REALIZATION OF LENGTH TRACEABILITY BY A FEMTOSECOND OPTICAL FREQUENCY COMB

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Abstract:

The national standard of length in Japan changed from the iodine-stabilized He–Ne laser to a femtosecond optical frequency comb (FOFC) in 2009. This means that there are different frequencies and their combination, adjacent pulse repetition interval length (APRIL), which can be used as a scale since an FOFC is a phase-coherent combination of several hundreds of thousands of wavelengths. Thus, the following questions arise naturally: Which parameters can be used as a scale? What are their characteristics? The goal of this paper is to answer these questions. We show the similarity and the difference between wavelength and APRIL for length measurement, and we discuss new concepts of APRIL-based length measurement. The status of the experiment is reported.

Keywords: **Length ruler, Ambiguity, Laser optics, Length measurement, Metrology**

1. INTRODUCTION

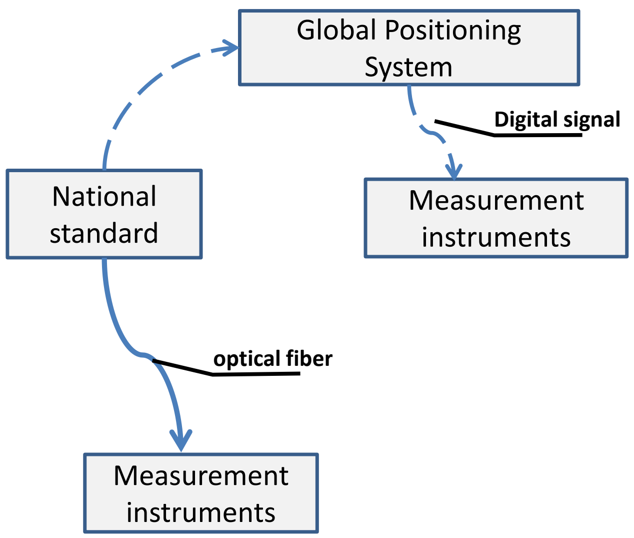

Fig. 1: Octopus-type traceability system.

Today, the unit of length, meter, is defined by the speed of light in vacuum. Its practical definition is based on the wavelength of frequency-stabilized lasers. The wavelength, λ_{vac}, and the frequency, f, have the relationship $\lambda_{\mathrm{vac}} = c_{\mathrm{vac}} / f$ (c_{vac} is the speed of light in vacuum, which has no assigned uncertainty). The traceability of length is required for applications requiring a high level of precision. For industrial applications, real objects, such as gauge blocks and line scales, are also widely used as reference standards.

The national standard of length in Japan changed from the iodine-stabilized He–Ne laser to a femtosecond optical frequency comb (FOFC) in 2009. This change has enabled us to transform the standard via optical fibers [1] and global positioning system (GPS) networks. As shown in Fig. 1, a new type of length traceability system is possible, in which electromagnetic signals and optical signals, rather than real objects, are used as reference standards and can be delivered to the measurement site via optical and GPS networks. Groups in Germany and Japan reported high-precision frequency transmission by fiber rings [2, 3]. The uncertainty of the frequency/meter can be expected to be of the order of 10^{-16}. Frequency transmission is also achieved via GPS networks. In such cases, the uncertainty of the frequency/meter can be expected to be of the order of 10^{-11}. With optical and GPS networks, the traceability tests and the measurement of length can by performed simultaneously. Our ultimate goal is to realize such a system.

To realize a cost-effective length traceability system, we are now developing a GPS-locked pulse repetition frequency-stabilized FOFC laser system. Since the offset frequency changes over time, the wavelength is unstable. The ruler for length measurement is the adjacent pulse repetition interval length (APRIL), which is inversely proportional to the stabilized pulse repetition frequency. In this paper, on the basis of previous works [4, 5], we provide a general overview of wavelength/APRIL-based length measurement and show the similarities and differences between them. We also discuss new concepts of APRIL-based length measurement.

2. PRINCIPLES

For convenience, we summarize the features of an FOFC, the details of which can be found elsewhere [6]. To express an FOFC, we need to know four parameters, which are shown in Fig. 2: the pulse repetition frequency, f_{rep}; the offset frequency, f_{CEO}; the pulse repetition period, $T_R = 1/f_{\mathrm{rep}}$; and the carrier phase slip which is the change in phase between the carrier and the carrier-envelope, $\Delta\varphi_{\mathrm{ce}} = 2\pi f_{\mathrm{CEO}} / f_{\mathrm{rep}}$.

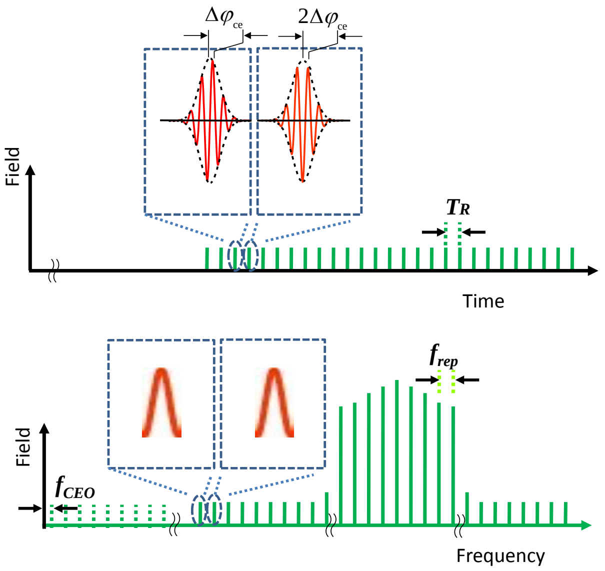

Fig. 2: FOFC. (a) Time domain. (b) Frequency domain.

One of the frequencies of an FOFC, f_P, may be expressed as $f_P = f_{CEO} + P \times f_{rep}$, where P is the number of comb lines that is of the order of 10^6. As a practical definition of the meter, the wavelengths of an FOFC can be written as $\lambda_{vac\_P} = c_{vac}/f_P$. An APRIL in vacuum is expressed as $\delta_{vac} = c_{vac}/f_{rep}$. If the frequency is stabilized, both the wavelength and APRIL can be used as units. When only the pulse repetition frequency is stabilized, only APRIL can be used. Below, we consider measuring the length using the APRIL when offset frequency is in a free-run mode.

Table 1: Analogies between wavelength and APRIL as scales in vacuum.

Ruler	Wavelength	APRIL
Relationship with frequency	$\lambda_{vac\_P} = c_{vac}/f_P$	$\delta_{vac} = c_{vac}/f_{rep}$
Uncertainty of length unit [5]	$\frac{u(\lambda_{vac})}{\lambda_{vac}} = \frac{u(f_P)}{f_P}$	$\frac{u(\delta_{vac})}{\delta_{vac}} = \frac{u(f_{rep})}{f_{rep}}$
Uncertainty of frequency [5]	$u(f_P)/f_P = \frac{[u(f_{CEO}) + P \times u(f_{rep})]}{(f_{CEO} + P \times f_{rep})}$ $u(f_{rep})/f_{rep} \approx u(f_P)/f_P$ $u(f_{CEO})/f_{CEO} \approx [u(f_P)/f_P] \times (P \times f_{rep}/f_{CEO})$	$u(f_{rep})/f_{rep} \approx u(f_P)/f_P$

We now start to organize the dualities between wavelength and APRIL. Table 1 lists the analogies between wavelength and APRIL as scales in vacuum. The subscripts "vac" and "air" denote vacuum and air, respectively. The uncertainty of x is denoted by $u(x)$. We find $u(\lambda_{vac})/\lambda_{vac} \approx u(\delta_{vac})/\delta_{vac}$, which implies that APRIL and wavelength can be stable to the same level. Therefore, APRIL can also be used as a scale in vacuum [5].

Length measurements in the industry are typically performed in air. Table 2 summarizes the analogies between both scales in air. The subscripts "*P*" and "*g*" denote phase and group, respectively. The subscript "cen_vac" denotes central wavelength in vacuum. Because $u(\lambda_{air})/\lambda_{air} \approx u(\delta_{air})/\delta_{air}$, APRIL and wavelength are stable to the same level in air. In our previous report [1], we showed that sensitivity coefficients for temperature, atmospheric pressure, and humidity of both refractive indexes are at same level. Based on these two points, we conclude that APRIL can also be used as a scale in air.

Next, we need to consider the length measurement in air. Table 3 gives a general overview of length measurements by wavelength and APRIL. M and N are the integral and fractional parts, respectively. The subscripts "λ" and "δ" denote wavelength and APRIL, respectively. In this regard:

1. Using the phase information, the resolution of measurement can be increased [12, 13]. In this experiment, there are no plans to lock the offset frequency. Therefore, it is not possible to use the phase information of the interference fringes. We cannot expect length measurement with high resolution by APRIL in this experiment.
2. To calculate the value of a ruler, we need the typical values of pulse repetition frequency. The pulse repetition frequency of a fiber laser is basically limited to a range of [40, 80] MHz because fiber forms all components of the laser cavity. It is

possible to adjust the laser cavity with a mirror in air to increase the pulse repetition frequency to several hundred megahertz. Use of the Fabry–Pérot etalon to select high-frequency parts of the repetition-frequency mode has also been reported [14].

3. Because the ambiguity of APRIL is 6 orders greater than that of wavelength, it will be useful for absolute long-distance measurement [5].

Table 2 Analogies between wavelength and APRIL as scale in air.

Ruler	Wavelength	APRIL
Conversion	$\lambda_{air} = \lambda_{vac} / n_p(\lambda_{vac})$	$\delta_{air} = \delta_{vac} / n_g(\lambda_{cen\_vac})$
Uncertainty of length unit [5]	$\frac{u(\lambda_{air})}{\lambda_{air}} \approx \frac{u(n_p(\lambda_{vac}))}{n_p(\lambda_{vac})}$	$\frac{u(\delta_{air})}{\delta_{air}} \approx \frac{u(n_g(\lambda_{cen\_vac}))}{n_g(\lambda_{cen\_vac})}$
Refractive index	$n_p(\lambda_{vac}) = f(\lambda_{vac}, T, P, H, t)$ [7-10]	$n_g(\lambda_{cen\_vac}) = f(\lambda_{cen\_vac}, T, P, H, t)$ [11]
Uncertainty of the parameters	$\frac{u(\lambda_{vac})}{\lambda_{vac}} = \frac{u(f_P)}{f_P} = 10^{-11}$ (Assumption) $\frac{u(n_p(\lambda_{vac}))}{n_p(\lambda_{vac})} \approx 30\times10^{-9}$ [7-10]	$\frac{u(\delta_{vac})}{\delta_{vac}} = \frac{u(f_{rep})}{f_{rep}} = 10^{-11}$ (Assumption) $\frac{u(n_g(\lambda_{cen\_vac}))}{n_g(\lambda_{cen\_vac})} \approx 30\times10^{-9}$ [1]

Table 3 Analogies between wavelength and APRIL based length measurements.

Ruler	Wavelength	APRIL
Representation	$L_{air}(t) = (M_\lambda + N_\lambda)\times\lambda_{air}(t) = (M_\lambda + N_\lambda)\times\lambda_{vac}/n_p(\lambda_{vac})$	$L_{air}(t) = (M_\delta + N_\delta)\times\delta_{air}(t) = (M_\delta + N_\delta)\times\delta_{vac}/n_g(\lambda_{cen\_vac})$
Detection parameters	Phase, brightness, and darkness of fringe	Peak (or center of gravity) of envelope
Resolution	0.1 nm [15] $< \lambda_{air}/1000$ [16]	0.3 nm [17]
Ruler value	$[400, 1700]$ nm	$[0.3, 7.5]$ m
Ambiguity	$\lambda_{air}/4$	$\delta_{air}(t)/4$

3. PREPARATION OF EXPERIMENTS

To verify the properties of APRIL, a GPS-calibrated FOFC system was built, as shown in Fig. 3. To obtain a cost-effective system, only the repetition frequency was stabilized by the GPS frequency standard. The laser source system consisted of a fiber laser (Neoaku, OCLS), a GPS-controlled frequency standard (Pendulum, GPS-12R), a frequency synthesizer, a set of repetition rate synchronization electronics (Neoaku, OCLS), and a frequency counter (Pendulum, CNT-90). Depending on the position of the satellites, we obtain at least 6 GPS signals. Figure 4 shows the Allan variance of the frequency signal generated from the frequency standard (output port no. 1). The highest stability of the APRIL that can be achieved in our system is of the order of 10^{-11}.

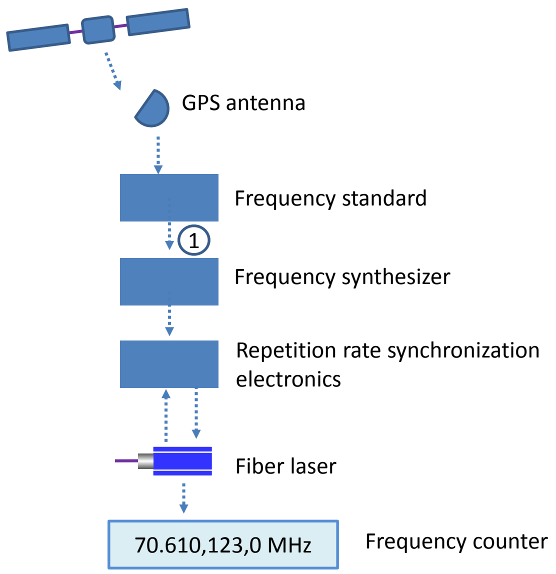

Fig. 3: Schematic of GPS-calibrated FOFC laser source. Dotted arrows indicate the flow of electrical signals.

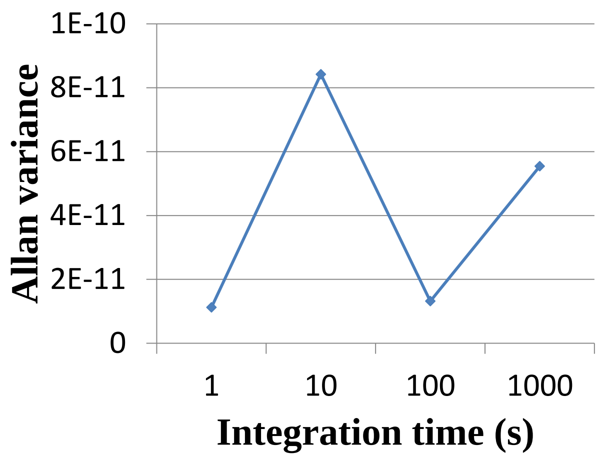

Fig. 4: Allan variance of the frequency signal generated from the frequency standard (output port no. 1).

4. DISCUSSION

Frequency stability obtained by the GPS is of the order of 10^{-11}. Frequency stability of an Iodine-stabilized He–Ne laser is also of the order of 10^{-11}. The advantage of using FOFC for length measurement is that calibration and measurement can be simultaneously performed. The calibration of laser frequency and measurement of length cannot be done simultaneously when an He–Ne laser is used.

The FOFC locked to GPS via the network enables length measurements and frequency calibration to be performed simultaneously. The precision of all length measurement values is validated by the frequency value recorded simultaneously.

5. CONCLUSIONS

We proposed a length traceability system that operates via optical and GPS networks. In this system, wavelength and APRIL can be used as scales for length measurement. We have shown the similarity and differences between wavelength and APRIL for length measurement. We understand that wavelength can provide higher resolution and accuracy, and APRIL can been used without ambiguity in a longer range. We also reported the status of the experiment. We anticipate that the results will contribute to the construction of a new length traceability system.

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