u(L_{vac})}{L_{vac}}\right)^2 + \left(\frac{u(L_{air})}{L_{air}}\right)^2 + \left(\frac{u(n_{vac})}{n_{vac}}\right)^2 \tag{21}.$$

In above equation, the resonance frequency is measured by a stabilized optical comb system which can provide an uncertainty of $1.6 \times 10^{-12}$ or less for the frequency measurement [12]. Therefore, the uncertainty can be estimated by Eq. (21) as

$$U_{n_{air}} = \sqrt{2 \times \left(1.6 \times 10^{-12}\right)^2 + 2 \times \left(10^{-10}\right)^2 + \left(10^{-12}\right)^2}$$
$$\approx 1.4 \times 10^{-10} \tag{22}.$$

The air refractive index detection with the resonance frequency measurement by an optical comb shows the best uncertainty of approximate $1.4 \times 10^{-10}$, which satisfied the measurement requirement. The FSR measurement method can reach $10^{-8}$ uncertainty at current time, and is possible to be improved to $10^{-10}$ order in a near future. In the proposed system, a combination method of the optical comb frequency detection and the FSR measurement will be used to measure the absolute air refractive index, which allows us to evaluate and compensate the measurement error.

## 5. CONCLUSIONS

In this paper, the air refractive index measurement method has been proposed. We presented a compensation method for the deformation of the FP cavity length under pressure change. Using the tracking of the resonance frequency of the FP cavity in vacuum and in air with the optical comb, an uncertainty of air refractive index measurement of $10^{-10}$ order can be expected.

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