ERROR PROPAGATION FOR STRAIGHTNESS EVALUATION USING 3-POINT METHOD CONSIDERING LONGITUDINAL POSITIONING ERROR

Tatsuya KUME[1], Kazuhiro ENAMI[2], Yasuo HIGASHI[3], Kenji UENO[2], and Masashi YAMANAKA[2]

1 High Energy Accelerator Research Organization (KEK), 1-1, Oho, Tsukuba, Ibaraki, 305-0801, JAPAN, tatsuya.kume@kek.jp
2 High Energy Accelerator Research Organization (KEK),
3 Okinawa Institute of Science and Technology Graduate University (OIST)

Abstract:

Straightness evaluation using 3-point method has an advantage for evaluating large objects being not affected by straightness references; however, error introduced by each measurement and propagating to the derived straightness should also be taken into account for evaluating large objects.

In this paper, errors propagated to the straightness derived by 3-point method were estimated analytically based on error propagating models and the estimated errors were evaluated by experiments. The results show that each longitudinal positioning error for the straightness detector should be considered for evaluating large object as well as error in each measurement.

The error estimation which considers the positioning error shows existence of optimum sampling interval which minimizes error in the derived straightness. Then the estimated optimum sampling interval is compared with experiment.

Finally, error for the straightness evaluation with the evaluation distance of 1 km was estimated and its optimum sampling interval was also derived.

Keywords: Straightness Evaluation, 3-Point Method, Error Propagation, Longitudinal Positioning Error

1. INTRODUCTION

Straightness evaluation using 3-point method[1-2], which detects the second-order difference of the straightness, has an advantage for evaluating large objects, because it is not affected by transferring locus or azimuth of the detector, which are usually used as an evaluation reference and difficult to be defined and maintained enough accurately as the object becomes large. On the other hand, error introduced by each measurement should also be taken into account for large objects, because the straightness deriving process of 3-point method tends to pile up the error into the derived straightness.

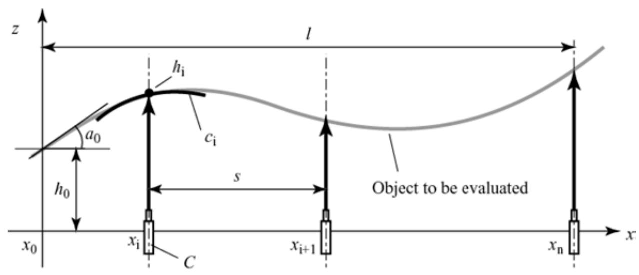
Generally, error in measurements of the straightness detector is considered to be dominant for evaluating quasi-straight objects; however, errors estimated by using a previous model, which only considers the error, differed from experiments as the evaluation distance became longer. The previous model also showed that error propagated to the straightness can be reduced simply by using small sampling interval; however, experiments showed that there exists minimum limitation for the error in the derived straightness.

In this paper, errors for the straightness derived by 3-point method were estimated by using newly proposed model, which considers longitudinal positioning error of the detector in addition. The newly estimated errors were evaluated by comparing with experiments. Then, by using the new model, error for longer evaluation distance and effects of the measurement parameters were estimated.

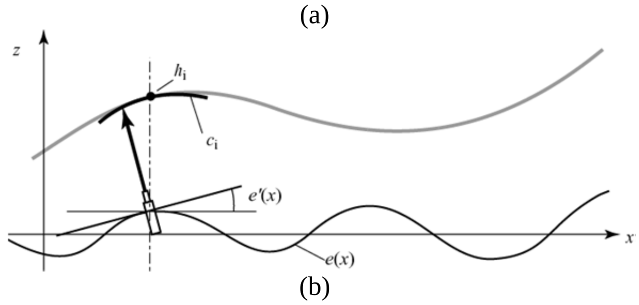
2. ERROR ESTIMATION

2.1 Straightness Evaluation Using 3-point Method

Figure 1 (a) shows schematics of straightness evaluation using 3-point method. The second-order difference c_i (i = 1 to n) of the straightness h_i for each measurement point x_i is detected by a detector C. Here, c_i corresponds to the curvature of the object at each measurement point. The detector typically consists of three displacementmeters or two anglemeters aligned toward the evaluation direction. Straightness evaluation is performed by transferring the detector toward which the object to be evaluated extends. We define x and z axes as a longitudinal and a transversal direction for the evaluation, respectively. The transferring locus $e(x)$ or its azimuth $e'(x)$ of the detector is generally used as a reference for conventional straightness evaluations, which detect the straightness or the first-order difference of the straightness. While straightness evaluation using 3-point method is not affected by $e(x)$ nor $e'(x)$, because c_i is not affected by them (Fig. 1(b)).



(a)



(b)

Fig. 1(a): Straightness evaluation using 3-point method, where h_0 and a_0 stands for arbitrarily defined straightness and its first-order difference at the first point x_0. (b): Neither transferring locus $e(x)$ nor its azimuth $e'(x)$ for the detector affects the detected c_i and consequently the derived straightness h_i.

The straightness h_i to be evaluated is derived as

$$h_i = h_{i-1} + s \cdot a_i \,, \tag{1}$$

where s stands for the distance between each measurement point, that is the sampling interval and a_i stands for the first-order difference of the straightness h_i, while a_i is derived as

$$a_i = a_{i-1} + s \cdot c_i \,. \tag{2}$$

It follows that the straightness h_n of the n-th measurement point can be derived by using the detected second-order differences c_i as

$$h_n = \left\{ \begin{matrix} n \cdot c_1 + (n-1) \cdot c_2 + \cdots + \\ (n-i+1) \cdot c_i + \cdots + 2 \cdot c_{n-1} + c_n \end{matrix} \right\} \cdot s^2 \,, \tag{3}$$

considering that both straightness h_0 and its first-order difference a_0 at the start point x_0 are 0.

Equation (3) is transformed into

$$h_n = k \cdot s^2 \tag{4}$$

by using k defined as

$$\begin{aligned} k = n \cdot c_1 + (n-1) \cdot c_2 + \cdots \\ + (n-i+1) \cdot c_i + \cdots + 2 \cdot c_{n-1} + c_n \end{aligned} \,. \tag{5}$$

2.2 Error Estimation

We evaluate error for the derived straightness by using the error σ_h propagated to the straightness h_n at the last measurement point. It is expressed as

$$\sigma_h = \sqrt{\left(\frac{\partial h_n}{\partial k} \cdot \sigma_k \right)^2 + \left(\frac{\partial h_n}{\partial s} \cdot \sigma_s \right)^2} \,, \tag{6}$$

where σ_k and σ_s stand for errors of k and s expressed by their standard deviations, respectively. Equation (6) is calculated as

$$\sigma_h = \sqrt{\left(s^2 \cdot \sigma_k \right)^2 + \left(2s \cdot k \cdot \sigma_s \right)^2} \,, \tag{7}$$

by using the relation expressed by Eq. (4).

If we consider that each c_i is independent with each other and that error in each c_i is random, the error σ_k propagated to k is expressed as

$$\sigma_k = \sqrt{\frac{n \cdot (n+1) \cdot (2n+1)}{6}} \cdot \sigma_c \,, \tag{8}$$

where σ_c stands for the error in each c_i expressed by its standard deviation.

If the detector for the 3-point method is composed of 3 displacementmeters aligned with an interval of d, the second-order difference c of the straightness measured by the detector is expressed as

$$c = \frac{m_1 - 2 \cdot m_2 + m_3}{d^2} \,, \tag{9}$$

where m_j (j = 1, 2, 3) stand for measurements of the 3 displacementmeters. Then, the error σ_c propagated to c is derived as

$$\sigma_c = \frac{\sqrt{6}}{d^2} \cdot \sigma_d \,, \tag{10}$$

where σ_d stands for the error in each measurement m_j, assuming that d has no error.

In order to simplify the following calculation, we assume that each second-order difference c_i is expressed by their representative value c_r as

$$c_i \approx c_r \,. \tag{11}$$

Then, Eq. (5) is transformed into

$$k \approx \left\{ \frac{n \cdot (n+1)}{2} \right\} \cdot c_r \,. \tag{12}$$

By using the relations expressed by Eqs. (7), (8), and (12), error σ_h propagated to the straightness derived by using 3-point method can be expressed as

$$\sigma_h \approx \sqrt{\begin{matrix} \left(\frac{s \cdot l^3}{3} + \frac{s^2 \cdot l^2}{2} + \frac{s^3 \cdot l}{6} \right) \cdot {\sigma_c}^2 \\ + \left(\frac{l^4}{s^2} + \frac{2l^3}{s} + l^2 \right) \cdot {c_r}^2 \cdot {\sigma_s}^2 \end{matrix}} \,, \tag{13}$$

where, l stands for the evaluation distance, which has a relation with n and s as

$$l = n \cdot s \,. \tag{14}$$

Generally, error σ_c in measurements is considered to be dominant comparing to the positioning error σ_s for evaluating quasi-straight objects. If we neglect σ_s in Eq. (13), it can be simplified as

$$\sigma'_h = \sqrt{\left(\frac{s \cdot l^3}{3} + \frac{s^2 \cdot l^2}{2} + \frac{s^3 \cdot l}{6} \right)} \cdot \sigma_c \,. \tag{15}$$

It shows the error previously estimated for the straightness derived by using 3-point method considering only σ_c.

3. EVALUATION OF THE ERROR ESTIMATION

The estimated errors were evaluated by comparing with experiments. The experimental values were derived from 10-times of repeat measurements for a turned cupper column (length, 1400 mm; diameter, 60 mm) by a straightness measurement system shown in Fig. 2[2]. The system consists of two sets of 3-point-method detectors facing each other, which detect second-order differences of straightness for both sides of the object at a time. Each of the detectors consists of 3 displacementmeters (range, 30 mm; resolution, 0.1 μm; model LS-7030, Keyence Co. Ltd.). The 3 displacementmeters are mounted with an interval d of 140 mm. The detectors are transferred by a ball screw-driven vertical stage (range, 1500 mm; reproducibility, better than 0.01 mm) against the object. The object is mounted on a rotational stage with its one end face at lower part of the system. The rotational stage is set with its rotation axis vertical in order to turn the object for reversal measurement. The system can evaluate straightness without affected by zero difference of each 3-point-method detectors through "Zero-adjustment method" [1-2].

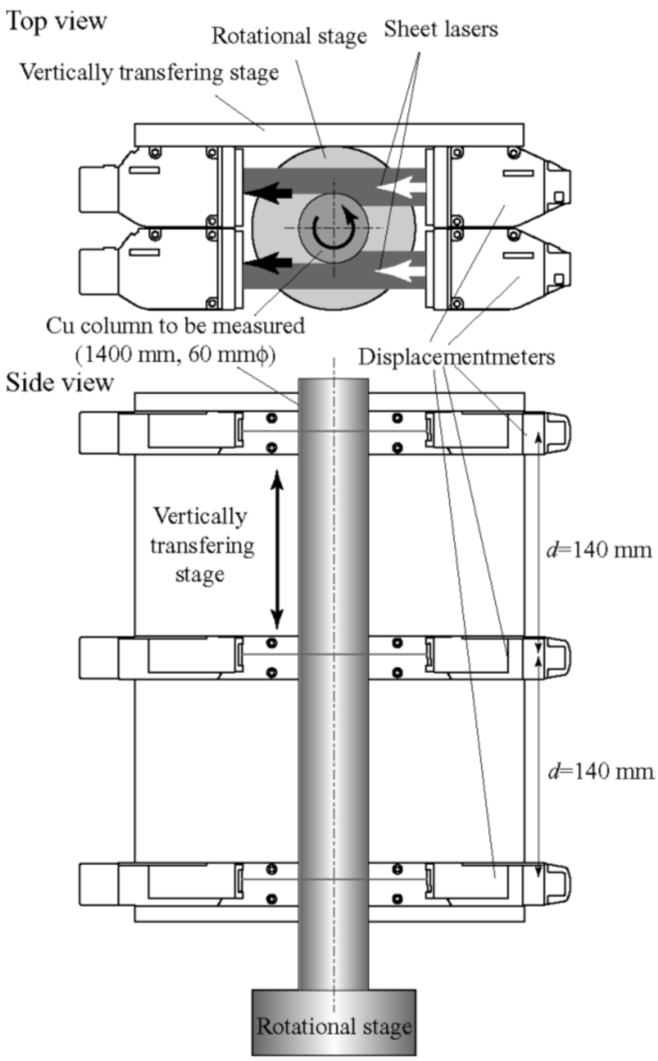


Fig. 2: Straightness measurement system used for evaluating the error estimation.

Figure 3 shows the error σ_h expressed by standard deviations for the straightness against the evaluation distance l. It is derived from the 10-times of repeat measurements under the condition of s = 0.1 mm. It also shows the estimated values by using the relations expressed by Eqs. (13) and (15). They were derived by using experimental values of $\sigma_c = 2\times10^{-8}$ mm^{-1}, σ_s = 2 μm, and $c_r = 1.6\times10^{-7}$ mm^{-1}. The representative value c_r for each c_i is the average of the total measured c_i. The value estimated by using Eq. (13), which considers both σ_c and σ_s, agrees well with the experiment, while that estimated by using Eq. (15), which only considers σ_c, leaves from the experiment as the evaluation distance l becomes long. It shows that the longitudinal positioning error σ_s should be considered especially for evaluating large object by using 3-point method.

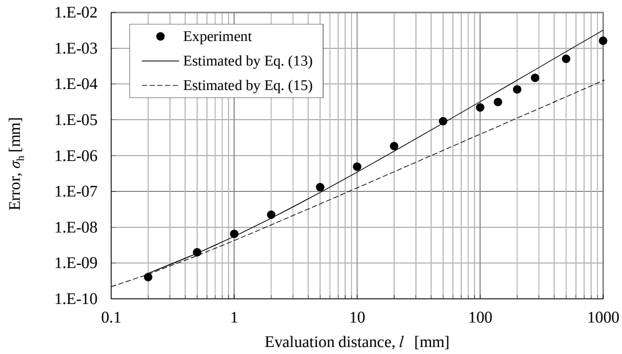


Fig. 3: Errors in the derived straightness as a function of the evaluation distance l for the case s = 0.1 mm.

Figure 4 shows the error σ_h against the sampling interval s. The two estimated values are also obtained by using the relations expressed by Eqs. (13) and (15) for the case l = 0.98 m, $\sigma_c = 2\times10^{-8}$ mm^{-1}, σ_s = 2 μm, and $c_r = 1.6\times10^{-7}$ mm^{-1}, respectively. They show that error σ_c in each measurement is dominant for the case using relatively large sampling interval, while longitudinal positioning error σ_s is dominant for the case using relatively small sampling interval. It follows that there exists optimum sampling interval which minimizes the error σ_h in the derived straightness.

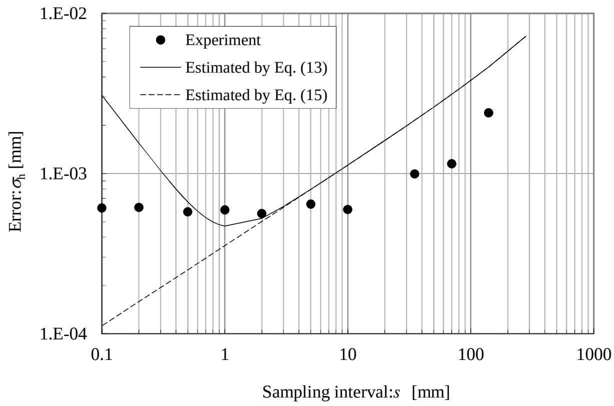


Fig. 4: Errors in the derived straightness as a function of the sampling interval s for the case l = 0.98 m.

Figure 4 also shows the error obtained by experiments. There could be seen minimum error not clearly in the experiment. However, the model which considers σ_s can account for the reason why the error obtained by the experiment does not decrease with the decrease of the sampling interval for the case using relatively small sampling interval. The difference from the estimated values may be caused by decrease of the error σ_s along with decrease of the sampling interval.

4. DISCUSSION

Figures 3 and 4 show that longitudinal positioning error σ_s should be considered for evaluating straightness by using 3-point method especially in cases of relatively large l and/or small s. It comes from the relation expressed by Eq. (13) which considers σ_s. For the cases of large l and/or small s, the second term in the square root for the right hand side of Eq. (13) is dominant for the error σ_h in the derived straightness, and σ_h is proportional to l^2 and $1/s$. While for the cases of small l and/or large s, the first term in the square root is dominant and σ_h is proportional to $l^{3/2}$ and $s^{1/2}$, which is also expressed by Eq. (15).

In the previous error estimation expressed by Eq. (15), which only considers σ_c as an error source, the error propagation σ_h is proportional to $l^{3/2}$ and it can be reduced simply by using small s. However, in the newly proposed model expressed by Eq. (13), which considers σ_s in addition, the error propagation σ_h increases more steeply (proportional to l^2) and there exists limitation for minimizing the error by adjusting s. It follows that the conditions for adopting 3-point method into evaluating straightness of large objects becomes more severe.

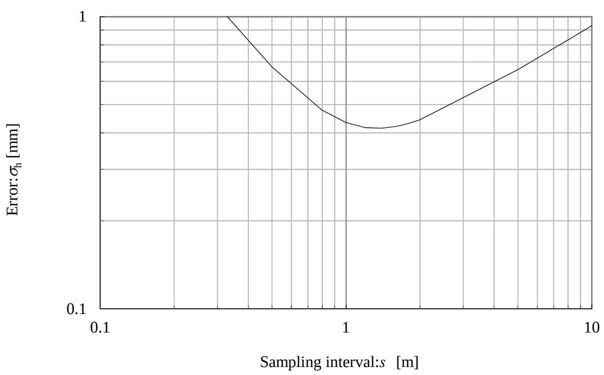


Fig. 5: Errors in the derived straightness as a function of the sampling interval s for the case l = 1 km.

Figure 5 shows error estimated for the l = 1 km of straightness as a function of a sampling interval s. It is derived under the same conditions as for obtaining the relations expressed by Figs. 3 and 4, that is σ_s = 2 μm, c_r = 1.6×10^{-7} mm^{-1}, and $\sigma_c = 1.6\times10^{-11}$ mm^{-1}. The former two are same as those for Figs. 3 and 4. The last one, $\sigma_c = 1.6\times10^{-11}$ mm^{-1} is scaled for d = 5 m from the experimental value of σ_c = 2×10^{-8} mm^{-1} obtained for d = 140 mm, where we consider that σ_c is proportional to $1/d^2$ as expressed by Eq. (10).

Figure 5 shows that error propagated to the derived straightness can be minimized to approximately 0.4 mm by using s = 1.4 m. It shows that 3-point method can be adopted for the straightness evaluation of l = 1 km with accuracy expressed by their tow standard deviation (2σ) of better than 1 mm.

5. CONCLUSION

Errors in the straightness derived by 3-point method were estimated analytically based on newly proposed error propagating model, which considers longitudinal positioning error σ_s of the straightness detector as well as the error σ_c, in each measurements of the detector, and the estimated errors were evaluated by comparing with experiments.

The estimation which considers both σ_c and σ_s, agrees well with experiments, while that only considers σ_c leaves from the experiments as the evaluation distance l becomes large. It follows that σ_s as well as σ_c should be considered for evaluating large object by using 3-point method.

The newly proposed estimation also shows existence of optimum sampling interval s which minimizes error in the derived straightness. There could be seen minimum error not clearly in experiment; however, the model account for the reason why the error does not decrease simply with the decrease of the sampling interval.

Error for the straightness evaluation with the evaluation distance l of 1 km was estimated and its optimum sampling interval was also derived to be 1.4 m. The results showed that 3-point method can be adopted for the straightness evaluation of l = 1 km with accuracy better than 1 mm (2σ).

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